① $\int dy = \int 20(5x+6)^3 dx$

$u = 5x+6$

$du = 5 \cdot dx$

$\frac{du}{5} = dx$

$\int dy = \int 20(u)^3 \frac{du}{5}$

$\int dy = \int 4u^3 du$

$y = u^4 + c$

$\boxed{y = (5x+6)^4 + c}$

② $\int dy = \int (x^5+x-1)^2 (5x^4+1)\,dx$

$\int dy = \int (u)^2 (5x^4+1)\cdot \frac{du}{5x^4+1}$

$\int dy = \int u^2\,du$

$y = \frac{1}{3}u^3 + c$

$y = \frac{1}{3}(x^5+x-1)^3 + c$

$u = x^5+x-1$

$du = 5x^4+1\,dx$

$\frac{du}{5x^4+1} = dx$

③ $\int dy = \int \cos(2x^3 - 3x)(6x^2 - 3)\,dx$

$\int dy = \int \cos(u) \cdot (6x^2 - 3) \cdot \frac{du}{6x^2 - 3}$

$\int dy = \int \cos(u) \cdot du$

$y = \sin(u) + c$

$y = \sin(2x^3 - 3x) + c$

$u = 2x^3 - 3x$

$du = 6x^2 - 3\,dx$

$\frac{du}{6x^2 - 3} = dx$

④ $\int dy = \int 5\sin^4(x)\cdot\cos(x)\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$\frac{du}{\cos(x)} = dx$

$\int dy = \int 5u^4 \cdot \cos(x) \cdot \frac{du}{\cos(x)}$

$\int dy = \int 5u^4\,du$

$y = u^5 + C$

$\boxed{y = \sin^5(x) + C}$

⑤ $\int dy = \int \frac{27}{4}(9x-6)^{-1/4}\,dx$

$u = 9x-6$

$du = 9\,dx$

$\frac{du}{9} = dx$

$\int dy = \int \frac{27}{4} u^{-1/4} \frac{du}{9}$

$\int dy = \int \frac{3}{4} u^{-1/4}\,du$

$y = u^{3/4} + C$

$y = (9x-6)^{3/4} + C$

⑥ $\int dy = \int -8x\cos^3(x^2)\cdot\sin(x^2)$

$u = \cos(x^2)$

$du = -\sin(x^2)\cdot 2x\,dx$

$\int dy = \int -8x\,u^3\cdot\sin(x^2)\,\frac{du}{-2x\sin(x^2)}$

$\frac{du}{-2x\cdot\sin(x^2)} = dx$

$\int dy = \int 4u^3\,du$

$y = u^4 + C$

$y = \cos^4(x^2) + C$