

Name \_\_\_\_\_

Review (1)

Derivative of each of the following.

$$\begin{aligned} & -3x^2 \csc^2(x^3) \\ & -\csc^2(x^3) \cdot 3x^2 \end{aligned}$$

$$-\frac{4}{x^3}$$

$$ix$$

$$\frac{3x^2}{(x-1)^3}$$

$$2. f(x) = \cot x^3$$

$$5. y = \sqrt[3]{9x^2 - 5}$$

$$8. y = (x^2 - 5)^{-3}(8x + 3)^2$$

$$3. h(x) = 2x^5 - \frac{4}{3}$$

$$6. g(x) = x^4 \sec x$$

$$9. g(x) = \frac{\tan^2 x}{\sec^3 x}$$

$$\begin{aligned} y' &= \frac{(4x-1)^{-1} \cdot (6x - 3x^2) \cdot 3 \cdot (4x-1)^{-2} \cdot 4}{(4x-1)^{-4}} \\ &= \frac{6x(4x-1)}{(4x-1)^3} \\ &= 24x^2 - 6x - 36x^3 = \frac{-12x}{(4x-1)^3} \end{aligned}$$

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Limit Def'n of a Derv

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Use Limit Definition of a Derivative to find the derivative of the following functions.

$$= 5x^2 + 3x - 8 \quad \lim_{h \rightarrow 0} \frac{5(x+h)^2 + 3(x+h) - 8 - (5x^2 + 3x - 8)}{h}$$

$$\frac{5x^2 + 10xh + 5h^2}{h} + \frac{3x + 3h - 5 - 5x^2 - 3x + 8}{h}$$

$$= 3x^2 - 4x$$

$$\lim_{h \rightarrow 0} \frac{10xh + 5h^2 + 3h}{h} = 10x + 5h + 3$$

$$= 5x - 8$$

$$m_{+n} = 10x + 3$$

Determine the first derivative of the following functions.

$$r = (2x^4 + 5x)^5$$

$$r = \sqrt{3x^2 + 2x - 1}$$

$$g(x) = x^2 \sin x - 2 \sec x$$

$$r = \cos(\sin(x^2))$$

$$f(x) = \cot^2(3x^2 - 6x)$$

$$2) \quad y = 2x^{-3} + \frac{3}{\sqrt[5]{x^3}} + \frac{2}{3x^2}$$

$$4) \quad f(x) = x^3(x^2 + 2)^{\frac{1}{2}}$$

$$6) \quad y = \tan(3x^4)$$

$$8) \quad h(x) = \frac{4}{(x^3 + 4)^3}$$

$$10) \quad y = \frac{\tan(2x+1)}{\sin(3x)} \text{ use quotient rule}$$

Find the first derivatives and simplify the answer for number 12.

$$y = (x^2 - x)(x^3 + 2)$$

$$12) \quad f(x) = \frac{(3x-4)}{(x^2+3)} \text{ use quotient rule}$$

4. Find the second derivative for the following functions.

$$y = 3x^5 - 2x^4 + 10x^{\frac{1}{5}} + 3x^{-3}$$

$$14) \quad y = \tan(5x)$$

For  $f(x) = (x^2 - 3x)^2$  find the equation of the tangent line when  $x = 1$ .

For  $y = x \cos^2(x)$  find the equation of the tangent line when  $x = \frac{\pi}{3}$ .

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## The Rules of Differentiation

Find the first derivative of the following functions.

$$(2x^4 + 5x)^5$$

$$5(2x^4 + 5x)^4(8x^3 + 5)$$

$$2) y = 2x^{-3} + \frac{3}{\sqrt[5]{x^3}} + \frac{2}{3x^2} = 2x^{-3} + 3$$

$$y' = -6x^{-4} - \frac{9}{5}x^{-\frac{8}{5}} - \frac{4}{3}x^{-\frac{5}{3}}$$

$$\begin{aligned} \sqrt{3x^2 + 2x - 1} &= (3x^2 + 2x - 1)^{1/2} \\ y' &= \frac{1}{2}(3x^2 + 2x - 1)^{-1/2} (6x + 2) \end{aligned}$$

