ost - $C(x)$

Total cost	Average Cost	Marginal Cost
$C(x)$ • Amount of $ it takes to produce & market "x" items	$\bar{C}(x) = \frac{C(x)}{x}$ $= \frac{\text{Total cost for x-units}}{\text{x-units}}$	$C'(x)$ • Cost of each item • finds the cost of the • $C'(x)$ = Marginal Cost of $(x+1)$

Minimal cost: ① Set $C'(x)=0$, ② Solve for "x", ③ Plug "x" into $C(x)$ to fi

EXAMPLE 8 Suppose $C(x)$ is the total cost that a company incurs in producing x units of a certain commodity. The function C is called a **cost function**. If the number of items produced is increased from x_1 to x_2, the additional cost is $\Delta C = C(x_2) - C(x_1)$, and the average rate of change of the cost is

$$\frac{\Delta C}{\Delta x} = \frac{C(x_2) - C(x_1)}{x_2 - x_1} = \frac{C(x_1 + \Delta x) - C(x_1)}{\Delta x}$$

The limit of this quantity as $\Delta x \to 0$, that is, the instantaneous rate of change of cost with respect to the number of items produced, is called the **marginal cost** by economists:

$$\text{marginal cost} = \lim_{\Delta x \to 0} \frac{\Delta C}{\Delta x} = \frac{dC}{dx}$$

[Since x can usually take on only integer values, it may not make literal sense to let Δx approach 0, but we can always replace $C(x)$ by a smooth approximating function as in Example 6.]

Taking $\Delta x = 1$ and n large (so that Δx is small compared to n), we have

$$C'(n) \approx C(n + 1) - C(n)$$

Thus the marginal cost of producing n units is approximately equal to the cost of producing one more unit [the $(n + 1)$st unit].

It is often appropriate to represent a total cost function by a polynomial

$$C(x) = a + bx + cx^2 + dx^3$$

where a represents the overhead cost (rent, heat, maintenance) and the other terms represent the cost of raw materials, labor, and so on. (The cost of raw materials may be proportional to x, but labor costs might depend partly on higher powers of x because of overtime costs and inefficiencies involved in large-scale operations.)

For instance, suppose a company has estimated that the cost (in dollars) of producing x items is

$$C(x) = 10{,}000 + 5x + 0.01x^2$$

Then the marginal cost function is

$$C'(x) = 5 + 0.02x$$

The marginal cost at the production level of 500 items is

$$C'(500) = 5 + (0.02)500 = \$15/\text{item}$$

This gives the rate at which costs are increasing with respect to the production level when $x = 500$.

The cost of producing the 501st item is

$$\begin{aligned} C(501) - C(500) &= [10{,}000 + 5(501) + 0.01(501)^2] \\ &\quad - [10{,}000 + 5(500) + 0.01(500)^2] \\ &= \$15.01 \end{aligned}$$

Notice that $C'(500) \approx C(501) - C(500)$. ■

venue - R(x)

- $ coming in from sales of "x" items

- R(x) = price * x ⟶ R(x) = p(x) · x

Marginal revenue - R'(x) - revenue for each individual item

Maximize revenue: ① Set R'(x) = 0, ② Solve for "x", ③ Plug into R(x) to f

revenue

fit - P(x)

- Amount of $ actually made

- $P(x) = R(x) - C(x)$

Profit Revenue Cost

Marginal profit - $P'(x)$ = profit for each individual item

Maximize profit: ① $P'(x) = 0$, ② Solve for "x", ③ Plug into $P(x)$ to find profit

A company estimates that it can sell 1000 units per week if it sets the price at \$3, but that its weekly sales will increase by 100 units for each \$.10 decrease in price. If x is the number of units sold each week ($x \geq 1000$) find the following.

Price Function

3) (1100, 2.90)

$\frac{2.9-3}{1100-1000} = \frac{-.10}{100} = -.001$

$y = mx + b$

$3 = -.001(1000) + b$

$3 = -1 + b$

$4 = b$

$y = -.001x +$

$\boxed{p(x) = -.001x +}$

Find the number of units and the corresponding price that will maximize weekly revenue.

$R(x) = p(x) \cdot x$

$= (-.001x + 4)x$

$R(x) = -.001x^2 + 4x$

$R'(x) = -.002x + 4 = 0$

$x = 2000$

$p(2000) =$

he production costs per week for producing x widgets is given by

$C(x) = 500 + 350x - 0.09x^2, 0 \le x \le 1000$. Answer the following questions.

What is the total cost to produce the 301st widget?

$C(301) - C(300)$

$97,695.91 - 97,400 = \boxed{\$295.91}$

What is the rate of change of the cost at x = 300?

$C'(x) = 350 - .18x$

$C'(300) = \boxed{\$296}$

m determines that x units of its product can be sold daily at p dollars per unit, where $1000 - p$. The cost of producing x units per days is $C(x) = 3000 + 20x$.

$= 1000 - x$

Find the revenue function.

$R(x) = p(x) \cdot x$

$= (1000 - x) \cdot x$

Find the profit function.

$R(x) = 1000x - x^2$

$P(x) = R(x) - C(x)$

$= 1000x - x^2 - 30$

$P(x) = -x^2 + 980x - .$

Find the cost, revenue and profit at the following 10 units, 100 units, 250 units, 500 units.

