KEY

Precalculus Spiral

Final exam preparation questions

Algebra

Graphing

Exponential

Trigonometry

Stat/probability

Functions: find f(2) and f(-3)

- 1.) $f(x) = -3x^3 + 4x^2 + 10x - 7$ $f(2) = 5$ $f(-3) = 80$
- 2.) $g(x) = 2x^4 - 5x^3 - 6x^2 + 7$ $f(2) = -25$ $f(-3) = 250$
- 3.) $h(x) = 6x^3 - 3x^2 - 8x - 2$ $f(2) = 18$ $f(-3) = -167$

Composition: find f(g(x))

- 1.) $f(x)=3x+2.....g(x)=5x-2$ $f(g(x))=15x-4$
- 2.) $f(x)=x^2-5.....g(x)=3x-4$ $f(g(x))=9x^2-24x+11$
- 3.) $f(x)=x^2-4x+2.....g(x)=x+2$ $f(g(x))=x^2-2$

Operations: Subtract and multiply

- 1.) $f(x) = 2x - 4 \ldots\ldots g(x) = 3x - 5$
- 2.) $f(x) = x^2 + 4 \ldots. g(x) = 3x - 1$
- 3.) $f(x) = 5x + 2 \ldots\ldots g(x) = x^2 - 3$

① $(f-g)(x) = -x+1$ $\quad (f \cdot g)(x) = 6x^2 - 22x + 20$

② $(f-g)(x) = x^2 - 3x + 5$ $\quad (f \cdot g)(x) = 3x^3 - x^2 + 12x - 4$

③ $(f-g)(x) = -x^2 + 5x + 5$ $\quad (f \cdot g)(x) = 5x^3 + 2x^2 - 15x - 6$

Y = mx + b

- 1.) (-3, 2), (-4, 1) $y = x + 5$
- 2.) (5, -2), (2, 4) $y = -2x$
- 3.) (-3, 4), (2, 4) $y = 4$

Inverse: find f^{-1}

- 1.) $f(x) = -3x + 2$
- 2.) $g(x) = x^3 - 5$
- 3.) $h(x) = \dfrac{2x+4}{5}$

① $y = -3x + 2$

$x = -3y + 2$

$x - 2 = -3y$

$y = \dfrac{x-2}{-3}$

$f^{-1}(x) = \dfrac{x-2}{-3}$

② $y = x^3 - 5$

$x = y^3 - 5$

$x + 5 = y^3$

$y = \sqrt[3]{x+5}$

$f^{-1}(x) = \sqrt[3]{x+5}$

③ $y = \dfrac{2x+4}{5}$

$x = \dfrac{2y+4}{5}$

$5x = 2y + 4$

$5x - 4 = 2y$

$\dfrac{5x-4}{2} = y$

$f^{-1}(x) = \dfrac{5x-4}{2}$

Synthetic Division: divide

- 1.) $(3x^3 - 7x^2 + 4x - 2) \div (x - 2)$
- 2.) $(2x^4 - 3x^2 + 3x - 5) \div (x + 2)$
- 3.) $(3x^3 - 4x^2 - 5x + 2) \div (x - 4)$

(1.)

2	3	-7	4	-2
	↓	6	-2	4
	3	-1	2	2

$3x^2 - x + 2 + \frac{2}{x-2}$

(2.)

-2	2	0	-3	3	-5
	↓	-4	8	-10	14
	2	-4	5	-7	9

$2x^3 - 4x^2 + 5x - 7 + \frac{9}{x+2}$

(3.)

