Chain Rule

1. $f(x) = (2x^2 - 5x)^3$

   $f'(x) = 3(2x^2 - 5x)^2(4x - 5)$

2. $f(x) = (5x^3 - 2x)^{1/2}$

   $f'(x) = \frac{1}{2}(5x^3 - 2x)^{-1/2}(15x - 2)$

3. $y = 3\sin(x-3)$

   $y' = 3\cos(x-3)$

4. $y = -2\cos(x^2+2)$

   $y' = 2\sin(x^2+2)(2x)$

   $y' = 4x\sin(x^2+2)$

5. $g(x) = (\sin(3x^2))^2$

   $g'(x) = 2(\sin 3x^2)(\cos 3x^2)(6x)$

   $g'(x) = 12x\sin(3x^2)\cos(3x^2)$

6. $h(x) = (\sec(x^2-5))^3$

   $h'(x) = 3(\sec(x^2-5))^2(\sec(x^2-5)\tan(x^2-5))(2x)$

   $h'(x) = 6x\sec^3(x^2-5)\tan(x^2-5)$

7. $f(x) = 3x^3 e^{2x-5}$

   $f'(x) = (27x^2)e^{2x-5} + (3x^3)(e^{2x-5})(2)$

   $f'(x) = 27x^2 e^{2x-5} + 6x^3 e^{2x-5}$

   $f'(x) = 3x^2 e^{2x-5}(9 + 2x)$

8. $g(x) = -5x^2 e^{x^2+3x}$

   $g'(x) = (-10x)e^{x^2+3x} + (-5x^2)(e^{x^2+3x})(2x+3)$

   $g'(x) = -5xe^{x^2+3x}(2 + x(2x+3))$

   $g'(x) = -5xe^{x^2+3x}(2x^2+3x+2)$

9. $y = 3x^2(4x^2-5x+1)^{1/2}$

$y' = (6x)(4x^2-5x+1)^{1/2} + (3x^2)\frac{1}{2}(4x^2-5x+1)^{-1/2}(8x-5)$

$y' = 6x(4x^2-5x+1)^{1/2} + \frac{3}{2}x^2(8x-5)(4x^2-5x+1)^{-1/2}$

$y' = \frac{3}{2}x(4x^2-5x+1)^{-1/2}\left[4(4x^2-5x+1) + x(8x-5)\right]$

10. $h(t) = \frac{2}{3}t^3(3t^3-5t)^{1/2}$

$h'(t) = 2t^2(3t^3-5t)^{1/2} + \frac{2}{3}t^3 \cdot \frac{1}{2}(3t^3-5t)^{-1/2}(27t^2-5)$

$h'(t) = 2t^2(3t^3-5t)^{1/2} + \frac{1}{3}t^3(27t^2-5)(3t^3-5t)^{-1/2}$

$h'(t) = \frac{1}{3}t^2(3t^3-5t)^{-1/2}\left[6(3t^3-5t) + t(27t^2-5)\right]$

11. $y = (x^3-4x^2+1)^{-1/3}$

$y' = -\frac{1}{3}(x^3-4x^2+1)^{-4/3}(3x^2-8x)$

$$y' = \frac{-(3x^2-8x)}{3(x^3-4x^2+1)^{4/3}}$$

12. $g(t) = -3(2t^3+5t-3)^{-1/4}$

$g'(t) = \frac{3}{4}(2t^3+5t-3)^{-5/4}(6t^2+5)$

$$g'(t) = \frac{3(6t^2+5)}{4(2t^3+5t-3)^{5/4}}$$

13. $g(m) = \sin(\cos(m))$

$g'(m) = \cos(\cos(m)) \cdot -\sin(m)$

$g'(m) = -\cos(\cos(m)) \cdot \sin(m)$

14. $f(x) = \cos(\tan(x))$

$f'(x) = -\sin(\tan(x)) \cdot \sec^2(x)$

15. $h(x) = (x^3+2)^{1/2}(x^2-1)^4$

$h'(x) = \frac{1}{2}(x^3+2)^{-1/2}(3x^2)(x^2-1)^4 + (x^3+2)^{1/2} \cdot 4(x^2-1)^3(2x)$

