AP Free Response 2018

(1)

(a) When there are 0 customers in line in front of the selected person, the time to check out is estimated to be 72.95 seconds, on average

(b) 73.33% of the change (or variability) in the time to checkout is due to (or explained by) the change in the number of customers in line in front of the selected person.

(c) the data point at approximately (3,0) is the outlier. It is an outlier because it's y-value (the time to check out) is significantly lower than the predicted time would be for that number of customers.

Grading:

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 72.95 as the intercept.
2. Communicates the concept of a y-intercept in a context that includes both time and zero customers.
3. Indicates that the value of the intercept is a prediction by using language such as "predicted," "estimated," or "average" value of y.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 73.33% as the coefficient of determination.
2. Provides a correct (possibly generic) interpretation of r^2.
3. Interpretation includes context.

Partially correct (P) if the response satisfies only two of the three components;

OR

if the response satisfies the three components, but reverses the roles of number of customers in line and time to finish checkout in the interpretation.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Correctly identifies the outlier.
2. Describes an unusual feature of the identified scatter plot point, relative to the remaining data points, that is sufficient to identify it as the outlier. Examples include:
   - The combination of x and y values is unusual compared to the other points.
   - The value of y is much lower than would be expected (or predicted), given the remaining data.
   - The residual for the point is unusually large relative to the other residuals.

Partially correct (P) if the response satisfies component 1 but does not satisfy component 2.

(2)

(a)

find: $p = 0.584 + 0.816 \div 2 = 0.70$

$m = 0.816 - 0.70 = 0.116$

$$1.96\sqrt{\frac{0.7(1-0.7)}{n}} = 0.116$$

n = 60 students

Grading:

E: 1) Shows correct formula (numbers filled in)
2) Shows that p = 0.7
3) shows that m = 0.116
4) shows that Z* = 1.96
5) Answer is a single, positive, whole number

P: missing 1 or 2 of the things above

I: missing 3 or more of the things above

(B)

Response bias. Students were asked to go to the environmental science teacher's office and asked about recycling. The students would be able to infer that an enviro science teacher would care about recycling, so they might say YES when they really don't recycle.

This would result in a sample proportion that would be higher than what is true.

GRADING:

E: 1) student explains WHY the survey might differ from the truth (in context).

Ex: response bias

2) student explains HOW the responses differ from the truth (in context).

Ex: says that people say YES when they don't

3) Describes the effect of the bias on the $\hat{p}$

P: missing 1 of the parts above

(C) (i) 300 * 0.50 = 150

(ii) 213/300 said no. Since we expected 150 to say no due to the coin flip, then the number that ACTUALLY said no (truthfully) was 213 - 150 = 63.

Thus, out of the 150 kids who answered truthfully, then only (150 - 63) = 87 said yes truthfully.

87/150 = 0.58 = 58%

GRADING:

E: student says "150" in part (i) and 0.58 in part (ii)

P: student says "150" in part (i) and 0.42 in part (ii)
OR
student does not say "150" in part (i) and does say 0.58 in part (ii)

Note: work needs to be shown in part (ii).

SCORING:

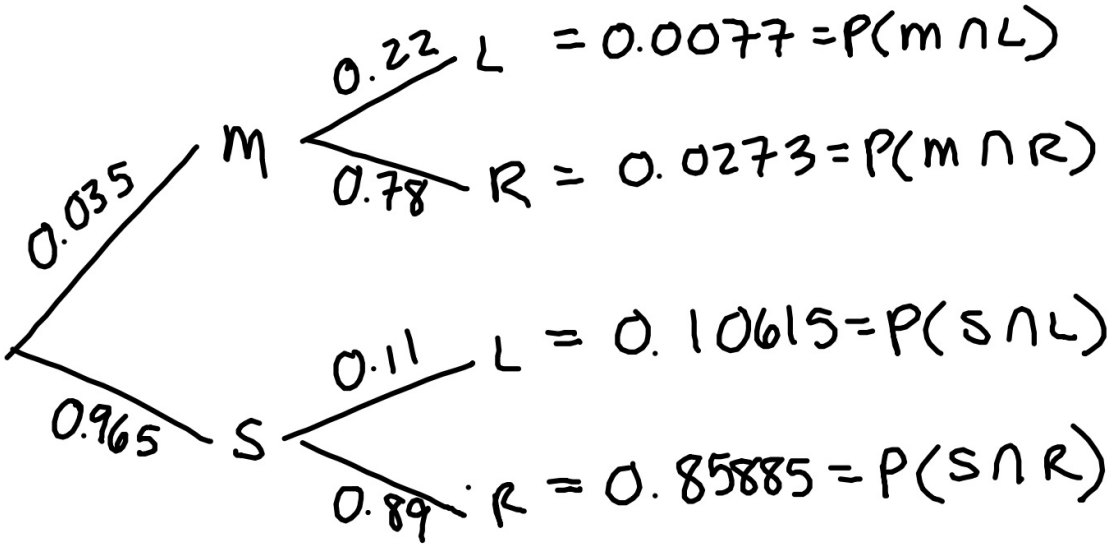
4: EEE

3: EEP

2: EEI EPP EPI PPP

1: EII PPI PII

(3)



