Slope of a Tangent Line – Limit Definition

$$\frac{-f(x_1)}{-x_1} \qquad m_{tan} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \textit{where } h=(x+h)-x$$

definition to find (a) the equation for the slope of any tangent line for each slope of the tangent line at the given value of x, and (c) the equation of the at the given value of *x*.

^{2} at $x=1$

$(x+h)($

x^2+x

x^2

$$\frac{(x+h)^2-3x^2}{h}=\lim_{h\to 0}\frac{3(x^2+2xh+h^2)-3x^2}{h}=\lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3}{h}$$

$$\lim_{h\to 0} 6x+3h=\boxed{6x=m_{tan}}$$

b) $m=6(1)=6$

$$\boxed{y=6x-3}$$

$x^2 + x \quad at \quad x = 2$

$$\frac{(x+h)^2+(x+h)-(x^2+x)}{h} = \lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h-\cancel{x^2}-\cancel{x}}{h} = \lim_{h\to 0}\frac{2x\cancel{h}+\ldots}{\cancel{h}}$$

$$\ldots+1 = \boxed{2x+1=m_{tan}}$$

b.) $m = 2(2)+1 = 5$

$\ldots+b$

$\ldots+b$

$$\boxed{y=5x-4}$$

$\frac{1}{x+1}$ at $x = -5, 2,$ [redacted]

$$\lim_{h\to 0}\frac{\frac{1}{x+h+1}\cdot\frac{(x+1)}{(x+1)} + \frac{-1}{x+1}\cdot\frac{(x+h+1)}{(x+h+1)}}{h} = \lim_{h\to 0}\frac{\frac{x+1}{(x+h+1)(x+1)} + \frac{-1(x+h+1)}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0}$$

$$\cdot\frac{1}{h} = \lim_{h\to 0}\frac{-1}{(x+h+1)(x+1)} = \boxed{\frac{-1}{(x+1)^2} = m_{tan}}$$

$\frac{-1}{16}$

c) $y = mx + b$

$\frac{-1}{4} = \frac{-1}{16}(-5) + b$

$\frac{-1}{4} = \frac{5}{16} + b$

$\frac{-4}{16} - \frac{5}{16} = b$

b) $m = \frac{-1}{(2+1)^2} = \frac{-1}{9}$

$\frac{1}{2+1}$

$y = mx + b$

$\frac{1}{3} = \frac{-1}{9}(2) +$

$\frac{1}{3} = \frac{-2}{9} + b$

$\frac{3}{9} + \frac{2}{9} = b$

$y =$

$x) = x^3 \quad at \quad x = 3$

$$\frac{(x+h)^3 - x^3}{h}$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$\frac{3xh^2 + h^3}{} = \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$(x+h)(x^2 + 2xh + h^2)$

$x^3 + 2x^2h + xh^2 + x^2h +$

b) $m = 3(3)^2 = 27$

c) $y = mx + b$

$27 = 27(3) + b$

$27 = 81 + b$

$-54 = b$

$$\boxed{y = 27x - 54}$$

$^2+1$ at $x=2$

$$\frac{(x+h)^2+1-(x^2+1)}{h} = \lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{+1}-\cancel{x^2}\cancel{-1}}{h} = \lim_{h\to 0}\frac{2xh+}{h}$$

$x+h = \boxed{2x = m_{tan}}$

$=4$

c) $y=mx+b$

$5=4(2)+b$

$5=8+b$

$^-3=b$

$\boxed{y=4x-3}$

$^3-2$ at $x=1$

$$\frac{(x+h)^3-2-(x^3-2)}{h} = \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{2}-\cancel{x^3}+\cancel{2}}{h}$$

$$\frac{3xh^2+h^3}{} = \lim_{h\to 0} 3x^2+3xh+h^2 = \boxed{3x^2 = m_{tan}}$$

3

c) $y=mx+b$

$-1=3(1)+b$

$-1=3+b$

$-4=b$

$\boxed{3x-4}$