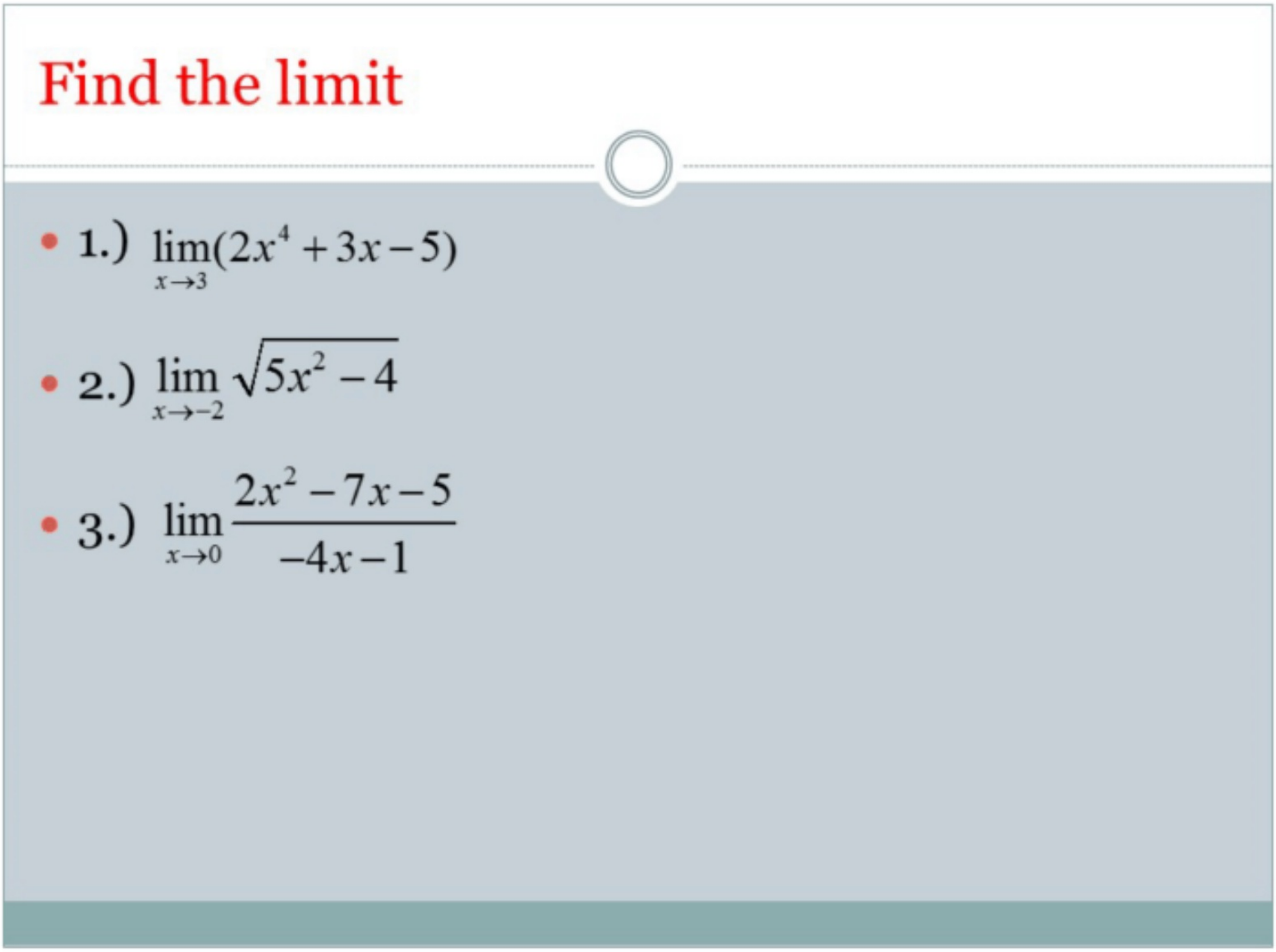
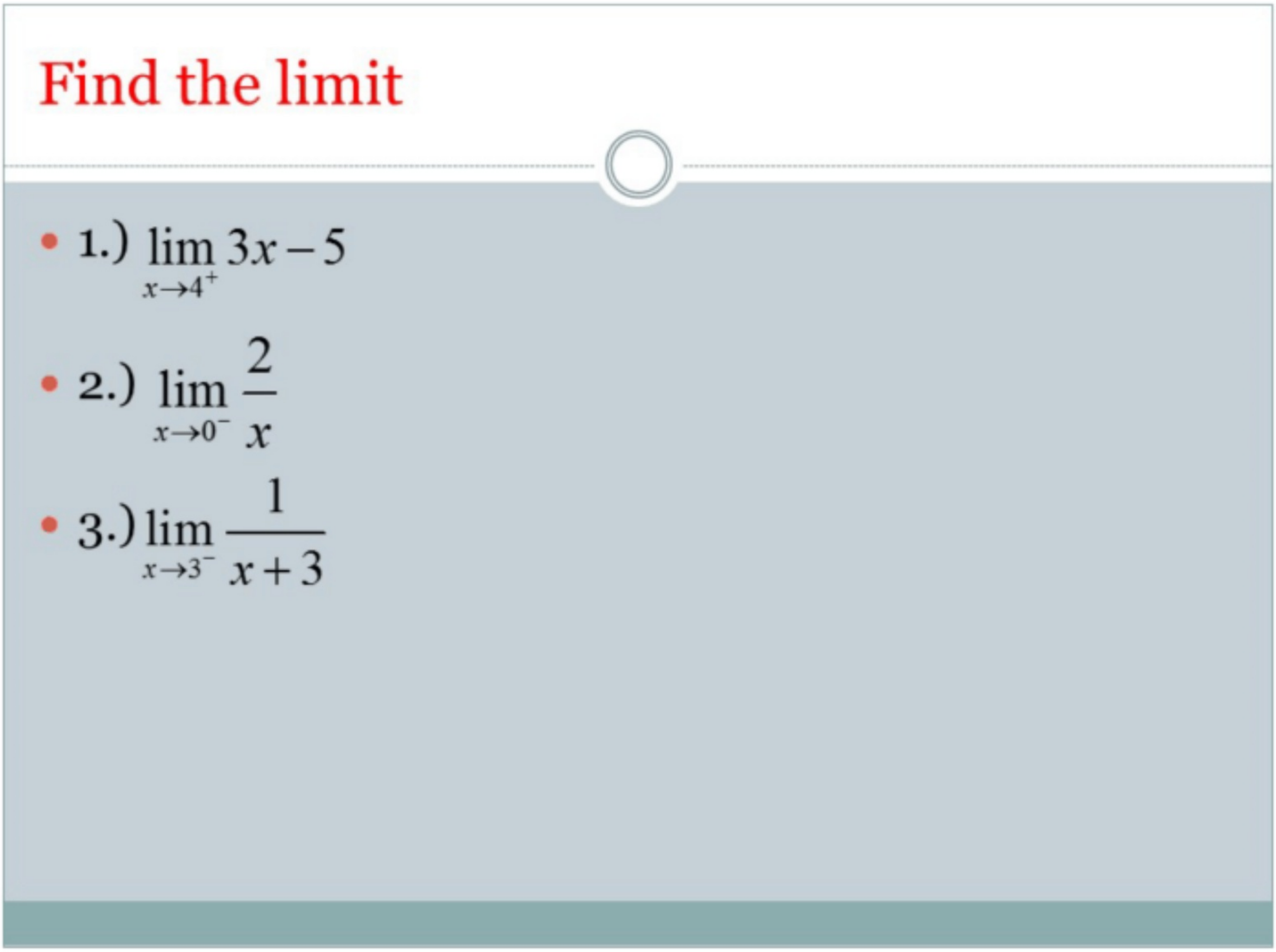
CALCULUS SPIRAL

