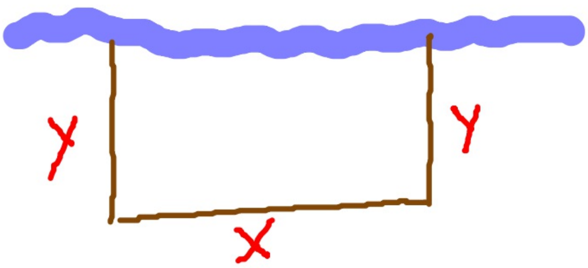
Calculus 1 Name ______________________________

Optimization Notes Block ____________Date ____________

1. A rectangular piece of land is bordered on one side by a river. The other 3 sides are to be enclosed by 300 feet of fencing. What is the maximum area that can be enclosed?



$x + 2y = 300$

$x = 300 - 2y$

$x = 300 - 2(75)$

$x = 150\text{ ft.}$

$A = x \cdot y$

$A = (300 - 2y)y$

$A = 300y - 2y^2$

$A' = 300 - 4y = 0$

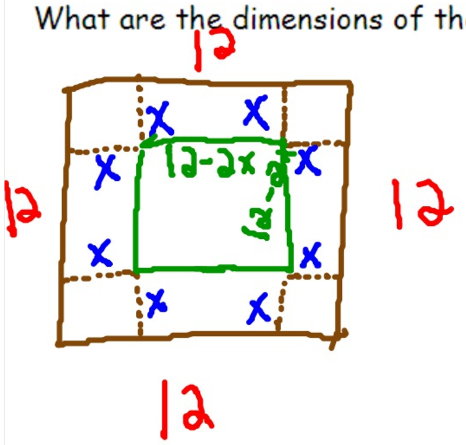
$300 = 4y$

$75\text{ ft} = y$

$A = 150 \cdot 75$

$A = 11{,}250\text{ ft}^2$

2. An open rectangular box is made by cutting 4 congruent squares from the corners of a square piece of cardboard and folding the sides up. The cardboard is 12 feet on each side. What are the dimensions of the box with the maximum volume?



$V = l \cdot w \cdot h$

$V = (12-2x)(12-2x) \cdot x$

$V = (144 - 48x + 4x^2)x$

$V = 144x - 48x^2 + 4x^3$

$V' = 144 - 96x + 12x^2 = 0$

$12(x^2 - 8x + 12) = 0$

$(x-6)(x-2) = 0$

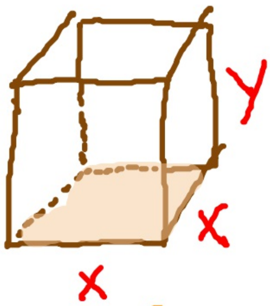
$x = 6, 2$

$V = 0 \cdot 0 \cdot 6 = 0$

$V = 8 \cdot 8 \cdot 2 = 128 ft^3$

$\boxed{8ft \times 8ft \times 2ft}$

3. An open box (no lid) with a square base has a volume of 4 cubic feet. What dimensions will minimize the surface area?



$V = x \cdot x \cdot y = 4$

$V = x^2 y = 4$

$y = \frac{4}{x^2}$

$2^2 y = 4$

$4y = 4$

$y = 1$

$SA = x^2 + 4xy$

$SA = x^2 + \frac{4\cancel{x} \cdot 4}{x^{\cancel{2}}}$

$SA = x^2 + \frac{16}{x}$

$SA = x^2 + 16x^{-1}$

$SA' = 2x - 16x^{-2} = 0$

$2x = \frac{16}{x^2}$

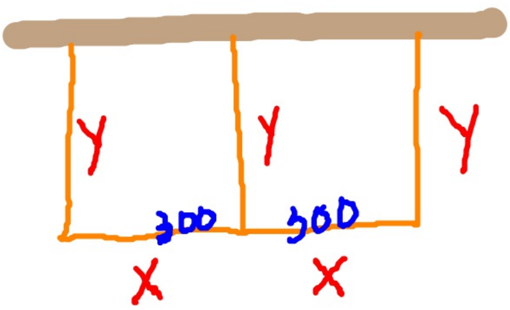
$2x^3 = 16$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

$\boxed{2ft \times 2ft \times 1ft}$

4. You have 1200 feet of fence. Your job is to construct two equal sized boat enclosures next to the dock and to maximize the area of the enclosures with the amount of fence you have. What are the dimensions of the enclosures?



