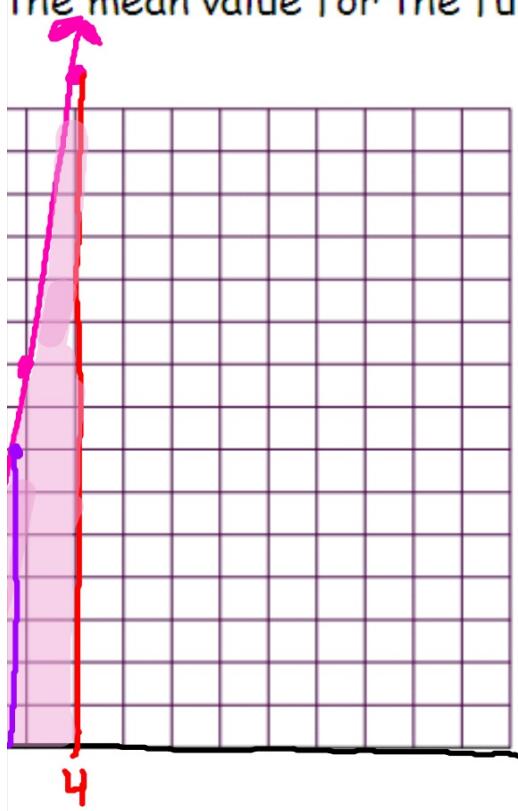


the mean value for the function  $f(x) = x^2$  on the interval  $[1, 4]$  and whe



$$\int_1^4 x^2 dx = f(c)(4-1)$$

$$\frac{1}{3} x^3 \Big|_1^4 = f(c) \cdot 3$$

$$F(4) - F(1)$$

$$21 = f(c) \cdot 3$$

$$7 = f(c) = \text{avg. value}$$

$$7 = \dots$$

$x = \dots$

$(2.6)$

## Mean Value Theorem for Integrals (Average Value Theorem for Integrals)

continuous on the closed interval  $[a,b]$ , then there is at least one number "c" at

$$\int_a^b f(x) dx = f(c) \cdot (b - a) \text{ where:}$$

$\int_a^b f(x) dx$  is the area under the curve,

$f(c)$  is the mean or average value, and

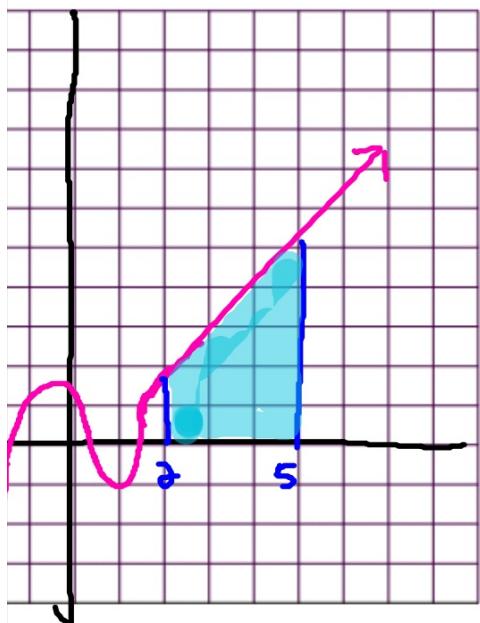
"c" is the value of "x" where the mean or average value occurs.

$f(c) \cdot (b - a)$  represents the area of the rectangle.

$f(c)$  is the Mean or Average value (height of the rectangle).