4	3	-4	-5	2
	↓	12	32	108
	3	8	27	110

$3x^2 + 8x + 27 + \frac{110}{x-4}$

Parent graphs

- Draw the parent graphs for: $x^2, x^3, x^4, \sqrt{x}, |x|, \sqrt[3]{x}$

x^2 x^3 x^4 $\sqrt{x}$ $|x|$ $\sqrt[3]{x}$

- Find Domain and Range for each graph

x^2 D: $(-\infty, \infty)$ R: $[0, \infty)$

x^4 D: $(-\infty, \infty)$ R: $[0, \infty)$

$|x|$ D: $(-\infty, \infty)$ R: $[0, \infty)$

x^3 D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

$\sqrt{x}$ D: $[0, \infty)$ R: $[0, \infty)$

$\sqrt[3]{x}$ D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

Transformations- describe them

- 1.) $y = \sqrt{x-4}+3$ RIGHT 4, UP 3
- 2.) $y = -x^2+5$ Reflect x-axis, up 5
- 3.) $y = |x+4|-7$ Left 4, Down 7

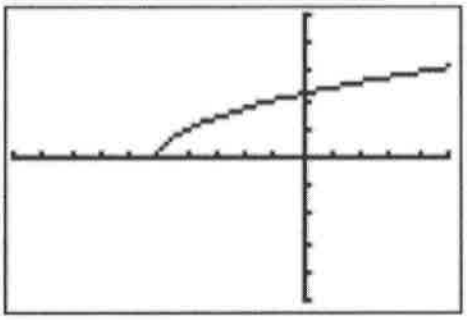
Transformations- write equation

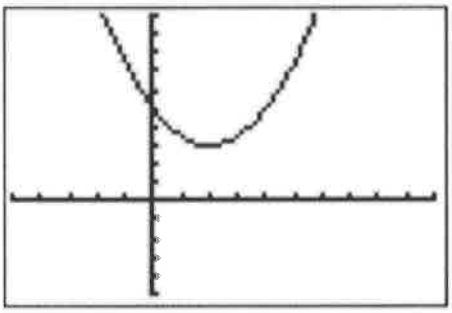
- 1.) x^2, up 2, left 5, reflect over **x-axis** $y = -(x+5)^2 + 2$
- 2.) absolute value, wider, right 2 $y = \frac{1}{2}|x-2|$
- 3.) square root, thinner, down 3