LIMITS
DERIVATIVES
GRAPHS
INTEGRALS
VOLUME



Find the limit

- 1.) $\lim_{x \to 3}(2x^4 + 3x - 5)$
- 2.) $\lim_{x \to -2} \sqrt{5x^2 - 4}$
- 3.) $\lim_{x \to 0} \frac{2x^2 - 7x - 5}{-4x - 1}$



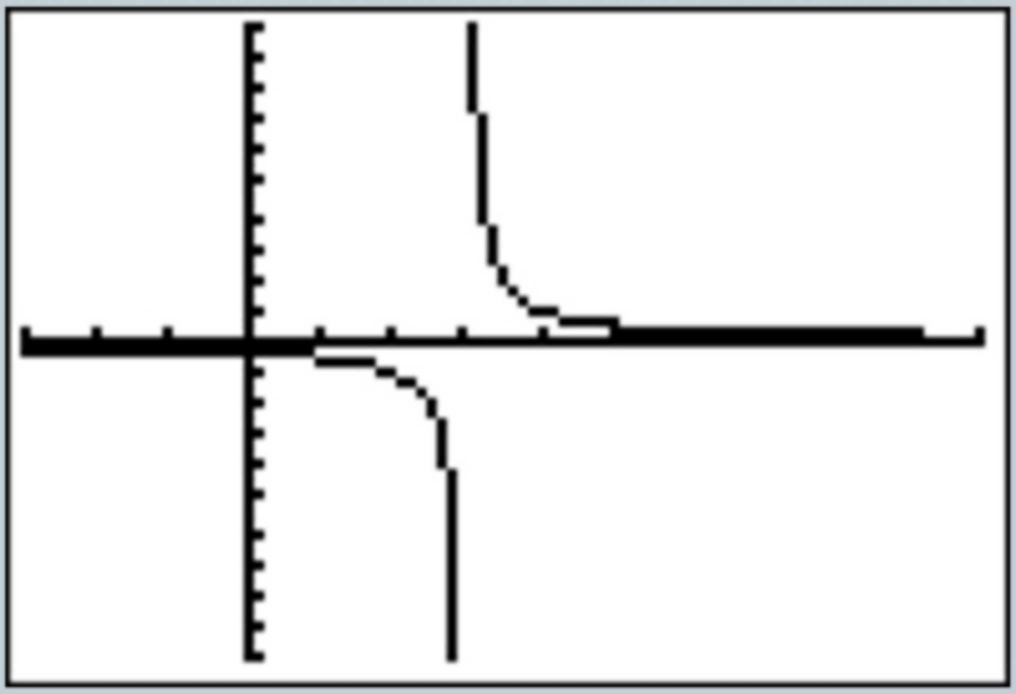
Find the limit

- 1.) $\lim_{x \to 4^+} 3x - 5$
- 2.) $\lim_{x \to 0^-} \frac{2}{x}$
- 3.) $\lim_{x \to 3^-} \frac{1}{x + 3}$