$(b - a)$  is the width of the base of the rectangle.

the average value for the function  $f(x) = x(x^2 - 2)^{1/3}$  on the interval from  $x=2$  to  $x=5$  and where it occurs.



$$v = x^2 - 2$$

$$\delta v = 2x \delta x$$

$$\int_2^5 x(x^2 - 2)^{1/3} dx = f(c)(5-2)$$

$$\int_2^5 x \cdot (v)^{1/3} \frac{dv}{dx}$$

$$\int_2^5 \frac{1}{2} v^{4/3} dv = f(c) \cdot 3$$

$$\frac{3}{8} v^{4/3} \Big|_2^3$$

$$F(23) - F(2) = f(c) \cdot 3$$

$$23.583 = f(c) \cdot 3$$

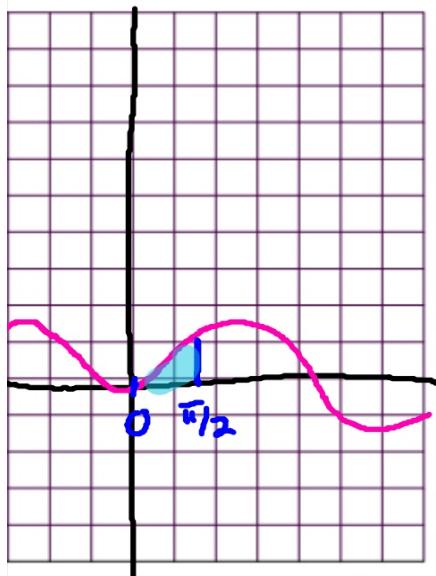
$$7.861 = f(c)$$

$$7.861 = x($$

$$x = 3.5$$

$$(3.566,$$

The mean value for the function  $f(x) = \sin^2 x \cos x$  on the interval  $[0, \pi/2]$  and where



$$U = \sin(x)$$

$$du = \cos(x) dx$$

$$\int_0^{\pi/2} \sin^2(x) \cdot \cos(x) dx = f(c) \cdot (\pi/2 - 0)$$

$$\int_0^{\pi/2} v^2 \cdot \cancel{\cos x} \frac{dv}{\cos x} = f(c) \cdot \pi/2$$

$$\int_0^{\pi/2} v^2 dv = f(c) \cdot \pi/2$$

$$\frac{1}{3} v^3 \Big|_0^1 = f(c) \cdot \pi/2$$

$$\frac{1}{3} = f(c) \cdot \pi/2$$

$$f(c) = .212$$

$$(.516, .212)$$

$$(1.346, .212)$$

ate the mean value and where it occurs for the following functions

4)  $f(x) = 3x$  for  $[1, 3]$

5)  $f(x) = \sin x$  for  $[0, \pi]$

$$f(x) = \frac{1}{x^2} \text{ for } [1, 3]$$

$$④ \int_1^3 3x \, dx = f(c) \cdot 2$$

$$\frac{3}{2} x^2 \Big|_1^3 = f(c) \cdot 2$$

$$12 = f(c) \cdot 2$$

$$6 = f(c)$$

$$\underline{\underline{f = 3x}} \quad (2, 6)$$

$$x = 2$$

$$7) f(x) = \sqrt{x} \text{ for } [0, 1]$$

$$⑤ \left. \begin{aligned} \int_0^\pi \sin(x) \, dx &= f(c) \cdot \pi \\ -\cos(x) &\Big|_0^\pi = f(c) \cdot \pi \\ 2 &= f(c) = \pi \end{aligned} \right\}$$

6.37 =  $f(c)$  (2.691, 6.37)  
6.37 =  $\sin(x)$  (2.451, 6.37)  
 $x = .691, 2.451$

$$f(x) = \frac{1}{x^2} \text{ for } [1, 3]$$

$$\int_1^3 x^{-2} dx = f(c) \cdot 2$$

$$-\frac{1}{x} \Big|_1^3 = f(c) \cdot 2$$

$$2/3 = f(c) \cdot 2$$

$$\underline{f(c) = .333}$$

$$.333 = \frac{1}{x^2}$$

$$x = 1.73$$

$$(1.73, .333)$$

$$7) f(x) = \sqrt{x} \text{ for } [0, 9]$$

$$\int_0^9 x^{1/2} = f(c) \cdot 9$$

$$\frac{2}{3} x^{3/2} \Big|_0^9 = f(c) \cdot 9$$

$$18 = f(c) \cdot 9$$

$$\underline{2 = f(c)}$$

$$2 = \sqrt{x}$$

$$x = 4$$

$$(4, 2)$$