$P(M) = 0.035$

$P(L|M) = 0.22$

$P(L|S) = 0.11$

(a) $P(L) = 0.0077 + 0.10615 = 0.11385$

(b) $P(M|L) = \frac{0.0077}{0.11385} = 0.0676$

(c) $P(L) = 0.11385$

$n = 20$

$B(20, 0.11385)$

$P(X \geq 3) = 0.4021$

1-binomcdf(20, 0.11385, 2)

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate strategy.

Partially correct (P) if the response provides a reasonable strategy for finding the probability, such as a formula or tree diagram, but uses one or more inappropriate values;
OR
if the response gives the correct probability but not enough work is shown to determine how that probability was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: A reasonable strategy needs to include summing the results of two multiplications.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes appropriate numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one appropriate term (numerator or denominator) and the other inappropriate.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Appropriate values include incorrectly calculated values from part (a).

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a calculation based on the binomial distribution to find the probability of the number of children in the sample who are left-handed.
2. Specifies appropriate values for n and p.
3. Uses correct endpoint value for the probability.
4. Uses correct direction to calculate the probability of at least three left-handed children.
5. Correctly calculates a binomial probability consistent with the previous work.

Partially correct (P) if the response satisfies component 1 and only two or three of the other four components;
OR
if components 2, 3, 4, and 5 are met, and the response does not explicitly indicate the binomial distribution is used by name or formula.

(4)

Intent of Question

The primary goals of this question were to assess a student's ability to (1) determine whether a cause-and-effect conclusion can be made based on how a study was conducted and (2) set up, perform, and interpret the results of a hypothesis test, in the context of the problem.

(a)

Yes, it would be reasonable to conclude that the new procedure causes a reduction in recovery time, for patients similar to those in the study. The patients in the study were randomly assigned to the two procedures, which reduces the chance that confounding variables will affect the results. Therefore the statistically significant reduction in mean recovery time can be attributed to the new procedure being superior to the standard procedure.

(b)

Hypotheses: $H_0 : \mu_S = \mu_N$

$H_a : \mu_S > \mu_N.$

Conditions:

1) Random: stated random samples in problem

2) Normality: $n_s = 110 > 30$ and $n_N = 100 > 30$

Conditions met, t distribution, 2 sample t test

Mechanics: $$t = \frac{\bar{x}_S - \bar{x}_N}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_N^2}{n_N}}} = \frac{217 - 186}{\sqrt{\frac{34^2}{110} + \frac{29^2}{100}}} \approx 7.13.$$

$P(t > 7.13) = 8.36 \times 10^{-12}$ df = 207.18

Conclusion:

We reject Ho b/c p-value of is < alpha = 0.05.

We have sufficient evidence that those who receive the new procedure will have less recovery time, on average, than those who receive the standard procedure.

(a)

Essentially correct (E) if the response satisfies the following three components:

1. Correctly states that it is reasonable to make a causal conclusion.
2. Justifies the causal conclusion based on random assignment of patients to procedures (or procedures to patients);
   OR
   justifies the causal conclusion by stating that a randomized experiment was conducted.
3. Includes the context of the situation.

Partially correct (P) if the response satisfies component 1 *AND* provides WEAK justification of the causal conclusion by stating that there was random assignment or a randomized experiment was conducted, but with no context;
OR
by stating that an experiment was conducted or there was assignment (without the word "randomized") *AND* the response includes context of the situation;
OR
by stating that the study design reduces the chance of confounding variables or balances the effects of uncontrolled variables across both groups in context without explicitly referring to the random assignment.

(b) hypotheses, conditions, test statistic

Essentially correct (E) if the response satisfies the following four components:

1. Parameters are defined correctly.
2. Hypotheses imply equality in the null and correct direction in the alternative.
3. Correct test is identified by name or formula.
4. Correct test statistic for a difference in means is calculated.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response satisfies at most one of the four components.

Notes:

- If standard symbols are used for the parameters with appropriate group labels (such as, μ_S, μ_N), component 1 is satisfied.
- If the correct test is identified, but the response states an incorrect formula or uses incorrect notation in the formula, component 3 is not satisfied.
- A pooled two-sample t-test is acceptable for component 3, but the student must also state and comment on the plausibility of the equal population variances assumption.
- If the response identifies a z-test for equal means as the correct test identification, component 3 is not satisfied but component 4 could be satisfied.