4)  $f(x) = x^3(x^2 + 2)^{\frac{1}{2}}$

$$y' = 3x^2(x^2 + 2)^{1/2} + x^3 \cdot \frac{1}{2}(x^2 + 2)^{-1/2} \cdot 2x$$

$$y' = 3x^2(x^2 + 2)^{1/2} + x^4(x^2 + 2)^{-1/2}$$

$$x) = x^2 \sin x - 2 \sec x$$

$$\therefore y' = 2x \sin x + x^2 \cos x - 2 \sec x \tan x$$

$$6) y = \tan(3x^4)$$

$$y' = \sec^2(3x^4) (12x^3)$$

$$= \cos(\sin(x^2))$$

$$= -\sin(\sin(x^2)) (\cos(x^2)) (2x)$$

$$y' = -2x (\sin(\sin(x^2))) (\cos(x^2))$$

$$8) h(x) = \frac{4}{(x^3 + 4)^3}$$

$$y' = \frac{(x^3 + 4)^3 - 4 \cdot 3(x^3 + 4)^2 \cdot 3x^2}{(x^3 + 4)^6}$$

$$= \frac{-36x^2}{(x^3 + 4)^4}$$

$$) = \cot^2(3x^2 - 6x)$$

$$10) \quad y = \frac{\tan(2x+1)}{\sin(3x)} \quad \text{use quotient rule}$$

$$f'(x) = 2 \cot(3x^2 - 6x) \cdot (-\csc^2(3x^2 - 6x)) \cdot (6x - 6)$$

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$$y' = \frac{\sin(3x) \cdot \sec^2(2x+1) \cdot 2 - \tan(2x+1) \cdot \cos(3x) \cdot 3}{\sin^2(3x)}$$

+ derivatives and simplify your answers.

$$\cdot x)(x^3 + 2) \quad 12) f(x) = \frac{(3x - 4)}{(x^2 + 3)} \text{ use quotient rule}$$

$$y' = (2x-1)(x^3+2) + (x^2-x)(3x^2)$$

$$y' = \cancel{2x^4} + 4x - x^3 - 2 + \cancel{3x^4} - \cancel{3x^3}$$

$$y' = 5x^4 - 4x^3 + 4x - 2$$

$$13) y' = \frac{(x^2+3) \cdot 3 - (3x-4) \cdot 2x}{(x^2+3)^2} = \frac{3x^2+9 - 6x^2 + 8x}{(x^2+3)^2}$$

$$y' = \frac{-3x^2 + 8x + 9}{(x^2+3)^2}$$

the second derivative for the following functions.

$$= 3x^5 - 2x^4 + 10x^{1/5} + 3x^{-3}$$

$$14) \quad y = \tan(5x)$$

$$y' = 15x^4 - 8x^3 + 2x^{-4/5} - 9x^{-4}$$

$$y'' = 60x^3 - 24x^2 - \frac{8}{5}x^{-9/5} + 36x^{-5}$$

$$\textcircled{14} \quad y' = \sec^2(5x) \cdot 5$$

$$y' = 5 \sec^2(5x)$$

$$y'' = 10 \sec(5x) \cdot \sec(5x) \cdot \underline{\tan(5x)} \cdot 5$$

$$y'' = 50 \cdot \sec^2(5x) \cdot \tan(5x)$$

or  $f(x) = (x^2 - 3x)^2$  find the equation of the tangent line when  $x =$

$$f'(x) = 2(x^2 - 3x) \cdot (2x - 3)$$

$$\begin{aligned} f'(1) &= 2(1-3) \cdot (2-3) \\ &= -4 \cdot -1 \end{aligned}$$

$$\boxed{-4 = m}$$

$$y = 4x + 0$$

$$\boxed{y = 4x}$$

$$f(1) = (1^2 - 3 \cdot 1)^2$$

$$= 4$$

$$(1, 4) \quad m = 4$$

$$y = mx + b$$

$$4 = 4 \cdot 1 + b$$

$$4 = 4 + b$$

$$0 = b$$

For  $y = x \cos^2(x)$  find the equation of the tangent line when  $x = \pi/4$

$$y' = 1 \cdot \cos^2 x + x \cdot 2 \cos(x) \cdot (-\sin x)$$

$$y' = \cos^2 x - 2x \cos x \sin x$$

$$y'(\frac{\pi}{3}) = \cos^2(\frac{\pi}{3}) - 2(\frac{\pi}{3}) \cdot \cos(\frac{\pi}{3}) \cdot \sin(\frac{\pi}{3})$$

$$= -0,203 = n$$

$$y = \frac{\pi}{3} \cdot \cos^2\left(\frac{\pi}{3}\right) = 0.262$$

$$m = -0.203$$

$$(143, 262)$$

$$y = -.203x + .475 \quad | \quad y = mx + b$$

$$.262 = -.203(\pi/3) + b$$

$$.475 = b$$

$$y = (x^2 - 5)^{-3} (8x + 3)^2$$

$$y' = -3(x^2-5)^{-4} \cancel{(2x)} (8x+3)^2 + (x^2-5)^{-3} \cdot 2(8x)$$

$$y' = -6x(x^2-5)^{-4} (8x+3)^2 + 16(x^2-5)^{-3} (8x)$$

$$= 3x^2 - 4x$$

$$\lim_{h \rightarrow 0} \frac{3(x+h)^2 - 4(x+h) - (3x^2 - 4x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6xh + \cancel{3h^2} - 4x - 4h - \cancel{3x^2} + \cancel{4x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{(6xh + 3h^2 - 4h)}{h} = 6x + 3h - 4$$

$$m_{tan} = 6x - 4$$

$$= 5x - 8$$

$$\lim_{h \rightarrow 0} \frac{5(x+h) - 8 - (5x - 8)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{5x+5h-8} - \cancel{5x+8}}{h} = \frac{5h}{h} = 5$$

$$\boxed{m_{tan} = 5}$$