$y = 2\sqrt{x} - 3$

Transformations: find them from graph, then find the domain

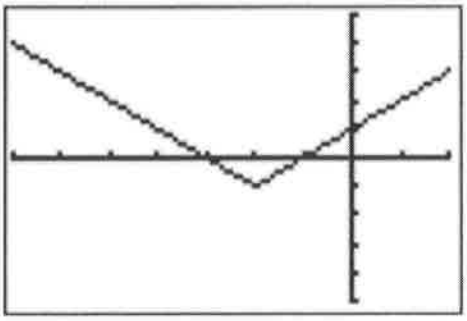
Left 5 D: $[-5, \infty)$





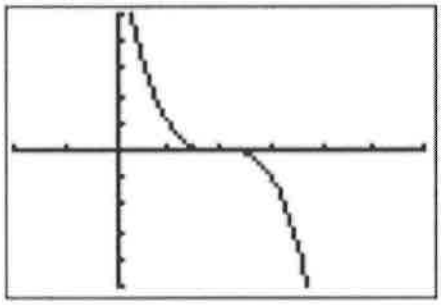
RIGHT 2
UP 3

D: $(-\infty, \infty)$



Left 2
Down 1

D: $(-\infty, \infty)$

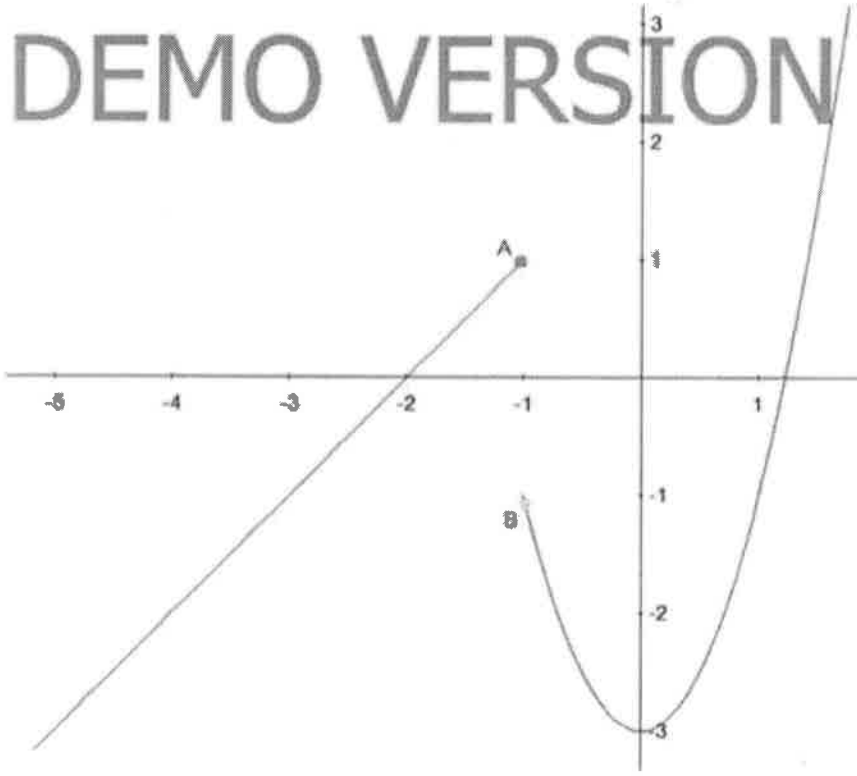


Reflect x-axis
Right 2

D: $(-\infty, \infty)$

Piecewise- find equation

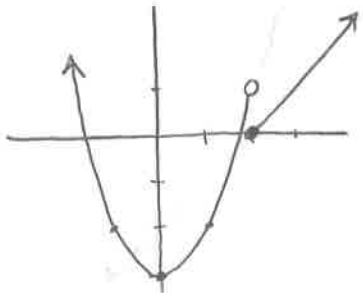
DEMO VERSION



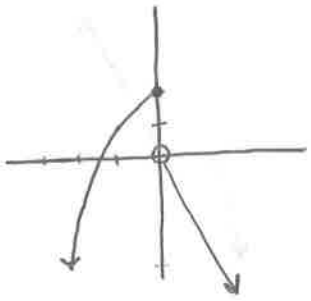
$$y = \begin{cases} x+2, & x \leq -1 \\ x^2-3, & x > -1 \end{cases}$$

Piecewise: Graph

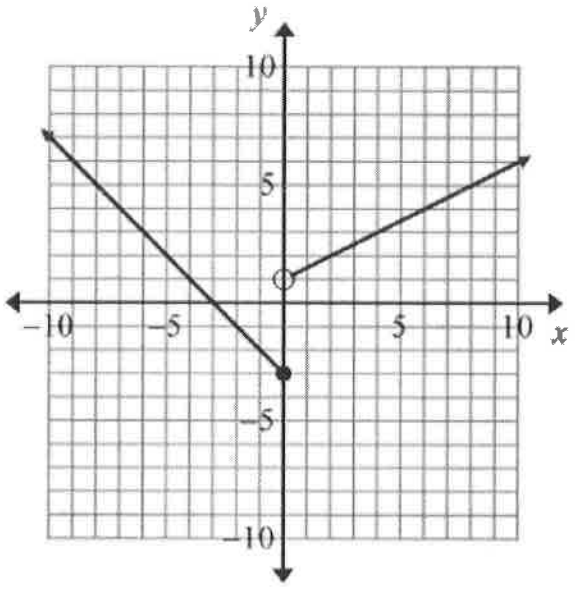
- 1.) $f(x) = \begin{matrix} x-2, x \geq 2 \\ x^2-3, x < 2 \end{matrix}$



- 2.) $g(x) = \begin{matrix} -3x, x > 0 \\ -x^2+2, x \leq 0 \end{matrix}$



Which function is represented by the graph?



