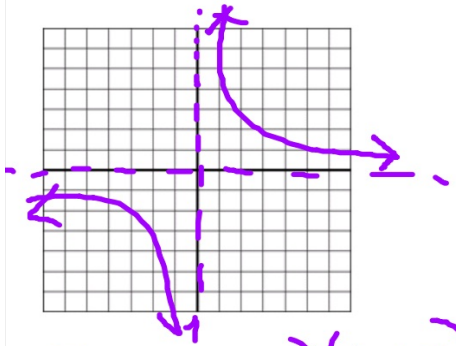
Determine the domain of the following functions. Draw a sketch of the graph.

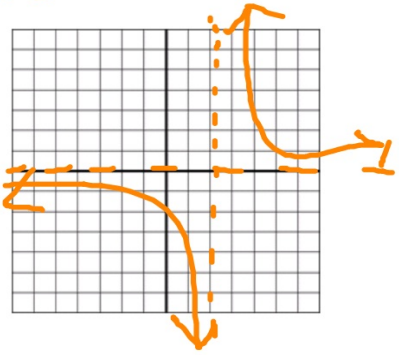
1. $y = \frac{1}{x}$



D: $(-\infty, 0)(0, \infty)$

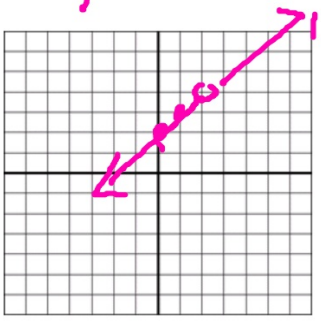
2. $y = \frac{1}{x-2}$ R 2

D $(-\infty, 2)(2, \infty)$



3. $y = \frac{x^2-4}{x-2}$ $(x-2)($ $x-$

$y = x + 2$



## Discontinuities

**Holes**
(removable discontinuity)

- occur when there is a value of x that makes the denominator equal to zero, but we can cancel out the factor

| | |
|---|---|
| 1. Set the canceled factor equal to 0. | 2. Solve for x. |
| 3. Plug the answer into the simplified equation to get *y*. | 4. (*x*, *y*) is the hole |

Examples of functions with holes:

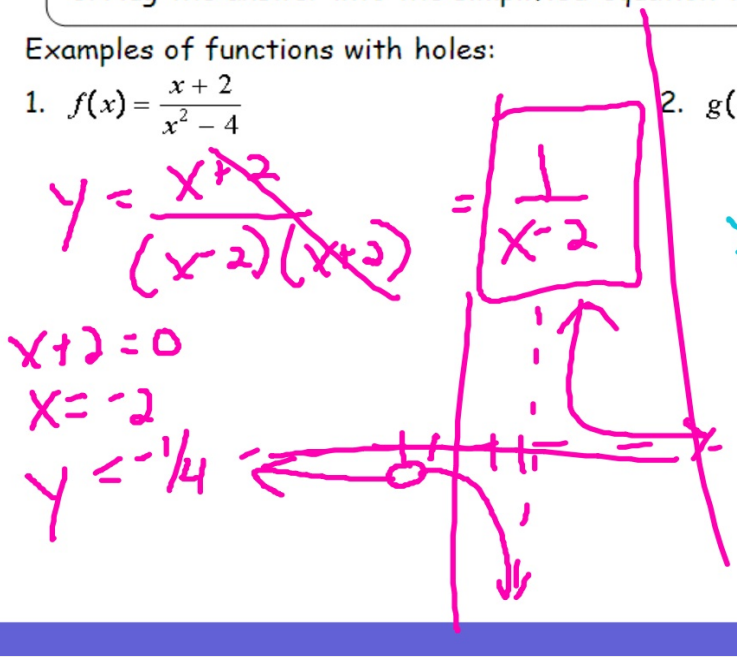
1. $f(x) = \dfrac{x+2}{x^2-4}$

$y = \dfrac{x+2}{(x-2)(x+2)} = \dfrac{1}{x-2}$

$x+2=0$

$x=-2$

$y=-1/4$



2. $g(x) = \dfrac{x+3}{x^2-2x-15}$

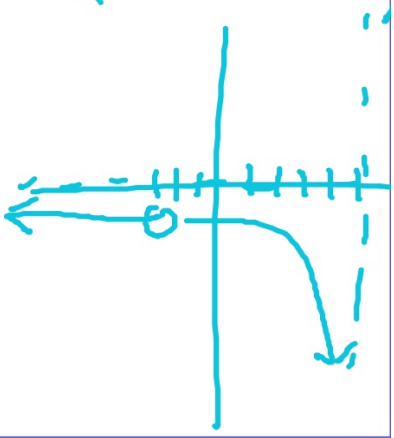
$(x-5)(x+3)$

$= \dfrac{1}{x-5}$

$x+3=0$

$x=-3$

$y=-1/8$



**Vertical Asymptotes**
(non-removable discontinuity)

- occur when there is a value of x that would make the denominator equal to zero, but we can't get rid of the factor

Examples of functions with vertical asymptotes:

