AP Calculus AB
Continuity Worksheet

Name Key

State whether or not each of the following functions is continuous. If not, state where the discontinuity occurs and whether or not it is removable. Is the discontinuity is an asymptote, a hole, or a jump? If it is an asymptote, what is its equation?

1) $f(x)=\dfrac{x}{x^2+1}$

* continuous

2) $f(x)=\dfrac{x}{2x^2-x-1}=\dfrac{x}{(2x+1)(x-1)}$

* not continuous at $x=\frac{-1}{2}$ & $x=1$

VA: $x=\frac{-1}{2}$, $x=1$
(non-removable)

3) $f(x)=\dfrac{2x+3}{x^2-x-6}=\dfrac{2x+3}{(x-3)(x+2)}$

* not continuous at $x=3$ and $x=-2$

VA: $x=3$, $x=-2$
(non-removable)

4) $f(x)=\dfrac{x-4}{x^2-16}=\dfrac{\cancel{x-4}}{(x+4)\cancel{(x-4)}}=\dfrac{1}{x+4}$

* not continuous at $x=-4$ & $x=4$

VA: $x=-4$ (non-removable)

hole: $(4,\frac{1}{8})$ (removable)

5) $f(x)=\begin{cases}\dfrac{x^2-9}{x-3} & \text{if } x\neq 3\\ 8 & \text{if } x=3\end{cases}$

$\dfrac{x^2-9}{x-3}=\dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}=(x+3)$

hole: $(3,6)$
(removable discontinuity)

* not continuous at $x=3$

6) $f(x)=\begin{cases}2x-3 & \text{if } x\le 2\\ x^2 & \text{if } x>2\end{cases}$

$f(2)=2(2)-3$
$f(2)=1$

$\lim_{x\to 2} f(x)=\text{dne}$

$\lim_{x\to 2^-} f(x)=1$ $\quad\lim_{x\to 2^+} f(x)=4$

Since $\lim_{x\to 2} f(x)$ does not exist, $f(x)$ is not continuous at $x=2$
(jump discontinuity)

7) $f(x)=\begin{cases}x^3 & \text{if } x<-1\\ x & \text{if } -1\le x<1\\ 1-x & \text{if } x\ge 1\end{cases}$

$f(-1)=-1$ $\quad\lim_{x\to -1} f(x)=-1$ $\quad\lim_{x\to -1}=f(-1)$

$\lim_{x\to -1^-} f(x)=-1$ $\quad\lim_{x\to -1^+} f(x)=-1$

$\therefore$ continuous at $x=-1$

$f(1)=0$ $\quad\lim_{x\to 1} f(x)=\text{dne}$

$\lim_{x\to 1^-} f(x)=1$ $\quad\lim_{x\to 1^+} f(x)=0$

$\therefore$ not continuous at $x=1$
(jump discontinuity)

8) $f(x)=\dfrac{x}{|x|-3}$

* not continuous at $x=3$ & $x=-3$

VA: $x=3$, $x=-3$
(non-removable)

Find the value of "a" and/or "b" for which the function is continuous.

9)
$$f(x) = \begin{cases} 7x - 2 & \text{if } x \le 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

$7x - 2 = ax^2$

$7(1) - 2 = a(1)^2$

$5 = a$

10)
$$f(x) = \begin{cases} ax^2 & \text{if } x \le 2 \\ 2x + a & \text{if } x > 2 \end{cases}$$

$ax^2 = 2x + a$

$a(2)^2 = 2(2) + a$

$4a = 4 + a$

$3a = 4$

$a = \frac{4}{3}$

11)
$$f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ ax + b & \text{if } 1 \le x < 2 \\ 3x & \text{if } x \ge 2 \end{cases}$$

$x + 1 = ax + b$

$(1) + 1 = a(1) + b$

$2 = a + b$

$\downarrow$

$a = 2 - b$

$a = 2 - (-2)$

$a = 4$

$ax + b = 3x$

$a(2) + b = 3(2)$

$2a + b = 6$

$2(2 - b) + b = 6$

$4 - 2b + b = 6$

$4 - b = 6$

$-b = 2$

$b = -2$

Are the following functions continuous at all points in the natural domain? If the function is not continuous, what kind of discontinuity exists? Rewrite the function to make it continuous.

12) $f(x) = \frac{x^2 - 16}{x + 4} = \frac{(\cancel{x+4})(x-4)}{(\cancel{x+4})} = x - 4$

hole: $(-4, -8)$
(point discontinuity)

$$g(x) = \begin{cases} \frac{x^2-16}{x+4} & x \neq -4 \\ -8 & x = -4 \end{cases}$$

13) $f(x) = \frac{2x^2 - x - 1}{x - 1} = \frac{(2x+1)(\cancel{x-1})}{(\cancel{x-1})} = 2x+1$

hole: $(1, 3)$
(point discontinuity)

$$h(x) = \begin{cases} \frac{2x^2-x-1}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$$

4) $f(x) = \frac{9x^2 - 4}{3x + 2} = \frac{(\cancel{3x+2})(3x-2)}{(\cancel{3x+2})} = 3x - 2$

hole: $(-\frac{2}{3}, -4)$
(point discontinuity)

$$k(x) = \begin{cases} \frac{9x^2-4}{3x+2} & x \neq -\frac{2}{3} \\ -4 & x = -\frac{2}{3} \end{cases}$$

15) $g(t) = \frac{\sin t}{t}$

not continuous at $t = 0$

$$\lim_{t \to 0} \frac{\sin t}{t} = 1$$

$$j(t) = \begin{cases} \frac{\sin t}{t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$