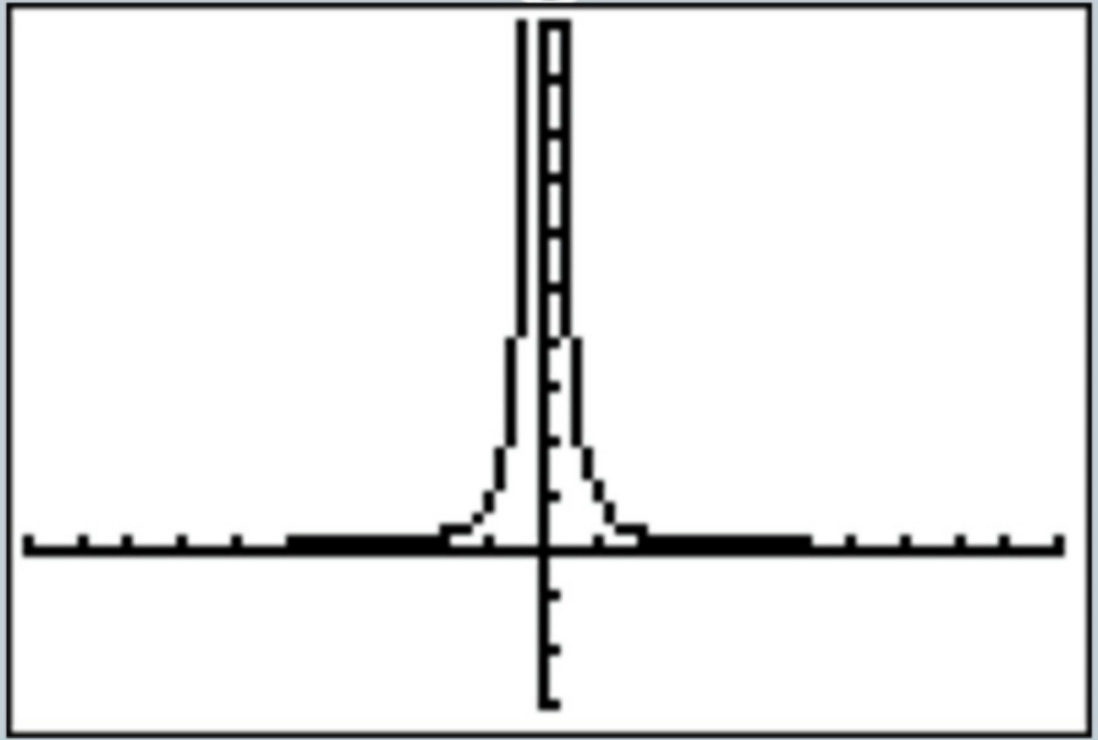
Find the limit

- 1.) $\lim_{x\to\infty}\frac{3x^2-4x+5}{3x+7}$
- 2.) $\lim_{x\to\infty}\frac{5x-x^3}{9-3x^3}$
- 3.) $\lim_{x\to\infty}\frac{-9x^2-6x-1}{3-2x^3}$
- 4.) Do # 1-3 as x approaches -∞

