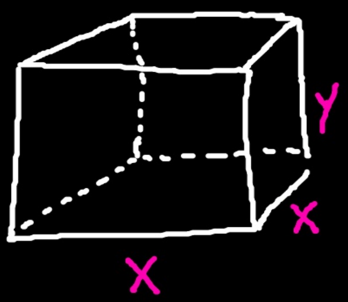
Calculus 1 Name________________________

Review – Optimization (1) Date________________________

1) You are to construct a box with an open top and a square base. The sides cost \$20 dollars per square foot and the base costs \$25 per square foot. The box must have a volume of 12.5 cubic feet. What dimensions will minimize the cost of the box?



$V = x^2 y$

$12.5 = x^2 y$

$\frac{12.5}{x^2} = y$

$\text{Cost} = 25x^2 + 20(4xy)$

$\text{Cost} = 25x^2 + 80x\left(\frac{12.5}{x^2}\right)$

$\text{Cost} = 25x^2 + 1000x^{-1}$

$\text{Cost}' = 50x - 1000x^{-2} = 0$

$50x = \frac{1000}{x^2}$

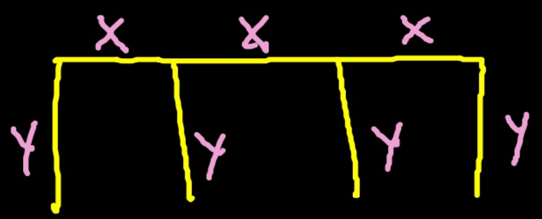
$x^3 = 20$

$x = 2.714$

$y = 1.7$

2.714ft x 2.714ft x 1.7ft

2) You have to construct three equal side by side storage bins. The storage bins will have fencing on three sides with one side open for delivery and pick up. The total area to be enclosed by the three storage bins is 2500 square feet. What are the dimensions of each storage bin if you minimize the amount of fencing used?



$$2500 = 3xy$$

$$\frac{2500}{3y} = x$$

$$F = 3x + 4y$$

$$F = \cancel{3}\left(\frac{2500}{\cancel{3}y}\right) + 4y$$

$$F = 2500y^{-1} + 4y$$

$$F' = -2500y^{-2} + 4$$

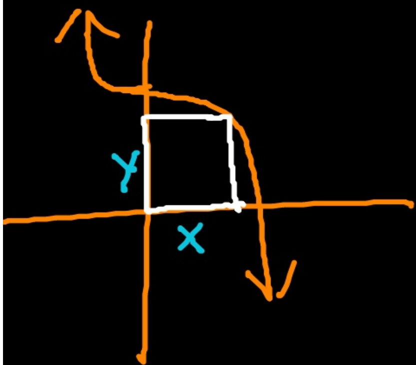
$$\frac{2500}{y^2} = 4$$

$$y^2 = 625$$

$$y = 25$$

$$x = 33.333$$

3) Find the largest rectangle (in the first quadrant) that can fit between $f(x) = -x^3 + 4$ and the x-axis.



$$A = x \cdot y$$

$$A = x \cdot (-x^3 + 4)$$

$$A = -x^4 + 4x$$

$$A' = -4x^3 + 4 = 0$$

$$4x^3 = 4$$

$$x^3 = 1$$

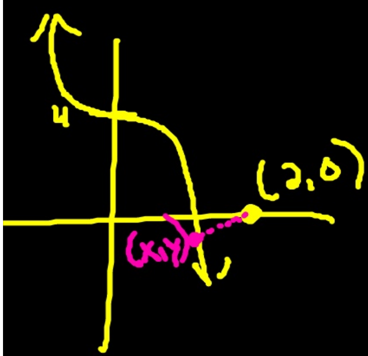
$$x = 1$$

$$y = -(1)^3 + 4$$

$$y = 3$$

$$\boxed{1 \times 3}$$

4) Find the closest distance to the graph $f(x) = -x^3 + 4$ from the point (2,0).



$$d = \sqrt{(x-2)^2 + (y-0)^2}$$

$$d = \sqrt{(x-2)^2 + (-x^3+4)^2}$$

$$d = \sqrt{x^2 - 4x + 4 + x^6 - 8x^3 + 16}$$

$$d = \sqrt{x^6 - 8x^3 + x^2 - 4x + 20}$$

$$d = \tfrac{1}{2}(x^6 - 8x^3 + x^2 - 4x + 20)^{-1/2}(6x^5 - 24x^2 + 2x - 4)$$

$$x = 1.594$$

$$y = {}^{-}.05$$

$$(1.594, {}^{-}.05)$$