Calculus I
Intro to Limits

Name ______________________
Block ________________ Date _

Given $f(x) = \dfrac{\sqrt{x^2+4}-2}{x^2}$ determine the following.

1) f(1) = $\sqrt{5} - 2$

2) f(0) = $\dfrac{0}{0} = U$

To find out what is happening to the graph of a function when we cannot evaluate it at a certain value we use a ___limit___.

Instead of asking what does f(x) equal when x is a certain value,
we ask *what does f(x) approach as x approaches a certain value.*

Limit Notation

$$\lim_{x \to c} f(x) = L$$

One Sided Limits

Left Hand Side: $\lim_{x \to c^-} f(x)$

Right Hand Side: $\lim_{x \to c^+} f(x)$

For a limit to exist:

Left side = Right side

Example 2:

$$\lim_{x \to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \frac{1-1}{1-1} = \frac{0}{0} = \cancel{\text{X}} \quad \frac{2}{3}$$

First, find the left & right hand side limits... if they are equal then the limit exists and is the value you got for each.

$$\lim_{x \to 1^-} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} =$$

$f(0) = \frac{-1}{-1} = 1$

$f(.5) = .7044$

$f(.9) = .6725$

$f(.99) = .667\cancel{3}$

$$\lim_{x \to 1^+} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} =$$

$f(2) =$

$f(1.5)$

$f(1.1)$

$f(1.01)$

You can also find limits by looking at the graph of a function.
Given the graph of f(x), find the following limits if they exist.