200	$R(10) = \$9900$	$P(10) = \$6,700$
000	$R(100) = \$90,000$	$P(100) = \$85,000$
000	$R(250) = \$187,500$	$P(250) = \$179,500$
,000	$R(500) = \$250,000$	$P(500) = \$237,000$

Find the average cost of producing 100 items. Find the cost of producing the 100th unit. Explain each in content of the problem.

$$0) = \frac{C(100)}{100} = \frac{5000}{100} = \$50$$

$$) - C(99)$$

$$0 - 4980$$

$$\$20$$

d the average revenue of producing 100 items. Find the marginal revenue equation. Then d the marginal revenue made on the 100 item.

$\overline{R}(100) = \frac{R(100)}{100} = \frac{90,000}{100}$

$= \$900$

$R(x) = 1000x - x^2$

$R'(x) = 1000 - 2x$

$R'(99) = \$802$

d the average profit of producing 100 items. Find the marginal profit equation. Then find marginal profit made on the 100 item.

$\overline{P}(100) = \frac{P(100)}{100} = \frac{85000}{100}$

$= \$850$

$P(x) = -x^2 + 980x - 3000$

$P'(x) = -2x + 980$

$P'(99) = \$782$

ssuming that the production capacity is at most 500 units per day, determine how many
ne company must produce and sell each day to maximize the profit.   Find the maximum p

$P'(x) = -2x + 980 = 0$

$x = 490$

$P(490) = \$237,100$

/hat price per unit must be charged to obtain the maximum profit?

$p(x) = 1000 - x$

$p(490) = 1000 - 490 = \boxed{\$510}$

manufacturing and selling x units of a certain commodity, the price function p and the cost nction C (in dollars) are given by $p(x) = 5 - .002x$ and $C(x) = 3 + 1.10x$.

Determine the revenue and profit functions.

$R(x) = p(x) \cdot x$

$= (5 - .002x)x$

$\boxed{R(x) = 5x - .002x^2}$

$P(x) = R(x) - C(x)$

$= 5x - .002x^2 - 3 - 1.1x$

$\boxed{P(x) = -.002x^2 + 3.9x - 3}$

Find the expression for the marginal cost, marginal revenue, and marginal profit.

$C'(x) = 1.10$

$R'(x) = 5 - .004x$

$P'(x) = -.004x + 3.9$

ind the cost, revenue, profit, average cost, marginal cost, average revenue, marginal reven
verage profit and marginal profit at 20 units, 800 units, and 1200 units. Interpret the
lifference between average and marginal for each item.

25
99.2
74.2

1.25
4.96
3.71

1.1
=4.92
=3.82

Total

$C(800) = 883$
$R(800) = 2720$
$P(800) = 1837$

Avg: $\bar{C}(800) = 1.1$
$\bar{R}(800) = 3.4$
$\bar{P}(800) = 2.3$

Marg. $C'(800) = 1.1$
$R'(800) = 1.8$
$P'(800) = .70$

Total

$C(1200) = 1323$
$R(1200) = 3120$
$P(1200) = 1797$

Avg: $\bar{C}(1200) = 1.1$
$\bar{R}(1200) = 2.6$
$\bar{P}(1200) = 1.5$

Marg: $C'(1200) = 1.1$
$R'(1200) = .20$
$P'(1200) = -.90$

rmine the production level that will produce the maximum total profit. Then determin
naximum profit.

$P'(x) = -.004x + 3.9 = 0$

$.004x = 3.9$

$x = 975$

$P(975) = \$1898.30$

partment complex has 250 apartments to rent. If they rent x apartments then their mor
it, in dollars, is given by $P(x) = -8x^2 + 3200x - 80,000$.

How many apartments should they rent to maximize their profit?

$P'(x) = {}^{-}16x + 3200 = 0$

$x = 200$

$P(200) = \$240,000$

he weekly cost to produce x widgets is given by $C(x) = 75000 + 100x - 0.03x^2 + 0.000004x^3$
$\leq x \leq 10000$, and the demand function for the widgets is given by, $p(x) = 200 - 0.005x$,
$\leq x \leq 10000$. Determine the marginal cost, marginal revenue, and marginal profit when 2500
idgets are sold and when 7500 widgets are sold. Assume that the company sells exactly what
ey produce.

$.06x + .000012x^2$

25

75

$\boxed{\text{Revenue}}$: $R(x) = p(x) \cdot x$

$R(x) = (200 - .005x)x$

$R(x) = 200x - .005x^2$

$R'(x) = 200 - .01x$

$R'(2500) = 175$

$R'(7500) = 125$

$\boxed{\text{Profit}}$ $P(x) = R(x) - C(x$

$P(x) = 200x - .005x^2 - (75000 + 100x$

$P(x) = -.000004x^3 + .025x^2 + 100x -$

$P'(x) = -.000012x^2 + .05x + 100$

$P'(2500) = 150$

$P'(7500) = -200$

commercial cattle ranch currently allows 20 steers per acre of grazing land; on the aver
eers weigh 2000 lb. at market. Estimates by the Agriculture Department indicate that
erage market weight per steer will be reduced by 50lbs for each additional steer added
re of grazing land. How many steers per acre should be allowed to get the largest possil
arket weight for its cattle? Use the min/max existence theorem to verify your answer i
aximum.

ers/acre

(21, 1950)

$\frac{1950}{21} = -50$

$0)$ lb

-3000

tion $= p(x)$

$R(x) = (-50x + 3000)x$

$R(x) = -50x^2 + 3000x$

$R'(x) = -100x + 3000 = 0$

$\boxed{x = 30}$

Proof

- Graph revenue fn.
- Bounds $\rightarrow 0 \rightarrow 60$

$R(0) = 0$

$R(30) = 45{,}000$

$R(60) = 0$