(b) p-value and conclusion

Essentially correct (E) if the response satisfies the following three components:

1. Makes reference to an approximately correct *p*-value that is consistent with the test statistic and alternative hypothesis for a difference in means.
2. Correctly justifies the conclusion based on the size of the *p*-value or the test statistic.
3. Correctly states the conclusion in context.

Partially correct (P) if the response satisfies only two of the three components.

(5)

Intent of Question

The primary goals of this question were to assess a student's ability to (1) determine which of two histograms represents data with a larger median; (2) calculate the mean of a combined data set when the separate means and sample sizes are known; and (3) calculate the probability that an individual randomly chosen from a finite population will have a value within one standard deviation of the mean, when provided with values for the mean, standard deviation, and all members of the population.

(a) method 1 = counting to the middle value

The median teaching year for High School A is any value with 100 data values at or below it and 100 data values at or above it. The median teaching year for High School B is the 111th value in the ordered list of values. For High School A the median is in the interval that starts at 7 and ends just before 10, because there are only 94 data values below 7 and 106 data values of at least 7. Therefore the median cannot be less than 7. For High School B the median is in the interval that starts at 4 and ends just before 7 because there are more than half (113) of the data values less than 7. Therefore the median must be less than 7. So High School A must be the one with a median of 7, and High School B must be the one with a median of 6.

method 2: shapes

Another way to determine which school has the median of 7 is to notice that the distribution for High School B is highly skewed to the right, whereas the distribution for High School A is bimodal with a few possible outliers on the right. A distribution that is highly right-skewed is likely to have a substantially larger mean than median. The mean of both distributions is given as 8.2 years, so it makes sense that the highly right-skewed distribution (High School B) is the one with the bigger gap between the mean and median and, therefore, the one with the lower median of 6.

(b)

$$\frac{(200)(8.2) + (18)(2.5)}{200 + 18} = \frac{1,640 + 45}{218} \approx 7.73 \text{ years.}$$

(c) $8.2 \pm 7.2 = (1, 15.4) \Rightarrow (1, 15)$

add up amount in each bar from 1-15:

$79 + 34 + 28 + 29 + 19 = 189$

$189/221 = 0.8552 = 85.52\%$

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. Provides a reasonable explanation of how the decision was made.
3. Provides the definition of the median or explicitly applies the definition of a median as a criterion in reaching their decision.

OR

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. States that High School B shows a skewed distribution (or High School A shows a less skewed distribution).
3. Provides a reasonable explanation of how the more skewed distribution (High School B) would be the one with a larger separation between the mean and median.

Partially correct (P) if the response satisfies the first component and only one of the other two components required for E.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. The correct answer that the mean is 7.73.
2. Enough work to show that the answer was obtained as a weighted average of the two individual means.

Partially correct (P) if the response satisfies only one of the two components.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Calculates that the appropriate interval is 1 to 15.4 or 1 to 15 teaching years.
2. Correctly sums the counts of data values in the numerator based on the intervals provided.
3. Computes the probability using 221 as the denominator.

Partially correct (P) if the response satisfies only two of the three components;
OR
if the response reports the correct probability (0.8552) without supporting work.

(6)

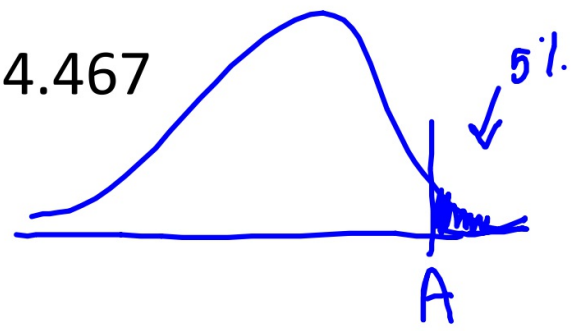
(a) Concluding that the average systolic blood pressure of the employees at the large corporation is not greater than the reported national mean (greater than 122), when really it is higher than 122.

(b) mean = 122 std dev = 1.5

Using alpha = 0.05, we would reject Ho if we find a value that is in the top 5% of the curve.

invNorm(0.95, 122, 1.5) = 124.467

P(X > A) = 0.05 A = 124.467



(c) mean = 125 std dev = 15/sqrt(100) = 1.5

P(X > 124.467) = 0.6388
normalcdf(124.467, E99, 125, 1.5)

(d) POWER!

(e) It would be greater. Increasing sample size increases the power of a test.