a) $\lim_{x \to 1^-} f(x) =$ 1

b) $\lim_{x \to 1^+} f(x) =$ 3

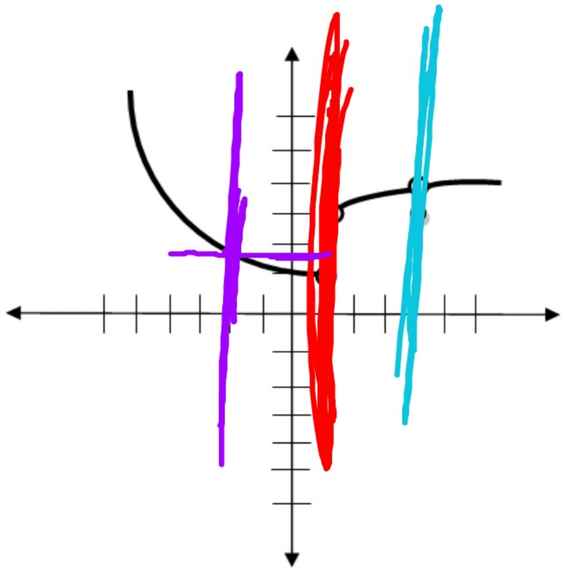
c) $\lim_{x \to 1} f(x) =$ DNE

d) $\lim_{x \to 4^-} f(x) =$ 4

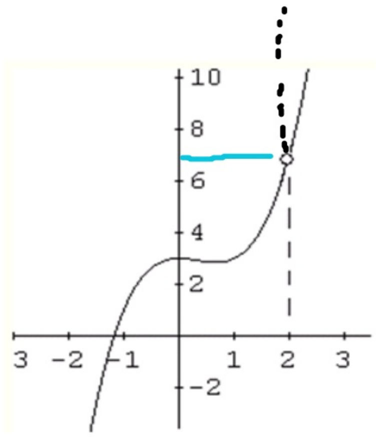
e) $\lim_{x \to 4^+} f(x) =$ 4

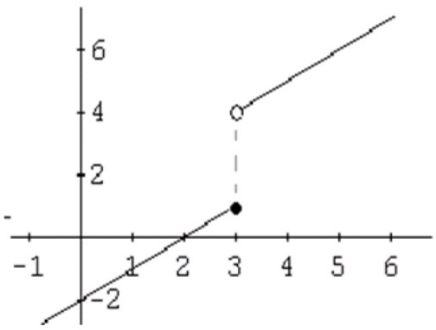
f) $\lim_{x \to 4} f(x) =$ 4

g) ~~$f(4) =$ 3~~

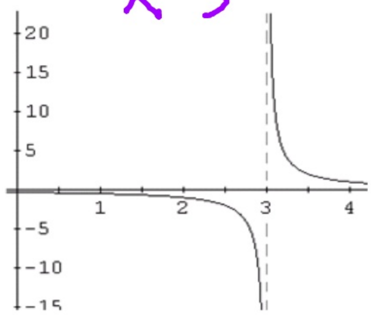


h) $\lim_{x \to -2} f(x) = 1.5$





$$\frac{1}{x-3}$$



$\lim_{x\to 2^-} f(x) = 7$

$\lim_{x\to 2^+} f(x) = 7$

$\lim_{x\to 2} f(x) = 7$

$f(2) =$ DNE

$\lim_{x\to 3^-} g(x) = 1$

$\lim_{x\to 3^+} g(x) = 4$

$\lim_{x\to 3} g(x) =$ DNE

$g(3) = 1$

$\lim_{x\to 3^-} h(x) = -\infty$

$\lim_{x\to 3^+} h(x) = \infty$

$\lim_{x\to 3} h(x) =$ DNE

$h(3) =$ DNE

$$g(x) = \begin{cases} x-2 & x \le 3 \\ x+2 & x > 3 \end{cases}$$

If a graph is continuous you can simply plug in the value to find the limit.

1) $\lim_{x \to 2} 2x + 3x =$

$2(2)+3(2)$

$4+6$

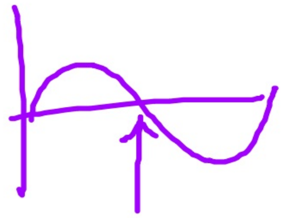
$\boxed{10}$

2) $\lim_{x \to -3} x^2 - 5 =$

$9-5$

$\boxed{4}$

3) $\lim_{\theta \to \pi} \sin\theta =$ $\boxed{0}$



4) $\lim_{x\to 4}\frac{3}{x+2}$

$\frac{3}{6}=\frac{1}{2}$

5) $\lim_{x\to \frac{1}{2}}\left(3x^2-2\right) = =$

$3\cdot\frac{1}{4}-2$

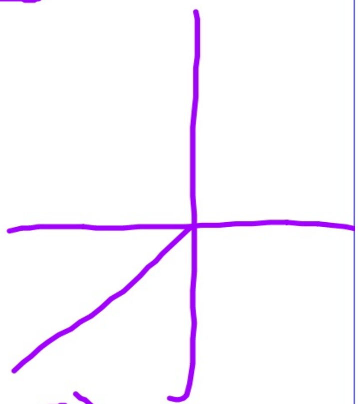
$\frac{3}{4}-2$

$\boxed{-\frac{5}{4}}$

6) $\lim_{\theta\to \frac{5\pi}{4}}\tan\theta =$

$\boxed{1}$

$\tan\left(\frac{5\pi}{4}\right)$



$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

The indeterminate form:

When direct substitution gives you $\frac{0}{0}$, simplify the function algebraically and substitution again.

Algebraic methods for finding limits:

1. Direct substitution

2. Simplify expressions

7) $\lim_{x\to -3} \frac{x^2-9}{x+3} = \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to -3} x-3$