A. $f(x)=\begin{cases}-x-3, & \text{if } x\le 0\\ \frac{1}{2}x+1, & \text{if } x>0\end{cases}$

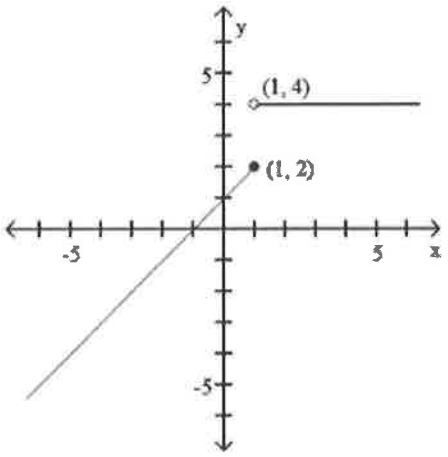
B. $f(x)=\begin{cases}x-3, & \text{if } x\le 0\\ -\frac{1}{2}x+1, & \text{if } x>0\end{cases}$

C. $f(x)=\begin{cases}-x+3, & \text{if } x\le 0\\ \frac{1}{2}x+1, & \text{if } x>0\end{cases}$

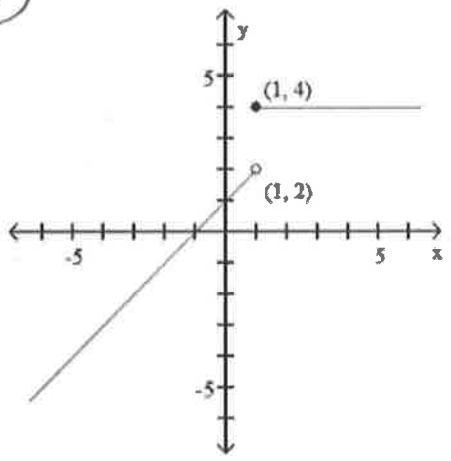
D. $f(x)=\begin{cases}x+3, & \text{if } x\le 0\\ -\frac{1}{2}x+1, & \text{if } x>0\end{cases}$

$$f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ 4 & \text{if } x \geq 1 \end{cases}$$

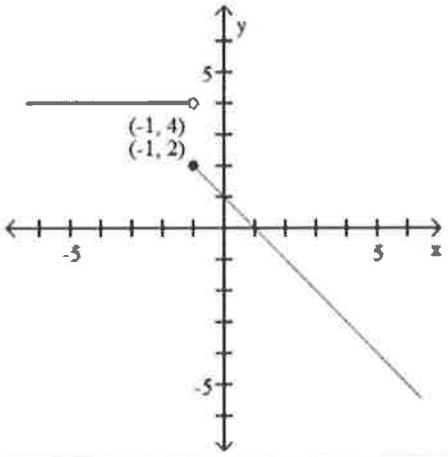
A)



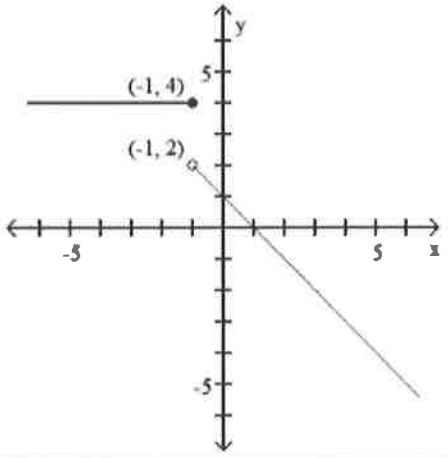
B)



C)

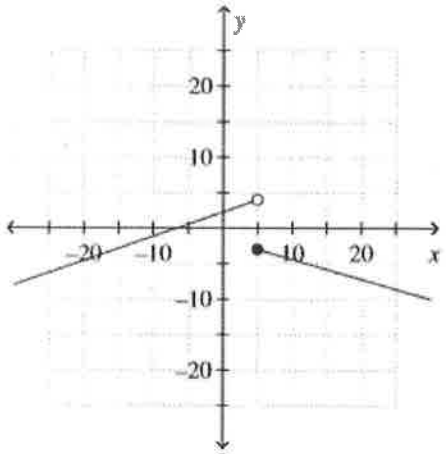


D)