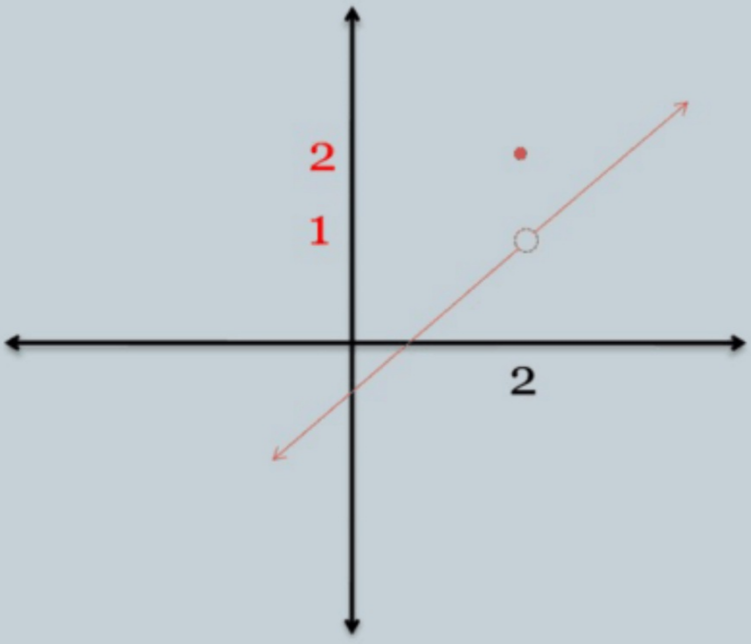
Find the Limit at $x=3^{+}$, 3^{-} , 3

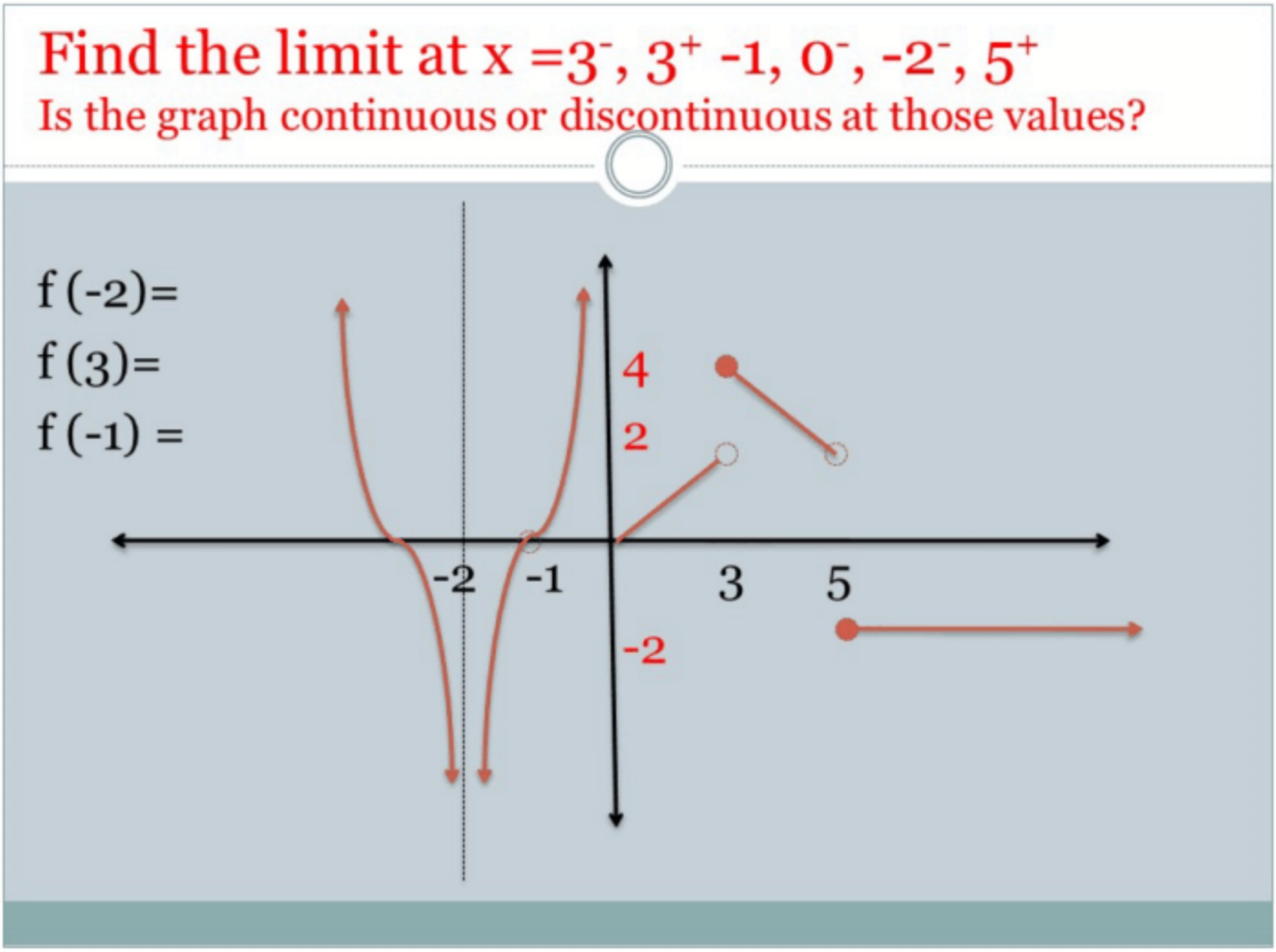


Find the Limit at x=0⁺, 0⁻ , 0



Find the limit at x = 2





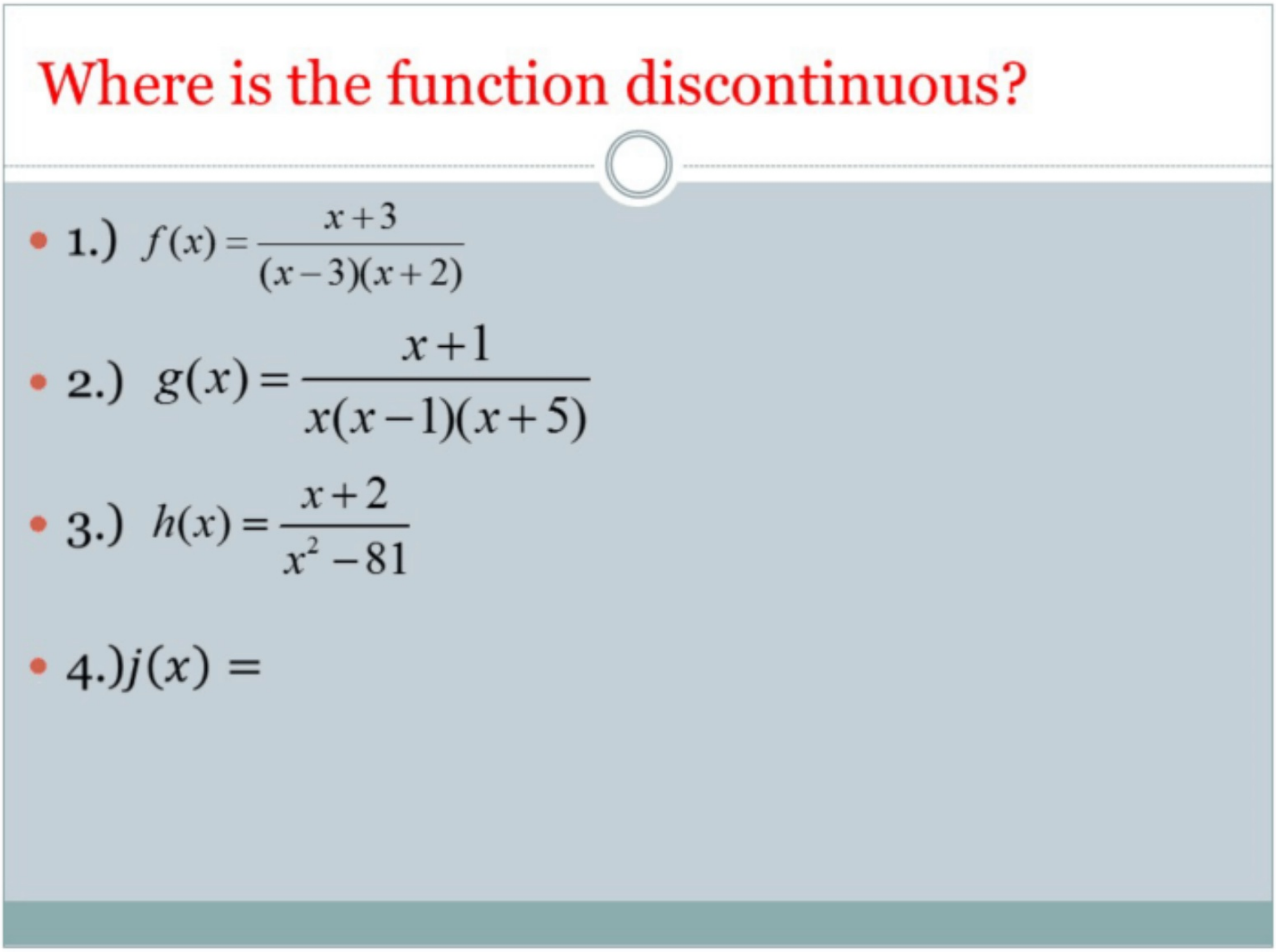
Find the limit at $x = 3^-, 3^+ -1, 0^-, -2^-, 5^+$