$\boxed{-6}$

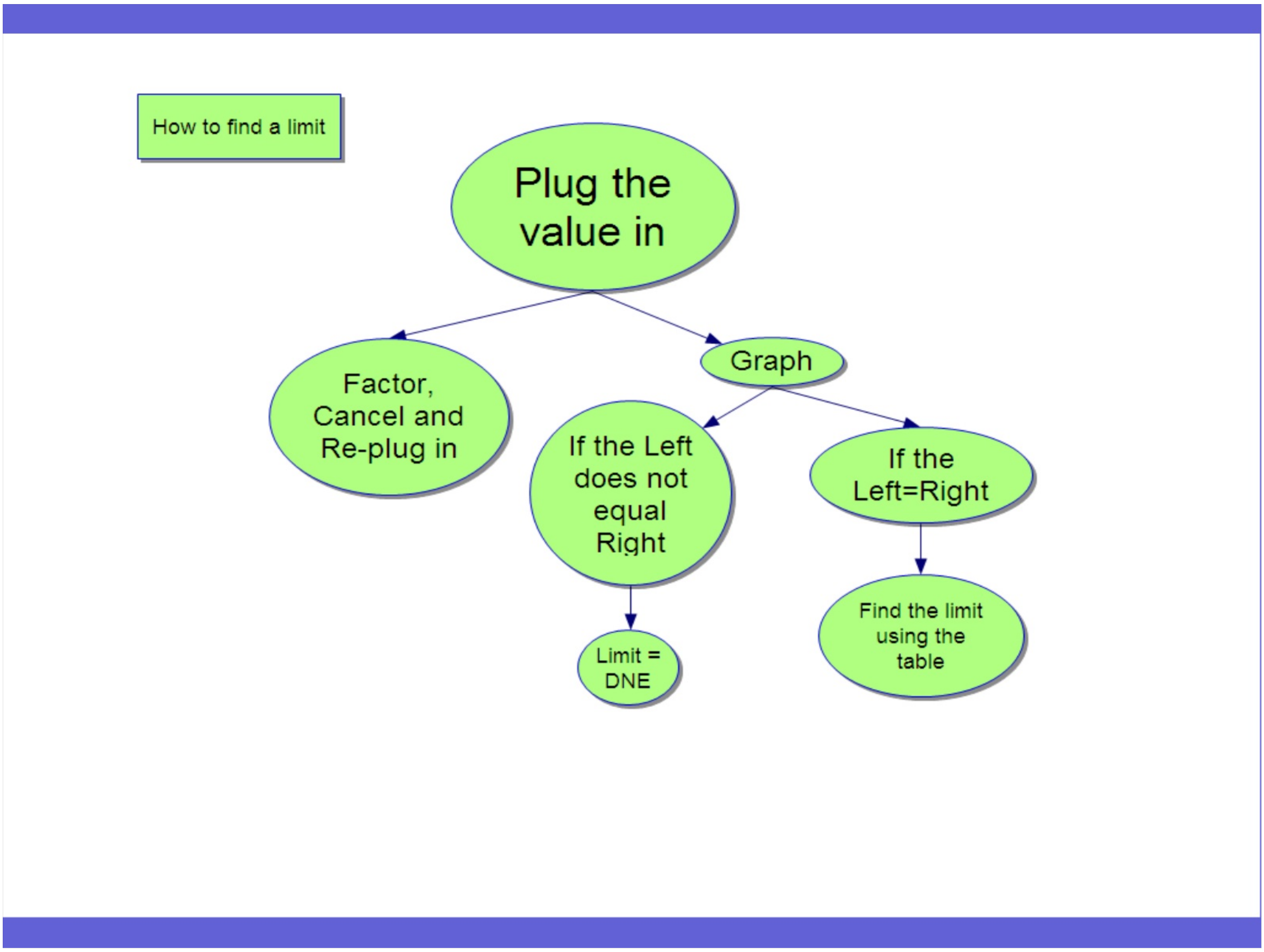
8) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1} = \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$\lim_{x\to 1} \frac{x+2}{x+1}$

$\boxed{\frac{3}{2}}$

9) $\lim_{x\to 4} \frac{x^2-7x+12}{x^2+8x-48} = \frac{\cancel{(x-4)}(x-3)}{\cancel{(x-4)}(x+12)}$

$\lim_{x\to 4} \frac{x-3}{x+12} = \boxed{\frac{1}{16}}$


How to find a limit
Plug the value in
Factor, Cancel and Re-plug in
Graph
If the Left does not equal Right
Limit = DNE
If the Left=Right
Find the limit using the table