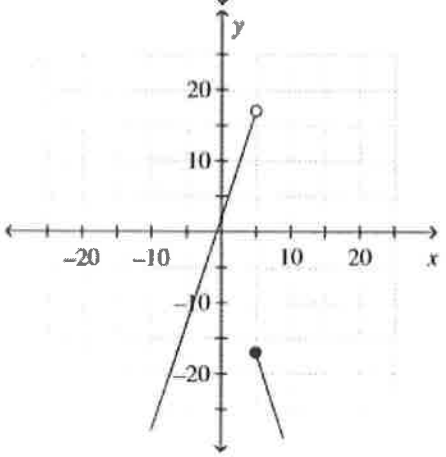


$$f(x) = \begin{cases} 3x+2 \text{ if } x < 5 \\ -3x-2 \text{ if } x \geq 5 \end{cases}$$

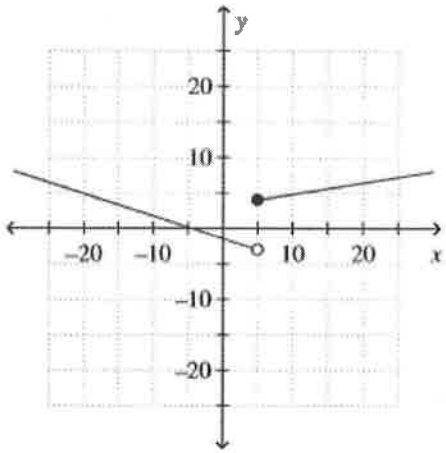
a.



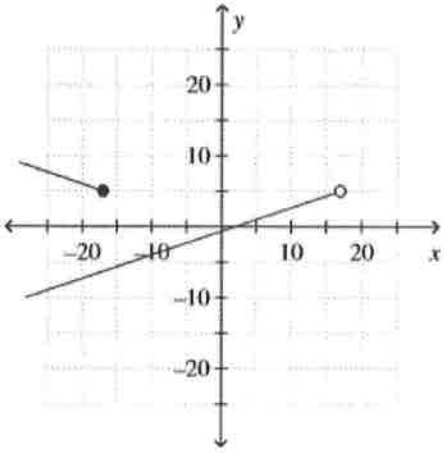
b.



c.



d.



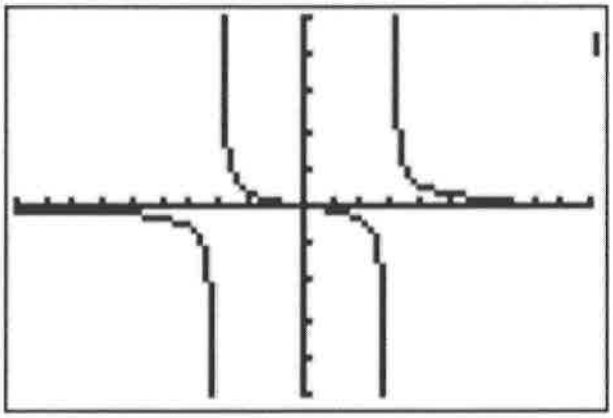
Domain- equation

- 1.) $f(x)=\dfrac{2x-4}{x^2-7x+12}$ D: $(-\infty,3)\cup(3,4)\cup(4,\infty)$
- 2.) $g(x)=\dfrac{x-5}{2x^2-x-15}$ D: $(-\infty,-5/2)\cup(-5/2,3)\cup(3,\infty)$
- 3.) $h(x)=\dfrac{3x-1}{x^3-25x}$ D: $(-\infty,-5)\cup(-5,0)\cup(0,5)\cup(5,\infty)$

Asymptotes- equation-VA &HA

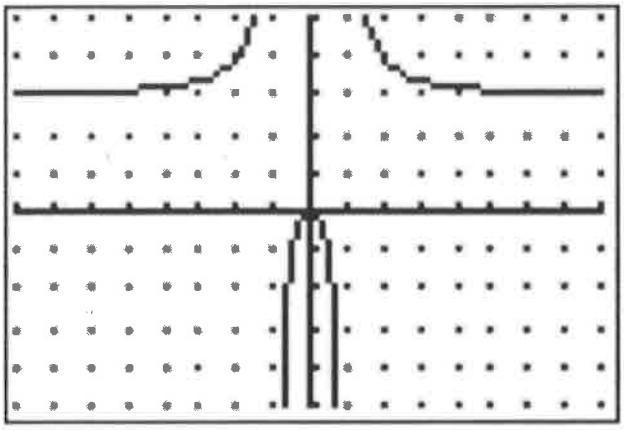
- 1.) $f(x)=\dfrac{x^3}{x^2-x-6}$ VA: $x=3$, $x=-2$ HA: none
- 2.) $g(x)=\dfrac{x+2}{6x^2-33x+42}$ VA: $x=2$, $x=7/2$ HA: $y=0$
- 3.) $h(x)=\dfrac{3x^2}{x^2-4}$ VA: $x=2$, $x=-2$ HA: $y=3$

