

Integration
"u du" Substitution

$$u = 6x + 7$$
$$du = 6 dx$$
$$\frac{du}{6} = dx$$

2) $\int (6x + 7)^3 dx$

1) $\int 3(3x - 1)^5 dx$

$$\int \cancel{3} u^5 \cdot \frac{du}{\cancel{3}}$$

$$\int u^5 du$$

$$y = \frac{1}{6} u^6 + C$$

$$y = \frac{1}{6} (3x - 1)^6 + C$$

$$u = 3x - 1$$

$$du = 3 dx$$

$$\frac{du}{3} = dx$$

$$\int u^3 \frac{du}{6}$$

$$\int \frac{1}{6} u^3 du$$

$$y = \frac{1}{24} u^4 + C$$

$$y = \frac{1}{24} (6x + 7)^4 + C$$

$$3) \int \sqrt{(1-4x)} dx$$

$$\int (1-4x)^{1/2} dx$$

$$\int u^{1/2} \frac{du}{-4}$$

$$\int -\frac{1}{4} u^{1/2} du$$

$$y = -\frac{1}{6} u^{3/2} + c$$

$$y = -\frac{1}{6} (1-4x)^{3/2} + c$$

$$\begin{aligned} u &= 1-4x & du &= -4dx \\ \frac{du}{-4} &= dx \end{aligned}$$

$$4) \int x(2x^2+1)^6 dx$$

$$\int x \cdot u^6 \frac{du}{4x}$$

$$\int \frac{1}{4} u^6 du$$

$$y = \frac{1}{28} u^7 + c$$

$$y = \frac{1}{28} (2x^2+1)^7 + c$$

$$5) \int (2x^2 + 1)^2 dx$$

$$\int u^2 \frac{du}{4x}$$

$$u = 2x^2 + 1$$

$$du = 4x dx$$

$$\frac{du}{4x} = dx$$

Problem \rightarrow x needs
to cancel out

$$\int (2x^2 + 1)(2x^2 + 1)$$

$$\int 4x^4 + 4x^2 + 1 dx$$

$$y = \frac{4}{5}x^5 + \frac{4}{3}x^3 + x + C$$

$$6) \int x^2(x^3 - 1)^{10} dx$$

$$u = x^3 - 1$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

$$\int x^{\cancel{2}} \cdot u^{10} \frac{du}{3x^{\cancel{2}}}$$

$$\int \frac{1}{3} u^{10} du$$

$$y = \frac{1}{33} u^{11} + C$$

$$y = \frac{1}{33} (x^3 - 1)^{11} + C$$

$$7) \int x^4 \sqrt{3x^5 - 5} dx$$

$$u = 3x^5 - 5$$

$$du = 15x^4 dx$$

$$\int x^4 (3x^5 - 5)^{1/2} dx$$

$$\frac{du}{15x^4} = dx$$

$$\int x^4 (u)^{1/2} \cdot \frac{du}{15x^4}$$

$$\int \frac{1}{15} u^{1/2} du$$

$$Y = \frac{2}{45} u^{3/2} + C$$

$$Y = \frac{2}{45} (3x^5 - 5)^{3/2} + C$$

$$8) \int \sin^{10}(x) \cos(x) dx$$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$\frac{du}{\cos(x)} = dx$$

$$\int u^{10} \cdot \cos(x) \cdot \frac{du}{\cos(x)}$$

$$\int u^{10} du$$

$$Y = \frac{1}{11} \sin^{11}(x) + C$$

$$9) \int 6x^2 \sin(x^3) dx$$

$$\int 6x^2 \cdot \sin(u) \cdot \frac{du}{3x^2}$$

$$\begin{aligned} u &= x^3 \\ du &= 3x^2 dx \\ \frac{du}{3x^2} &= dx \end{aligned}$$

$$\int 2 \sin(u) du$$

$$y = -2 \cos(u) + C$$

$$y = -2 \cos(x^3) + C$$

$$10) \int \sin^2(2x) \cos(2x) dx$$

$$u = \sin(2x)$$

$$du = 2 \cos(2x) dx$$

$$\frac{du}{2 \cos(2x)} = dx$$

$$\int u^2 \cdot \cos(2x) \cdot \frac{du}{2 \cos(2x)}$$

$$\int \frac{1}{2} u^2 du$$

$$y = \frac{1}{6} u^3 + C$$

$$y = \frac{1}{6} (\sin^3(2x)) + C$$