

1. $\int x^2(x^3 - 1)^4 dx$

2. $\int 5x \sqrt[3]{1 - x^2} dx$

3. $\int \frac{x^2}{(1 + x^3)^2} dx$

4. $\int \sin(2x) dx$

5. $\int x(x^2 + 1)^{3/2} dx$

6. $\int \sqrt[5]{3 - 5y} dy$

7. $\int \frac{3x}{\sqrt[3]{10 - x^2}} dx$

8. $\int x^3 \sqrt{5x^4 + 20} dx$

9. $\int t^2 \cos(1 - t^3) dt$

10. $\int (x^2 + 1)(x^3 + 3x)^{-5} dx$

11. $\int \frac{x}{\sqrt{x^2 + 1}} dx$

12. $\int x^2 \sec^2(x^3) dx$

13. $\int x^2(x^3 + 4)^{10} dx$

14. $\int \frac{\sin 2x}{(1 - \cos 2x)^3} dx$

15. $\int \sin(\sin x) \cos x dx$

$$1. \int x^2 (x^3 - 1)^4 dx$$

$$u = x^3 - 1$$

$$\int \cancel{x^2} (u)^4 \frac{du}{3\cancel{x^2}}$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

$$\int \frac{1}{3} u^4 du$$

$$y = \frac{1}{15} u^5 + C$$

$$y = \frac{1}{15} (x^3 - 1)^5 + C$$

$$2. \int 5x \sqrt[3]{1-x^2} dx$$

$$u = 1 - x^2$$

$$du = -2x dx$$

$$\frac{du}{-2x} = dx$$

$$\int 5\cancel{x} (u)^{1/3} \frac{du}{-2\cancel{x}}$$

$$\int \frac{-5}{2} u^{1/3} du$$

$$y = -\frac{15}{8} u^{4/3} + C$$

$$y = -\frac{15}{8} (1 - x^2)^{4/3} + C$$

$$3. \int \frac{x^2}{(1+x^3)^2} dx$$

$$\int x^2 (1+x^3)^{-2} dx$$

$$\int \cancel{x} u^{-2} \frac{du}{3\cancel{x}}$$

$$\int \frac{1}{3} u^{-2} du$$

$$y = -\frac{1}{3} u^{-1} + c$$

$$y = -\frac{1}{3} (1+x^3)^{-1} + c$$

$$u = 1+x^3$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

$$4. \int \sin(2x) dx$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{du}{2} = dx$$