Asymptotes- graph-VA &HA



VA: $x = -3$, $x = 3$

HA: $y = 0$



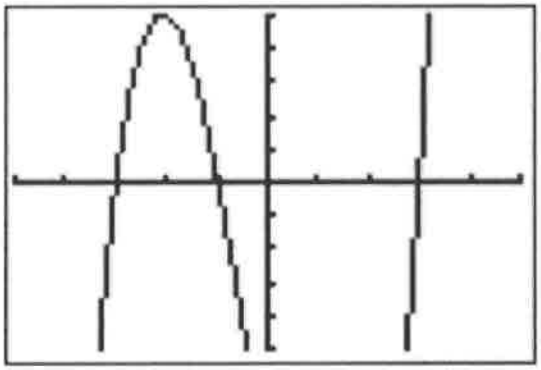
VA: $x = -1$, $x = 1$

HA: $y = 3$

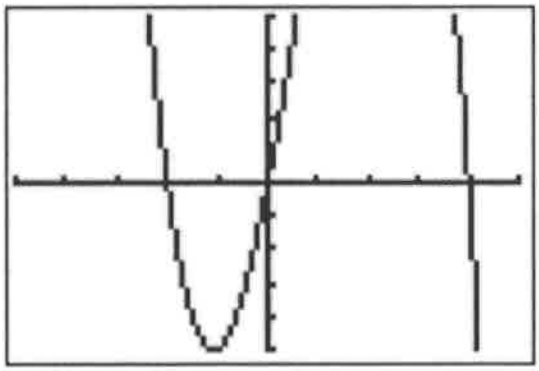
Roots-given roots- find equation, then graph

- 1.) roots of 3 and -5, $y = (x-3)(x+5) \rightarrow y = x^2 + 2x - 15$
- 2.) roots of 2, -2, 0 $y = x(x^2-4) \rightarrow y = x^3 - 4x$
- 3.) roots of 2, 2i, -2i $y = (x-2)(x^2+4) \rightarrow y = x^3 - 2x^2 + 4x - 8$

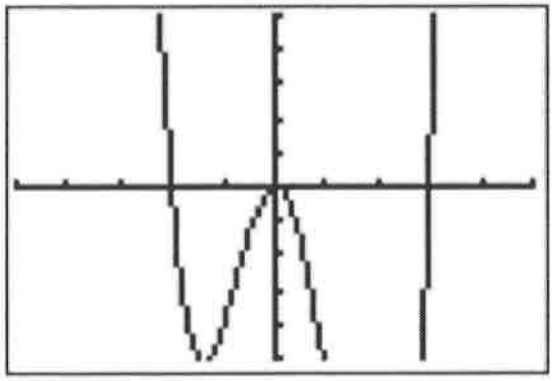
Roots- roots from graph



$x = -3, -1, 3$



$x = -2, 0, 4$



$x = -2, 0$ (D.R.), 3

Roots- Find roots from equation

- 1.) $f(x) = x^3 + 3x^2 - 4x - 12$ $x = -3, -2, 2$
- 2.) $g(x) = x^2 - 4x + 7$ $x = 2 \pm i\sqrt{3}$
- 3.) $h(x) = 4x^3 - 25x$ $x = 0, -5/2, 5/2$

Factor: trinomials and grouping

- 1.) $y = x^2 - 121$ $(x+11)(x-11)$
- 2.) $y = 12x^2 - 8x - 15$ $(2x-3)(6x+5)$
- 3.) $y = x^3 - 4x + 5x^2 - 20$ $(x+5)(x+2)(x-2)$
- 4.) $y = x^3 + 5x - x^2 - 5$ $(x-1)(x^2+5)$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