Is the graph continuous or discontinuous at those values?

f (-2)=

f (3)=

f (-1) =



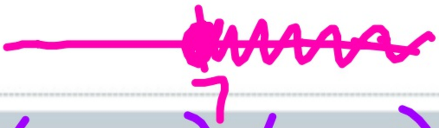
Where is the function discontinuous?

- 1.) $f(x) = \frac{x+3}{(x-3)(x+2)}$
- 2.) $g(x) = \frac{x+1}{x(x-1)(x+5)}$
- 3.) $h(x) = \frac{x+2}{x^2-81}$
- 4.) $j(x) =$

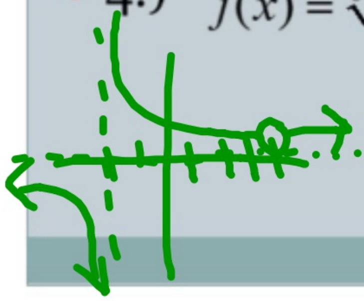
Continuity

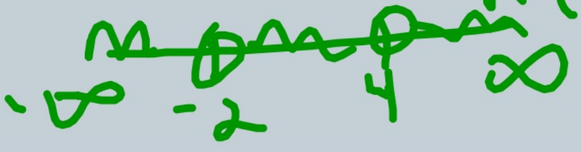
- 1.) Define f(2) in a way that extends $f(x)=\dfrac{x^2-4}{x-2}$ to be continuous at x = 2.
- 2.) Define g(-4) in a way that extends $g(x)=\dfrac{x^2-16}{x+4}$ to be continuous at x = -4.
- 3.) Define h(1) in a way that extends $h(x)=\dfrac{x^2-1}{x-1}$ to be continuous at x = 1.

