Precalculus/Trig 3 Name Key
Continuity Notes

A function is continuous at a point if $\lim_{x\to c} f(x) = f(c)$. This means that there are three conditions that must be met for a function to be continuous at a point.

1. $f(c)$ exists
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

A polynomial is continuous at every point on its domain.

A rational function is continuous at every point in the domain except for when the denominator equals zero.

1. $f(x) = x^2 - 4$

continuous

2. $f(x) = \dfrac{x^2 - 4}{x - 2} = \dfrac{(x+2)(x-2)}{x-2}$

$x \neq 2$

hole: $(2, 4)$

3. $f(x) = \dfrac{x+1}{x^2 - 1} = \dfrac{(x+1)}{(x+1)(x-1)}$

$x \neq 1, -1$

hole: $(-1, -\frac{1}{2})$

VA: $x = 1$

A piecewise function may not be continuous where the function changes.

4.

$$f(x) = \begin{cases} -3x + 4 & \text{if} \quad x \le 2 \\ -2 & \text{if} \quad x > 2 \end{cases}$$

$x = 2$:

1. $f(2) = -2$

2. $\lim_{x\to 2} f(x) = -2$

$\lim_{x\to 2^-} f(x) = -2$ $\quad$ $\lim_{x\to 2^+} f(x) = -2$

3. $f(2) = \lim_{x\to 2} f(x)$

$\therefore f(x)$ is continuous at $x = 2$

5.

$$f(x) = \begin{cases} x^2 & \text{if} \quad x < 0 \\ -x & \text{if} \quad 0 \le x \le 1 \\ x & \text{if} \quad x > 1 \end{cases}$$

$x = 0$:

1. $f(0) = 0$

2. $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 0^-} f(x) = 0$ $\quad$ $\lim_{x\to 0^+} f(x) = 0$

3. $f(0) = \lim_{x\to 0} f(x)$

$\therefore f(x)$ is continuous at $x = 0$

$x = 1$:

1. $f(1) = -1$

2. $\lim_{x\to 1} f(x) =$ dne

$\lim_{x\to 1^-} f(x) = -1$ $\quad$ $\lim_{x\to 1^+} f(x) = 1$

$\therefore f(x)$ is not continuous at $x = 1 \rightarrow$ jump discontinuity

Removable and Non-Removable Discontinuities

6. $f(x) = \dfrac{x^2 + 3x - 4}{x - 1} = \dfrac{(x+4)\cancel{(x-1)}}{\cancel{(x-1)}} = (x+4)$

$x \neq 1$

hole: $(1,5)$

↳ removable discontinuity
(point discontinuity)

7. $f(x) = \dfrac{x^3 - 8}{x - 2} = \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}} = x^2+2x+4$

$x \neq 2$

hole: $(2,12)$

8. $f(x) = \dfrac{x^2 - 16}{x - 3} = \dfrac{(x+4)(x-4)}{x-3}$

$x \neq 3$

↳ non-removable discontinuity
(infinite discontinuity)

VA: $x=3$

Find the value of "a" for which the function is continuous.

9.

$$f(x) = \begin{cases} 2x + a & \text{if } \ x < 4 \\ x^2 - a & \text{if } \ x \geq 4 \end{cases}$$

$$2x + a = x^2 - a$$
$$2(4) + a = (4)^2 - a$$
$$8 + a = 16 - a$$
$$2a = 8$$
$$a = 4$$

10.

$$f(x) = \begin{cases} ax + 1 & \text{if } \ x \leq 3 \\ ax^2 - 1 & \text{if } \ x > 3 \end{cases}$$

$$ax + 1 = ax^2 - 1$$
$$a(3) + 1 = a(3)^2 - 1$$
$$3a + 1 = 9a - 1$$
$$2 = 6a$$
$$a = \frac{1}{3}$$