$h'(x) = \frac{3}{2}x^2(x^3+2)^{-1/2}(x^2-1)^4 + 8x(x^3+2)^{1/2}(x^2-1)^3$

$h'(x) = x(x^3+2)^{-1/2}(x^2-1)^3\left[\frac{3}{2}x(x^2-1) + 8(x^3+2)^{1/2}\right]$

16. $h(m) = (m^2+1)^{1/2}(m^2+1)^3$

$$h'(m) = \tfrac{1}{2}(m^2+1)^{-1/2}(2m)(m^2+1)^3 + (m^2+1)^{1/2}\,3(m^2+1)^2(2m)$$

$$h'(m) = m(m^2+1)^{-1/2}(m^2+1)^3 + 6m(m^2+1)^{1/2}(m^2+1)^2$$

$$h'(m) = m(m^2+1)^{-1/2}(m^2+1)^2\left[(m^2+1) + 6(m^2+1)\right]$$

OR

$$h(m) = (m^2+1)^{7/2}$$

$$h'(m) = \tfrac{7}{2}(m^2+1)^{5/2}(2m)$$

$$h'(m) = 7m(m^2+1)^{5/2}$$

17. $f(t) = \dfrac{(t^2+2)^{1/3}}{(t^2-2)^{1/3}}$

$$f'(t) = \frac{\frac{1}{3}(t^2+2)^{-1/3}(2t)(t^2-2)^{1/3} - (t^2+2)^{1/3}\cdot\frac{1}{3}(t^2-2)^{-1/3}(2t)}{(t^2-2)^{2/3}}$$

$$f'(t) = \frac{\frac{2}{3}t(t^2+2)^{-1/3}(t^2-2)^{1/3} - \frac{2}{3}t(t^2+2)^{1/3}(t^2-2)^{-1/3}}{(t^2-2)^{2/3}}$$

$$f'(t) = \frac{\frac{2}{3}t(t^2+2)^{-1/3}(t^2-2)^{-1/3}\left[(\cancel{t^2}-2) + (\cancel{-t^2}-2)\right]}{(t^2-2)^{2/3}}$$

$$f'(t) = \frac{-4}{3(t^2+2)^{1/3}(t^2-2)}$$

18. $f(t) = \dfrac{(t^3+8)^{1/4}}{(t^3-8)^{1/4}}$

$$f'(t) = \frac{\frac{1}{4}(t^3+8)^{-3/4}(3t^2)(t^3-8)^{1/4} - (t^3+8)^{1/4}\cdot\frac{1}{4}(t^3-8)^{-5/4}(3t^2)}{(t^3-8)^{1/2}}$$

19. $h(x) = (2x+5)^7(3x^4-8)^5$

$h'(x) = 7(2x+5)^6(2)(3x^4-8)^5 + (2x+5)^7 \cdot 5(3x^4-8)^4(12x^3)$

$h'(x) = 14(2x+5)^6(3x^4-8)^5 + 60x^3(2x+5)^7(3x^4-8)^4$

$h'(x) = 2(2x+5)^6(3x^4-8)^4\left[7(3x^4-8) + 30x^3(2x+5)\right]$

20. $g(n) = (3x^2-2)(4x^3+1)$

$g(n) = 12x^5 + 3x^2 - 8x^3 - 2$

$g'(n) = 60x^4 - 24x^2 + 6x$

21. $f(t) = \csc^2(t^3)$

$f(t) = (\csc(t^3))^2$

$f'(t) = 2(\csc(t^3))(-\csc(t^3)\cot(t^3))(3t^2)$

$f'(t) = -6t^2\csc^2(t^3)\cot(t^3)$

22. $f(t) = (\cot(2t^2))^4$

$f'(t) = 4(\cot(2t^2))^3(-\csc^2(2t^2))(4t)$

$f'(t) = -16t\cot^3(2t^2)\csc^2(2t^2)$

23. $h(x) = e^{(2x^3-x^2)^{1/2}}$

$h'(x) = e^{(2x^3-x^2)^{1/2}} \cdot \frac{1}{2}(2x^3-x^2)^{-1/2}(6x^2-2x)$

24. $f(x) = e^{(4x^2-3x)^{1/2}}$

$f'(x) = e^{(4x^2-3x)^{1/2}} \cdot \frac{1}{2}(4x^2-3x)^{-1/2}(8x-3)$

25. $h(x) = \dfrac{3x}{(5+2x^2)^{1/2}}$