Find the domain and range

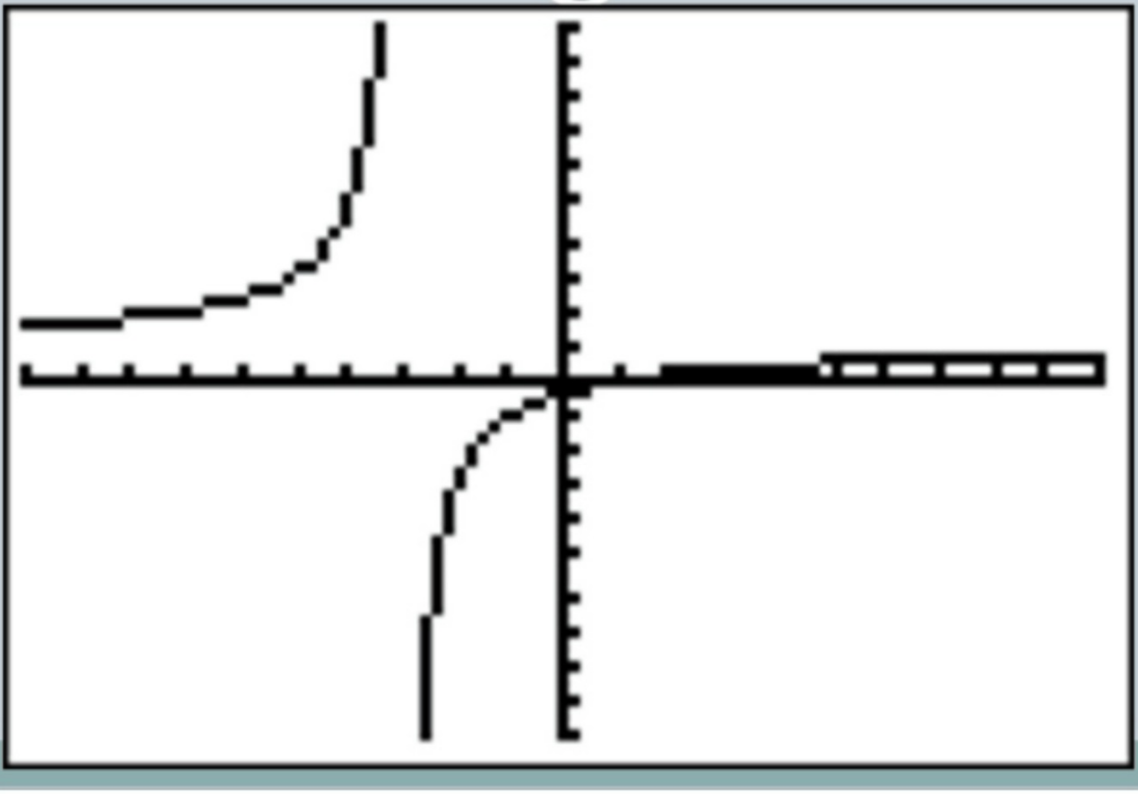


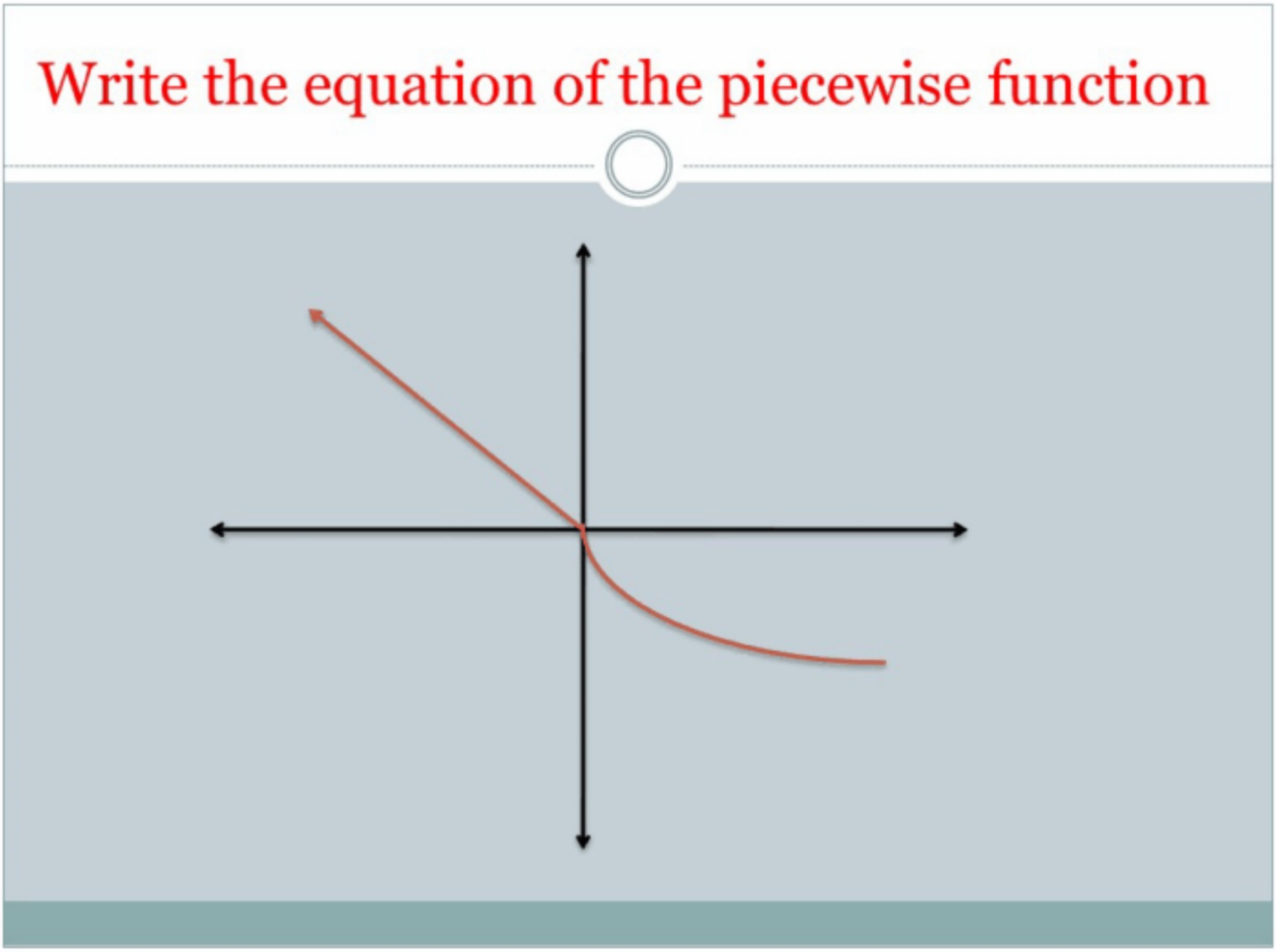
- 1.) $f(x) = \frac{x-3}{x+1} \neq 0$ $x \neq -1$ D: $(-\infty, -1) \cup (-1, \infty)$
  R: $(-\infty, 1) \cup (1, \infty)$
- 2.) $g(x) = \sqrt{x-7} \geq 0 \rightarrow x \geq 7$ D: $[7, \infty)$
  R: $[0, \infty)$
- 3.) $h(x) = \frac{x-4}{x^2-2x-8}$ $\frac{(x-4)}{(x-4)(x+2)} = \frac{1}{x+2}$
  $x \neq 4, -2$ D: $(-\infty, -2) \cup (-2, 4$
  R: $(-\infty, 0) \cup (0, 1/6),$
- 4.) $j(x) = \sqrt[5]{x}$





