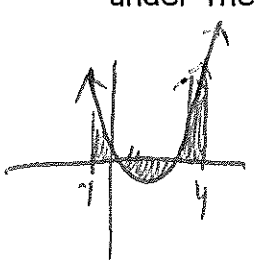
Calculus 1: Integration Review Name________________ Date ___________

1) Calculate the area between the given limits, then calculate the total area by treating all areas as positive) for the graph of $\int_{-4}^{3}(x^3 - 3x + 2)dx$. Sketch the graph and shade the area under the curve.

2) Calculate the area between the given limits, then calculate the total area by treating all areas as positive for the graph of $\int_{-1}^{4}(x^2 - 3x + 1)dx$. Sketch the graph and shade the area under the curve.



$\int_{-1}^{4} x^2 - 3x + 1\, dx$

$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x \Big|_{-1}^{4}$

$\boxed{4.167}$

$x^2 - 3x + 1 = 0$

$x = .382, 2.618$

$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x \Big|_{-1}^{.382}$

3.015

$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x \Big|_{.382}^{2.618}$

1.863

$\boxed{7.893}$

$F(x) = \frac{1}{3}x^3 - 3/2\, x^2 + x \Big|_{2.618}^{4}$

3.015

3) Calculate the area under the curve of $g(x) = x^4 - 2x^3 + 3$ from $x = 1.2$ to $x = 4.4$ using 18 rectangles.

4) Calculate the area under the curve of $f(x) = x^3 - 3x^2 + x + 4$ on the interval $[0.5, 3.1]$ using 24 rectangles.

LH: sum(seq($(x^3-3x^2+x+4)(13/120)$, x, .5, 3.1-13/120, 13/120)) ≈ 8.272

RH: sum(seq($(x^3-3x^2+x+4)(13/120)$, x, .5+13/120, 3.1, 13/120)) ≈ 8.725

$\boxed{\approx 8.499}$

Integrate the following. Make sure you show all your work and set-ups as required for each problem.

5) $\int (x^3 - 3x^2 + x + 4)dx$

6) $\int \left(\frac{1}{4x^3} - 3\sqrt[5]{x^2}\right)dx$

$\int \frac{1}{4}x^{-3} - 3x^{2/5}dx$

$\boxed{y = -\frac{1}{8}x^{-2} - \frac{15}{7}x^{7/5} + C}$

7) $\int (6x^2 + 8x)(x^3 + 2x^2)^4 dx$

8) $\int \tan x \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x dx$

$\int u \cdot \cancel{\sec^2 x} \frac{du}{\cancel{\sec^2 x}}$

$\int u\, du$

$\frac{1}{2}u^2 + C$

$y = \frac{1}{2}(\tan^2(x)) + C$

$\boxed{y = \frac{1}{2}\tan^2(x) + C}$

9) $\int \frac{1}{\sqrt{x}} \sin(\sqrt{x})dx$

10) $\int \frac{x+2}{\sqrt{x^2 + 4x + 7}}dx$

$x^2 + 4x + 7 = u$

$2x + 4\, dx = du$

$\int (x+2)(x^2+4x+7)^{-1/2}$

$\int \cancel{(x+2)}\, u^{-1/2} \frac{du}{2\cancel{(x+2)}}$

$\int \frac{1}{2} u^{-1/2} du$

$y = u^{1/2} + C$

$\boxed{y = (x^2+4x+7)^{1/2} + C}$

Solve for the constant of integration given the derivatives and a point of the original function.

11) $f'(x)=\int \sqrt[3]{x}\,dx \quad (1,2)$

12) $f'(x)=\int\left(\sec^2 x-\sin x\right)dx \quad \left(\frac{\pi}{4},1\right)$

$y = \tan x + \cos x + c$

$1 = \tan(\pi/4) + \cos(\pi/4) + c$

$-.707 = c$

$y = \tan x + \cos x - .707$

Use the Fundamental Theorem of Calculus for Area to calculate the definite integrals.

13) $\int_1^4 \left(2x^2-4x-5\right)dx$

14) $\int_{\pi/3}^{5\pi/6} (x-\sin x)dx$

$F(x) = \frac{1}{2}x^2 + \cos(x) \Big|_{\pi/3}^{5\pi/6}$

$F(5\pi/6) - F(\pi/3) =$

1.513

15) $\int_1^8 \left(5x^{2/3}-4x^{-2}\right)dx$

16) $\int_{0.6}^{1.3} \left(3x\sqrt{x^2+2}\right)dx$

$u = x^2+2$

$du = 2x\,dx$

$\int_{.6}^{1.3} 3\cancel{x}\, u^{1/2} \frac{du}{2\cancel{x}}$

$\int_{.6}^{1.3} \frac{3}{2} u^{1/2} du$

$F(x) = u^{3/2} du \Big|_{.6}^{1.3}$ $\quad F(x) = u^{3/2} \Big|_{2.36}^{3.69}$

$F(x) = (x^2+2)^{3/2} \Big|_{.6}^{1.3}$

$=$

3.463

Use the Fundamental Theorem of Calculus for Derivatives to find $G'(x)$.

17) $G(x) = \int_{8}^{4x^2+2x} (t^2 - 3t + 2)dt$

18) $G(x) = \int_{\cos(x^2)}^{3x+6} (\tan s + 2s^3)ds$

$G'(x) = 3(\tan(3x+6) + 2(3x+6)^3) + 2x\sin(x^2)($

$\tan(\cos(x^2)) + 2\cos^3(x^2))$

19) Find the mean (average) value and where it occurs for $f(x) = x^2 - 8x + 18$ for the interval $[2,6]$.

20) Find the mean (average) value and where it occurs for $f(x) = \sin x + \cos x$ for the interval $[0, 3\pi/4]$.

$\int_0^{3\pi/4} (\sin x + \cos x)\,dx = f(c)(3\pi/4 - 0)$

$-\cos x + \sin x \Big|_0^{3\pi/4} = f(c)(3\pi/4)$

$2.414 = f(c)(3\pi/4)$

$1.025 = f(c)$

$1.025 = \sin(x) + \cos(x)$

$x = .025$

$x = 1.545$