1. $y = \frac{x}{2x-6}$

$2x-6=0$

VA: $x=3$

2. $y = \frac{10}{x^2-4}$

$(x-2)(x+2)$

VA: $x = \pm 2$

3. $y = \frac{x+5}{x^3+5x^2+6x}$

$x(x^2+5x+6)$

$x(x+3)(x+2)$

VA: $\begin{cases} x=0 \\ x=-3 \\ x=-2 \end{cases}$

**Horizontal Asymptote** – occur in 2 out of the 3 cases

- to find a horizontal asymptote, examine what happens to the function as $x$ approaches infinity

**Conclusion (short cut)**

Case 1: $\frac{1}{x}, \frac{1}{x^2}, \frac{x^2}{x^3}$ → Denominator exponent is larger/higher

HA: $y = 0$

Case 2: $\frac{3x^2}{5x^2}, \frac{2x-5}{-4x+3}$ → Exponents are equal

$y = 3/5$ $y = -1/2$

HA: $y =$ Ratio of the coefficients of the hig

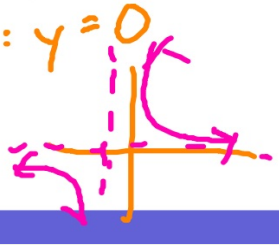
Case 3: $\frac{x^3}{x^2}, \frac{x^5+1}{x}$ → Numerator exponent is larger/higher

HA: none

1. $y = \frac{1}{x+1}$

VA: $x = -1$

HA: $y = 0$

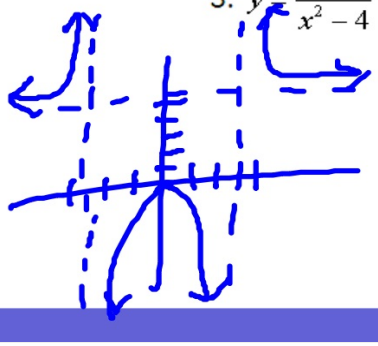


2. $y = \frac{5x^2+4x}{x^2-7}$

VA: $x = \pm\sqrt{7}$

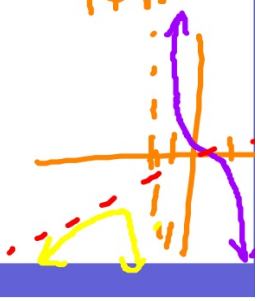
HA: $y = 5$

3. $y = \frac{x^3-2}{x^2-4}$



VA: x

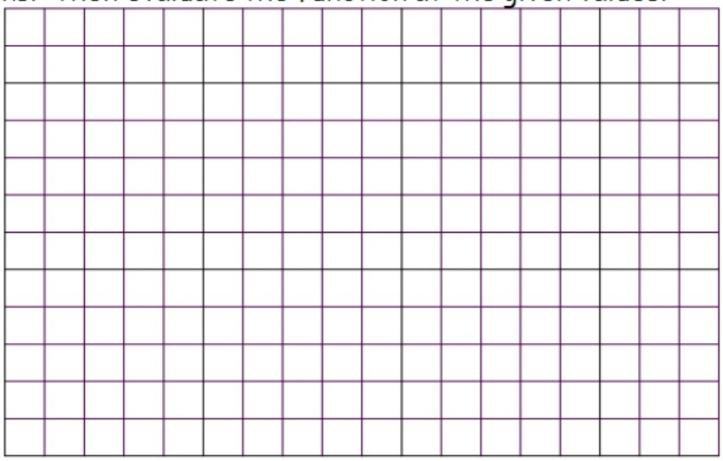
HA:



Graphing and Evaluating Piecewise Functions Block _____ Date _____

Plot the following piecewise functions. Then evaluate the function at the given values.

1) $f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ x+4 & \text{if } x > 3 \end{cases}$



$f(0) =$

$f(3) =$

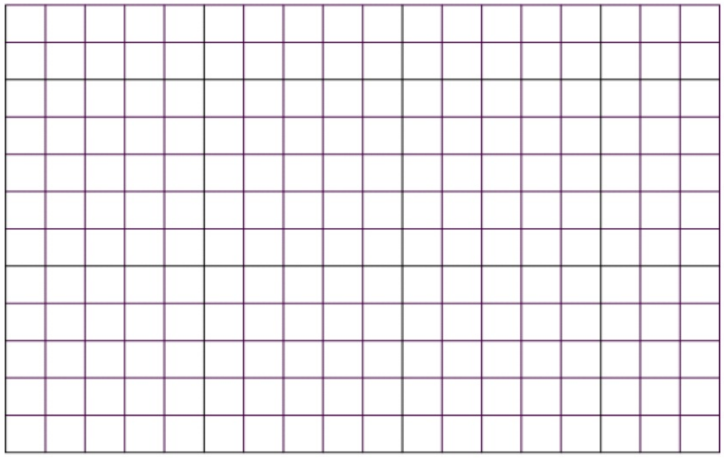
$f(-2) =$

2) $f(x)=\begin{cases} x^2 & \text{if } x>1 \\ 2 & \text{if } x\le 1 \end{cases}$

$f(1)=$

$f(3)=$

$f(-2)=$



3) $f(x)=\begin{cases} 2 & if \quad x \le -1 \\ -x+1 & if \quad -1 < x < 1 \\ \frac{1}{2}x-\frac{1}{2} & if \quad x \ge 1 \end{cases}$

$f(0)=$ $f(3)=$

$f(1)=$ $f\left(\frac{1}{2}\right)=$

$f(-1)=$ $f(-2)=$