$$\int \sin(u) \cdot \frac{du}{2}$$

$$\int \frac{1}{2} \sin(u) du$$

$$y = -\frac{1}{2} \cos(u) + c$$

$$y = -\frac{1}{2} \cos(2x) + c$$

$$5. \int x(x^2+1)^{3/2} dx \quad U = x^2+1$$

$$\int x(u)^{3/2} \frac{du}{2x} \quad \frac{du}{2x} = dx$$

$$\int \frac{1}{2} u^{3/2} du$$

$$Y = \frac{1}{5} u^{5/2} + C$$

$$Y = \frac{1}{5} (x^2+1)^{5/2} + C$$

$$6. \int \sqrt[5]{3-5y} dy \quad U = 3-5y$$

$$dU = -5 dy$$

$$\frac{du}{-5} = dy$$

$$\int u^{4/5} \frac{du}{-5}$$

$$\int -\frac{1}{5} u^{4/5} du$$

$$Y = -\frac{1}{6} u^{6/5} + C$$

$$Y = -\frac{1}{6} (3-5y)^{6/5} + C$$

$$7. \int \frac{3x}{\sqrt[3]{10-x^2}} dx$$

$$\int 3x(10-x^2)^{-1/3}$$

$$\int 3x(u)^{-1/3} \frac{du}{-2x}$$

$$\int -\frac{3}{2} u^{-1/3} du$$

$$y = \frac{-9}{4} u^{2/3} + C$$

$$y = \frac{-9}{4} (10-x^2)^{2/3} + C$$

$$u = 10 - x^2$$

$$du = -2x dx$$

$$\frac{du}{-2x} = dx$$

$$8. \int x^3 \sqrt{5x^4 + 20} dx$$

$$u = 5x^4 + 20$$

$$du = 20x^3 dx$$

$$\frac{du}{20x^3} = dx$$

$$\int x^3 (u)^{1/2} \frac{du}{20x^3}$$

$$\int \frac{1}{20} u^{1/2} du$$

$$y = \frac{1}{30} u^{3/2} + C$$

$$y = \frac{1}{30} (5x^4 + 20)^{3/2} + C$$

$$9. \int t^2 \cos(1-t^3) dt \quad \begin{array}{l} u = 1-t^3 \\ du = -3t^2 dt \end{array}$$

$$\int \cancel{t^2} \cdot \cos(u) \cdot \frac{du}{-3\cancel{t^2}} = \int -\frac{1}{3} \cos(u) du$$

$$\int -\frac{1}{3} \cos(u) du$$

$$y = -\frac{1}{3} \sin(u) + c$$

$$y = -\frac{1}{3} \sin(1-t^3) + c$$

$$10. \int (x^2+1)(x^3+3x)^{-5} dx$$

$$u = x^3+3x$$

$$du = 3x^2+3 dx$$

$$\frac{du}{3x^2+3} = dx = \frac{du}{3(x^2+1)}$$

$$\int (\cancel{x^2+1}) \cdot u^{-5} \frac{du}{3(\cancel{x^2+1})}$$

$$\int \frac{1}{3} u^{-5} du$$

$$y = -\frac{1}{12} u^{-4} + c$$

$$y = -\frac{1}{12} (x^3+3x)^{-4} + c$$

$$11. \int \frac{x}{\sqrt{x^2+1}} dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{du}{2x} = dx$$

$$\int x(x^2+1)^{-1/2}$$

$$\int x(u)^{-1/2} \frac{du}{2x}$$

$$\int \frac{1}{2} u^{-1/2} du$$

$$y = u^{1/2} + C$$

$$y = (x^2+1)^{1/2} + C$$

$$12. \int x^2 \sec^2(x^3) dx \quad u = x^3$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

$$\int x^2 \sec^2(u) \frac{du}{3x^2}$$

$$\int \frac{1}{3} \sec^2(u) du$$

$$y = \frac{1}{3} \tan(u) + C$$

$$y = \frac{1}{3} \tan(x^3) + C$$

$$13. \int x^2 (x^3 + 4)^{10} dx$$

$$x^3 + 4 = u$$

$$3x^2 dx = du$$

$$dx = du / 3x^2$$

$$\int \cancel{x^2} \cdot u^{10} \frac{du}{3\cancel{x^2}}$$

$$\int \frac{1}{3} u^{10} du$$

$$y = \frac{1}{33} u^{11} + C$$

$$y = \frac{1}{33} (x^3 + 4)^{11} + C$$

$$14. \int \frac{\sin 2x}{(1 - \cos 2x)^3} dx$$

$$\int \sin(2x) \cdot (1 - \cos(2x))^{-3} dx$$

$$u = 1 - \cos(2x)$$

$$du = 2 \sin(2x) dx$$

$$\frac{du}{2 \cdot \sin(2x)} = dx$$

$$\int \cancel{\sin(2x)} \cdot u^{-3} \frac{du}{2\cancel{\sin(2x)}}$$

$$\int \frac{1}{2} u^{-3} du$$

$$y = -\frac{1}{4} u^{-2} + C$$

$$y = -\frac{1}{4} (1 - \cos(2x))^{-2} + C$$

15. $\int \sin(\sin x) \cos x dx$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$\int \sin(u) \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = \int \sin(u) du$$

$$\int \sin(u) du$$

$$y = -\cos(u) + C$$

$$y = -\cos(\sin(x)) + C$$