Find the domain and range

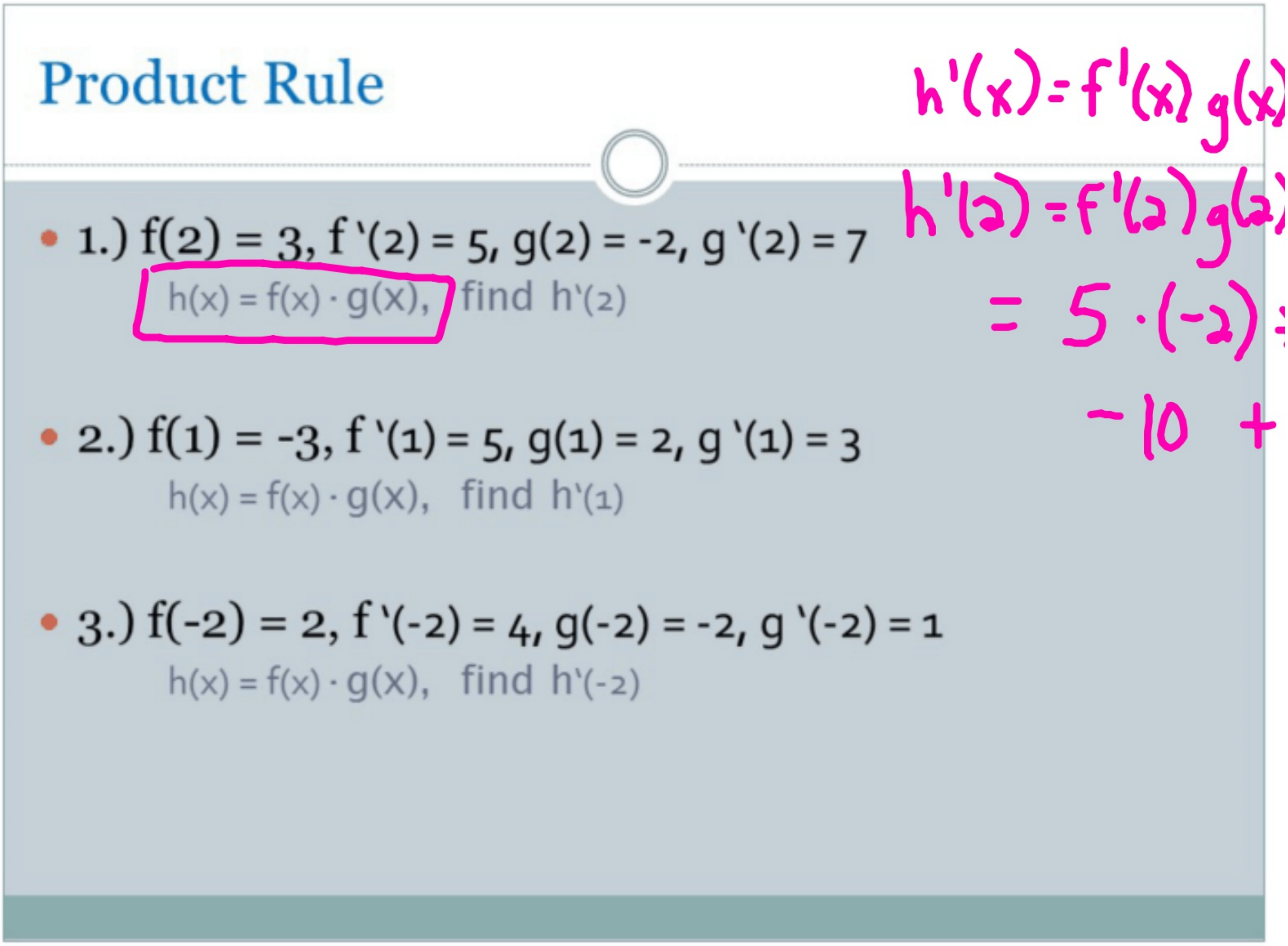




Write the equation of the piecewise function

Power Rule: Find the derivative

- 1.) $f(x) = 3x^3 - 4x^2 + 3x - 10$
- 2.) $g(x) = -2x^4 - 2x^3 + \sin x$
- 3.) $h(x) = \sqrt{x} + \frac{1}{2}x^2 - \frac{1}{x}$