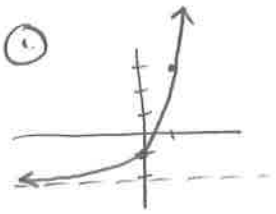
- 1.) $f(x) = x^2 - 6x + 4$ $3 \pm 2\sqrt{5}$
- 2.) $g(x) = 2x^2 + 4x - 3$ $\frac{-2 \pm \sqrt{13}}{2}$
- 3.) $h(x) = -3x^2 - 2x + 3$

$\frac{1 \pm \sqrt{10}}{-3}$

Graph

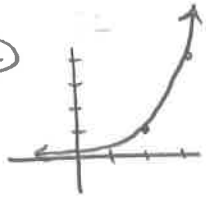
- 1.) $y = 5^x - 2$
- 2.) $y = 4^{x-2}$
- 3.) $y = -2^{x+2}$

1.



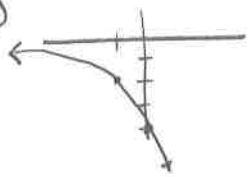
HA: $y = -2$

2.



HA: $y = 0$

3.



HA: $y = 0$

Solve- log and ln

- 1.) $\log_3 4 =$ 1.262
- 2.) $\log_7 12 =$ 1.277
- 3.) $7^{-2x} = 14$ -.678
- 4.) $11^{5x-2} = 25.75$ .671
- 5.) $8e^{-.5x} = 36$ -3.008
- 6.) $34.3 = 13e^{-1.4x}$ -.693
- 7.) $\log_4(3x-4) = 2$ 20/3
- 8.) $\log_2 x - \log_2 5 = 7$ 640
- 9.) $2\log_3 x = 5$ 15.588

Properties: expand

- 1.) $\log_4 x^2 y^3 \sqrt{z}$ $2\log_4 x + 3\log_4 y + \frac{1}{2}\log_4 z$
- 2.) $\ln \frac{x(y+5)}{z^4}$ $\ln x + \ln(y+5) - 4\ln z$
- 3.) $\log \frac{x^2-4}{x}$ $\log(x+2) + \log(x-2) - \log x$

Properties: condense

- 1.) $\log(x-3)+\log(x+3)-\log x$ $\log \frac{x^2-9}{x}$
- 2.) $6\log x-7\log y-4\log z$ $\log \frac{x^6}{y^7z^4}$
- 3.) $\ln 3+3\ln x-\frac{1}{2}\ln x$

$\ln \frac{3x^3}{\sqrt{x}}$

Word problems- interpret variables

Compounded $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuous $A = Pe^{rt}$

Population $P(t) = Ce^{kt}$

Word Problems

- 1.) If $3000 is invested at 3.5% interest compounded quarterly, what is the value after 10 years?

  $A = 3000\left(1 + \frac{.035}{4}\right)^{4 \cdot 10}$ ≈ 4250.72

- 2.) If $9500 is invested at 5% interest compounded daily, what is the value after 7 years?

  $A = 9500\left(1 + \frac{.05}{365}\right)^{365 \cdot 7}$ ≈ 13,480.81

- 3.) Problem # 2 compounded continuously.

  $A = 9500e^{.05(7)}$ ≈ 13,481.14

Word Problems (2)

- 4.) You invest $2500 in an account that earns 7% interest compounded continuously. When will it double?

$5000 = 2500e^{.07t}$ 9.9 years

- 5.) You invest $4550 in an account that earns 4.5% interest compounded continuously. When will it be worth $10,000?