$2x + 3y = 1200$

$3y = -2x + 1200$

$y = -\frac{2}{3}x + 400$

$A = 2x\,y$

$A = 2x\left(-\frac{2}{3}x + 400\right)$

$A = -\frac{4}{3}x^2 + 800x$

$A' = -\frac{8}{3}x + 800 = 0$

$800 = \frac{8}{3}x$

$2400 = 8x$

$300 = x$

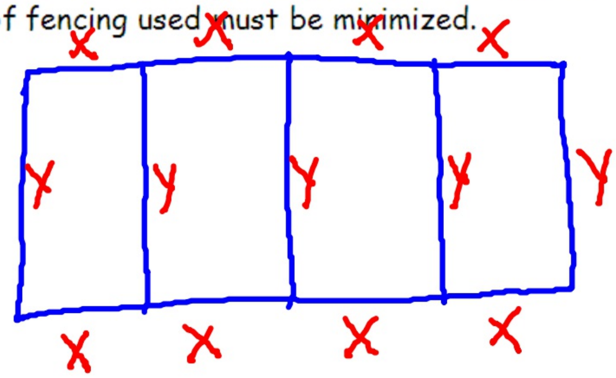
$600 + 3y = 1200$

$3y = 600$

$y = 200$

$\boxed{300\text{ ft} \times 200\text{ ft}}$

5. Four side by side equal sized pens must be constructed using fencing material. Each one is to have 500 square feet of area. Find the dimensions of one of the pens if the amount of fencing used must be minimized.



$$F = 8x + 5y$$

$$F = 8x + 5\left(\frac{500}{x}\right)$$

$$F = 8x + 2500x^{-1}$$

$$F' = 8 - 2500x^{-2} = 0$$

$$8 = \frac{2500}{x^2}$$

$$8x^2 = 2500$$

$$\sqrt{x^2} = \sqrt{312.5}$$

$$x = 17.678\text{ ft}$$

$$A = x \cdot y$$

$$500 = x \cdot y$$

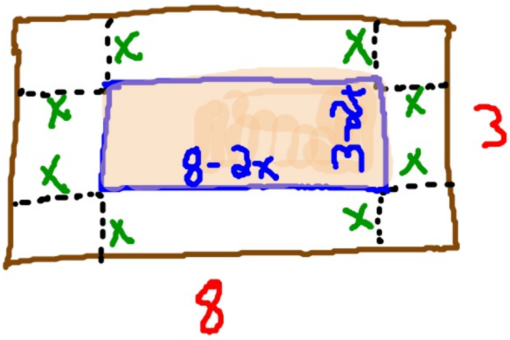
$$y = \frac{500}{x}$$

$$y = \frac{500}{17.678}$$

$$y = 28.284\text{ ft}$$

$$\boxed{17.678\text{ ft} \times 28.284\text{ ft}}$$

6. A cardboard box manufacturing company has to maximize the volume of a box made from an 8ft by 3ft sheet of cardboard. What are the dimensions of the box if the volume is maximized?



$V = l \cdot w \cdot h$

$V = (8-2x)(3-2x) \cdot x$

$V = (24 - 16x - 6x + 4x^2)x$

$V = 4x^3 - 22x^2 + 24x = 0$

$V' = 12x^2 - 44x + 24 = 0$

$4(3x^2 - 11x + 6) = 0$

$x^2 - 11x + 18 = 0$

$(x - \frac{9}{3})(x - \frac{2}{3}) = 0$

$(x-3)(3x-2)$

$x = 3, \ 2/3$

$V = 2 \cdot (-3) \cdot 3 = \cancel{-18}$

$V = \left(\frac{20}{3}\right)\left(\frac{5}{3}\right)\left(\frac{2}{3}\right) = \frac{200}{27}$

$\boxed{\frac{20}{3}\text{ft} \times \frac{5}{3}\text{ft} \times \frac{2}{3}\text{ft}}$