Calculus 1 = ☺ Name____________________

Worksheet – The Wonderful World of Related Rates Date____________________

$V_{cone} = \frac{\pi}{3} r^2 h$ $V_{sphere} = \frac{4}{3}\pi r^3$ $SA_{sphere} = 4\pi r^2$

1) A tumor in an animal's stomach is relatively spherical in shape. The radius of the tumor is growing at a rate of 0.001 centimeters per day. What is the rate of change of the volume of the tumor when the radius is 0.5 centimeters?

$\frac{dr}{dt} = .001 \text{ cm/day}$

$\frac{dV}{dt} = ?$

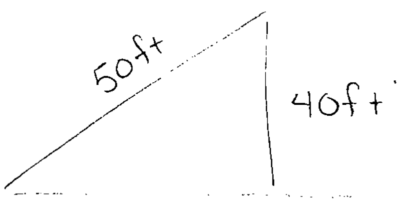
when $r = .5$ cm

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{3}\pi (3)(.5)^2(.001)$

$\frac{dV}{dt} = .00314 \text{ cm}^3\text{/day}$

2) A kite is flying at a height of 40 feet. A child is flying it so that it is moving horizontally at a rate of 3 feet per second. Assuming the string is taut, at what rate is the string being paid out when its length is 50 feet?



3 ft/sec

$50^2 = 40^2 + x^2$

$30 = x$

$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dx} = 2r\frac{dr}{dt}$

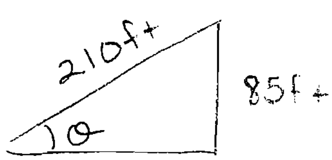
$2(30)(3) + 2(40)(0) = 2(50)\frac{dr}{dt}$

$1.8 \text{ ft/sec} = \frac{dr}{dt}$

3) Another kite is flying at a fixed height of 85 feet. It is flying horizontally at a rate of 2.7 feet per second. Assuming the string is taut, at what rate is the angle of elevation changing when the length of the string is 210 feet

$\tan\theta = \frac{y}{x}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$



2.7ft/sec

$210^2 = 85^2 + x^2$

$192.029 = x$

$\sin\theta = \frac{85}{210}$

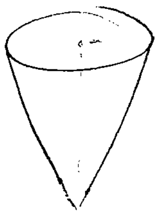
$.417 = \theta$

$\sec^2(.417)\frac{d\theta}{dt} = \frac{(192.029)(0) - 85(2.7)}{192.029^2}$

$\frac{d\theta}{dt} = -.005 \text{ rad/sec}$

$V_{cone} = \frac{\pi}{3} r^2 h$ $\quad V_{sphere} = \frac{4}{3}\pi r^3$ $\quad SA_{sphere} = 4\pi r^2$

4) A water tank in the form of an inverted cone is being emptied at a rate of 6 cubic meters per minute. The height of the cone is 24 meters and the radius of the top is 12 meters. Find the rate at which the water level is changing at the instant the water is 10 meters deep.



$h = 24$
$r = 12\,m$

$\frac{h}{r} = \frac{24}{12}$
$h = 2r$
$\frac{1}{2}h = r$

$\frac{dV}{dt} = -6\,m^3/min$

$\frac{dh}{dt} = ?$ $\quad h = 10m$

$V = \frac{\pi}{3} r^2 h$
$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$
$V = \frac{\pi}{12} h^3$
$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

$-6 = \frac{\pi}{12}(3)(10)^2 \ldots$
$-.0764\ m/min = \frac{dh}{dt}$

5) Using the information from problem 4, find the rate of change of the radius at the same instant.

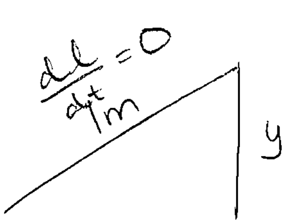
$\frac{h}{r} = \frac{24}{12}$
$\frac{10}{r} = \frac{24}{12}$
$r = 5$

$V = \frac{\pi}{3} r^2 h$
$V = \frac{\pi}{3}(r^2)(2r)$
$V = \frac{2\pi}{3} r^3$
$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$
$-6 = 2\pi(5)^2\frac{dr}{dt}$
$-.038\ m/min = dr/dt$

OR

$h = 2r$
$\frac{dh}{dt} = 2\frac{dr}{dt}$
$-.076 = 2\frac{dr}{dt}$
$-.038\ m/min = \frac{dr}{dt}$

6) A 7-meter ladder is leaning against a wall. The bottom of the ladder is pushed horizontally toward the wall at 1.5 meters per second. What is the rate of change of the top of the ladder when the bottom of the ladder is 2 meters away from the wall?