$10.000 = 4550e^{.045t}$ 17.5 years

- 6.) You invest $1000 in an account that is compounded monthly. In 5 years it is worth $1475. Find the rate.

$$1475 = 1000\left(1 + \frac{r}{12}\right)^{12 \cdot 5}$$

$$1.475 = \left(1 + \frac{r}{12}\right)^{60}$$

$$1.475^{1/60} = 1 + \frac{r}{12}$$

$$\frac{1.475^{1/60} - 1}{12} = r \qquad r = 7.8\%$$

Word Problems (3)

- 7.) The initial amount of bees is 50. They grow at a rate of 10% a day compounded continuously. Find the number of bees in 20 days. $A = 50e^{.1(20)}$ 369 bees

- 8.) The initial amount of a mass of radium is 4.5 tons. It decays at a rate of -.5% compounded continuously. Find the amount of radium in 100 years.

$A = 4.5e^{-.005(100)}$ 2.729 tons

Convert

- To degrees:
  - 1.) 3π $540°$
  - 2.) $\frac{4\pi}{5}$ $144°$
  - 3.) $\frac{-3\pi}{8}$ $-67.5°$
- To radians:
  - 1.) $225°$ $\frac{5\pi}{4}$
  - 2.) $-300°$ $-\frac{5\pi}{3}$
  - 3.) $280°$ $\frac{14\pi}{9}$

Evaluate

- 1.) $\sin 30^\circ$ $1/2$
- 2.) $\cos\frac{3\pi}{4}$ $-\frac{\sqrt{2}}{2}$
- 3.) $\tan 90^\circ$ UND.
- 4.) $\sec(-150^\circ)$ $-\frac{2\sqrt{3}}{3}$
- 5.) $\csc\frac{\pi}{6}$ $= 2$
- 6.) $\cot\pi$ UND

Given trig- find trig

- 1.) If $\sin\Theta = \frac{3}{4}$ in quadrant II, find $\cot\Theta$ $-\frac{\sqrt{7}}{3}$
- 2.) If $\cos\Theta = -2/7$ in quadrant III, find $\sin\Theta$ $\frac{3\sqrt{5}}{7}$
- 3.) If $\csc\Theta = 3$ in quadrant I, find $\tan\Theta$ $\frac{\sqrt{2}}{4}$

Graph- describe the graph

- 1.) $y = 3\cos\Theta - 2$ $a = 3$ $p = 2\pi$ down 2
- 2.) $y = \cos(\Theta - 2)$ phase shift 2 $a = 1$ $p = 2\pi$
- 3.) $y = 3\cos 2\Theta - 2$ $a = 3$ $p = \pi$ down 2
- 4.) $y = 3\cos(2\Theta - 2)$ $a = 3$ $p = \pi$ phase shift 1

- 1.) $y = -2\sin(3\theta) + 4$ reflect, $a = 2$ $p = \frac{2\pi}{3}$, up 4
- 2.) $y = 3\cos(\theta - \pi) - 5$ $a = 3$ $p = 2\pi$ phase shift π down 5
- 3.) $y = -4\csc(\theta + \frac{\pi}{2})$ reflect $a = 4$ phase shift $-\frac{\pi}{2}$

Simplify

- 1.) $\sin^2\theta\cos^2\theta\sec\theta\tan\theta$ $\sin^3\theta$
- 2.) $\dfrac{\sin^2\theta}{1+\cos\theta}$ $\cos\theta - 1$
- 3.) $\dfrac{\sin^2\theta\cos\theta\cot^2\theta}{\cos^3\theta}$ $= 1$

Solve

- 1.) $2\cos^2 x + \cos x = 0$ $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$
- 2.) $2\cos^2 x - \cos x - 1 = 0$ $0, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$
- 3.) $\sin^2 x - 1 = 0$ $\frac{\pi}{2}, \frac{3\pi}{2}$
- 4.) $25\tan^2 x - 25 = 0$ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Law of Sines

- 1.) $m\angle A = 20°, m\angle C = 100°, c = 10, a = ?$ 3.47
- 2.) $m\angle A = 35°, m\angle B = 75°, a = 5.4, b = ?$ 9.09
- 3.) $m\angle A = 23°, m\angle B = 34°, a = 3.23, c = ?$ 2.26

Law of Cosines

- 1.)a = 3, b = 5, c = 7, angle A = ? 21.79°
- 2.) a = 4, b = 8, c = 11, angle B = ? 33.95°
- 3.) a = 2, b=7, c = 8, angle B = ? 53.58°