$-1.5\,m/s$
$x = 2$
find $\frac{dy}{dt} =$ when $x = 2$
$7^2 = 2^2 + y^2$
$6.708 = y$

$x^2 + y^2 = l^2$
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2l\frac{dl}{dt}$
$2(2)(-1.5) + 2(6.708)\frac{dy}{dt} = 2(7)(0)$
$\frac{dy}{dt} = .447\ m/sec$

$V_{cone} = \frac{\pi}{3}r^2h$ $\quad V_{sphere} = \frac{4}{3}\pi r^3$ $\quad SA_{sphere} = 4\pi r^2$

7) A 13-meter ladder is leaning against a wall. The bottom of the ladder is pushed horizontally <u>toward</u> the wall at 2.6 meters per second. What is the rate of change of the angle between the ladder and the ground at the moment the bottom of the ladder is 8 meters away from the wall?

-2.6 m/sec

8 m

$13^2 = 8^2 + y^2$

$10.247 = y$

$\cos\theta = \frac{8}{13}$

$\cos^{-1}\left(\frac{8}{13}\right) = \theta$

$.908 = \theta$

$\cos\theta = \frac{x}{r} = \frac{x}{13}$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{\frac{1}{13}(-2.6)}{-\sin(.908)}$

$\frac{d\theta}{dt} = .254$ m/sec

8) A spherical snowball is rolling down a hill and its volume is changing at a rate of 8 cubic feet per minute. Find the rate at which the radius is changing at the instant the <u>diameter</u> is 4 feet. Find the rate of change of the surface area at the same instant.

$\frac{dV}{dt} = 8$ ft³/min

$d = 4$

$r = 2$ft

$\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

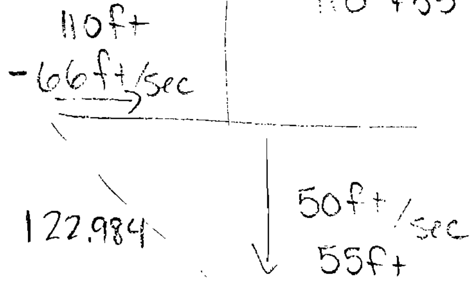
$\frac{dV}{dt} = \frac{4}{3}\pi\, 3r^2 \frac{dr}{dt}$

$8 = \frac{4}{3}\pi(3)(2)^2\left(\frac{dr}{dt}\right)$

$.159$ ft/min $= \frac{dr}{dt}$

9) A car is traveling toward an intersection at a rate of 66 feet per second and a bus traveling at a rate of 50 feet per second away from the intersection (the car and bus are perpendicular to each other). How fast is the distance between two vehicles changing at the instant the car is 110 feet from the intersection and the bus is 55 feet away?

$110^2 + 55^2 = c^2$



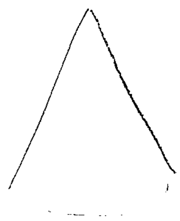
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$

$2(110)(-66) + 2(55)(50) = 2(122.984)\frac{dc}{dt}$

-36.671 ft/sec $= \frac{dc}{dt}$

$V_{cone} = \frac{\pi}{3} r^2 h$ $\quad V_{sphere} = \frac{4}{3}\pi r^3$ $\quad SA_{sphere} = 4\pi r^2$

10) Sand is being dropped at a rate of 10 cubic meters per minute forming a conical pile. If the height of the pile is always twice the base radius, what is the rate of change of the height when the pile is 8 meters?



$\frac{1}{2}h = r$

$h = 2r$

$\frac{dh}{dt} = ?$

$h = 8$

$\frac{dV}{dt} = 10\,m^3/min$

$V = \frac{\pi}{3} r^2 h$

$= \frac{\pi}{3} \left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$

$10 = \frac{\pi}{12} 3(8)^2 \frac{dh}{dt}$

$.199\,m/min = \frac{dh}{dt}$

11) Using the information from problem number 10, find the rate of change of the radius at the same instant.

$V = \frac{\pi}{3} r^2 (2r)$

and continue

$h = 2r$

$\frac{dh}{dt} = 2\frac{dr}{dt}$

$.199 = 2\frac{dr}{dt}$

$.0995$

$.1 = \frac{dr}{dt}$

12) A spherical balloon is being inflated at a rate of 5 cubic meters per minute. At what rate is the <u>diameter</u> changing at the instant the <u>diameter</u> is 12 meters?

$\frac{dV}{dt} = 5\,m^3/min$

$r = 6$ meters

$2r = d$

$2\frac{dr}{dt} = \frac{dd}{dt}$

$2(.0111) = \frac{dd}{dt} = .0222\,m/min$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

$.0111 = \frac{dr}{dt}$