Product Rule

- 1.) $f(2) = 3$, $f'(2) = 5$, $g(2) = -2$, $g'(2) = 7$

  $\boxed{h(x) = f(x) \cdot g(x),}$ find $h'(2)$

- 2.) $f(1) = -3$, $f'(1) = 5$, $g(1) = 2$, $g'(1) = 3$

  $h(x) = f(x) \cdot g(x)$, find $h'(1)$

- 3.) $f(-2) = 2$, $f'(-2) = 4$, $g(-2) = -2$, $g'(-2) = 1$

  $h(x) = f(x) \cdot g(x)$, find $h'(-2)$

h'(x) = f'(x) g(x

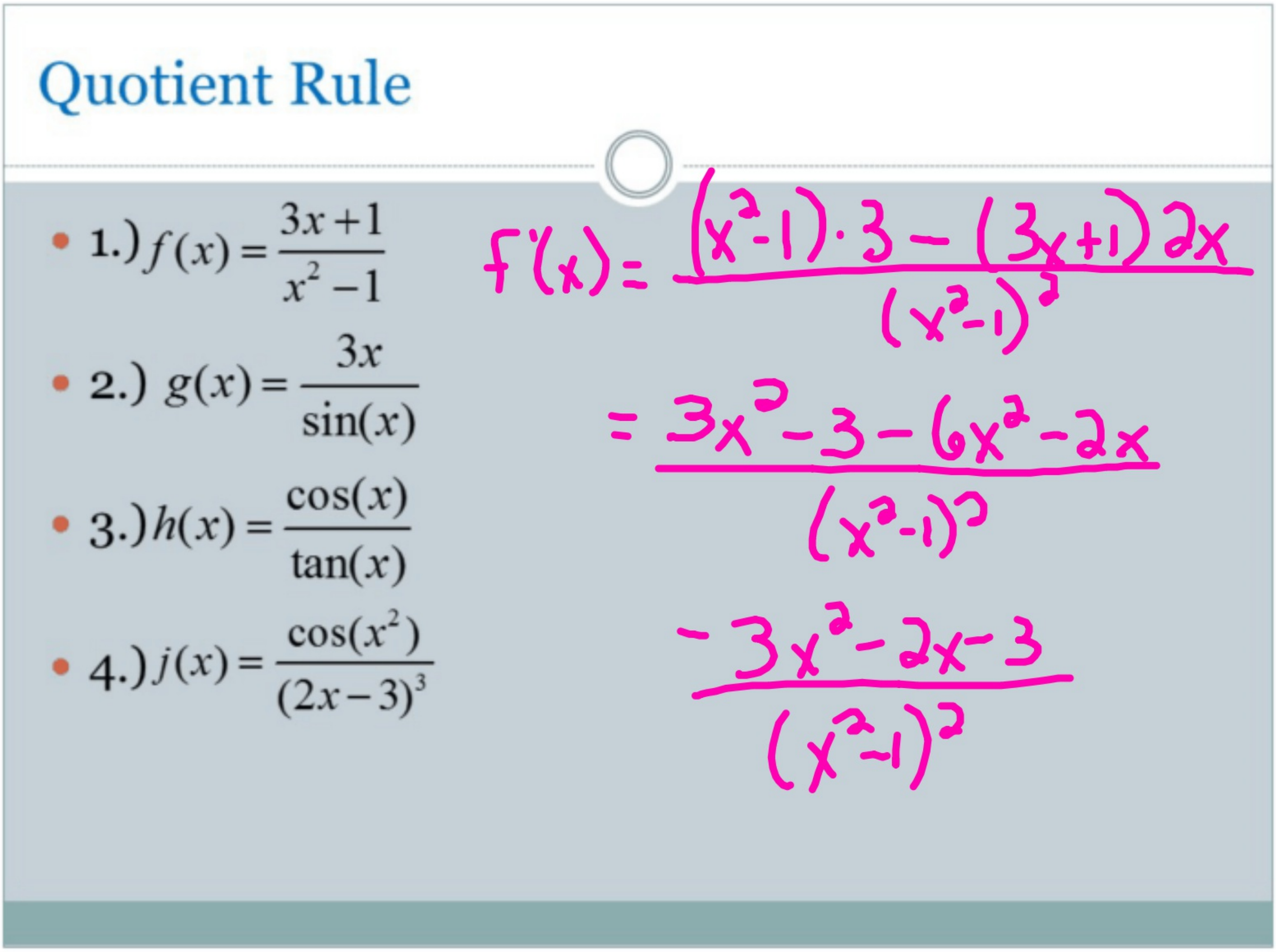
h'(2) = f'(2) g(2

= 5 · (-2)

-10 +

Product Rule: Find the derivative

- 1.) $f(x) = (3x)(x^2 - 4)$
- 2.) $g(x) = -2x\cos(x)$
- 3.) $h(x) = \tan x * \sec x$
- 4.) $j(x) = 3x^2(2x - 4)$
- 5.) $k(x) = \tan x \csc x$
- 6.) $l(x) = 4x^5 \sin(x^2)$



Quotient Rule

- 1.) $f(x)=\dfrac{3x+1}{x^2-1}$
- 2.) $g(x)=\dfrac{3x}{\sin(x)}$
- 3.) $h(x)=\dfrac{\cos(x)}{\tan(x)}$
- 4.) $j(x)=\dfrac{\cos(x^2)}{(2x-3)^3}$

$$f'(x)=\frac{(x^2-1)\cdot 3-(3x+1)2x}{(x^2-1)^2}$$

$$=\frac{3x^2-3-6x^2-2x}{(x^2-1)^2}$$

$$\frac{-3x^2-2x-3}{(x^2-1)^2}$$

Chain Rule

1.) $f(x) = (3x-2)^2$

2.) $g(x) = (-2x+3)^5$

3.) $h(x) = (4x^2-7)^3$

4.) $j(x) = \sin(x^2)$

5.) $k(x) = \cos^4(3x^2)$

6.) $l(x) = \tan(x)\sin(x^3)$

$g'(x) = 5(-2x+3)^4(-2)$

$= -10(-2x+3)^4$

$k'(x) = 4\cos^3(3x^2) \cdot -\sin(3x^2) \cdot$

$= -24x \cos^3(3x^2)\sin(3x$

Definition of a Derivative

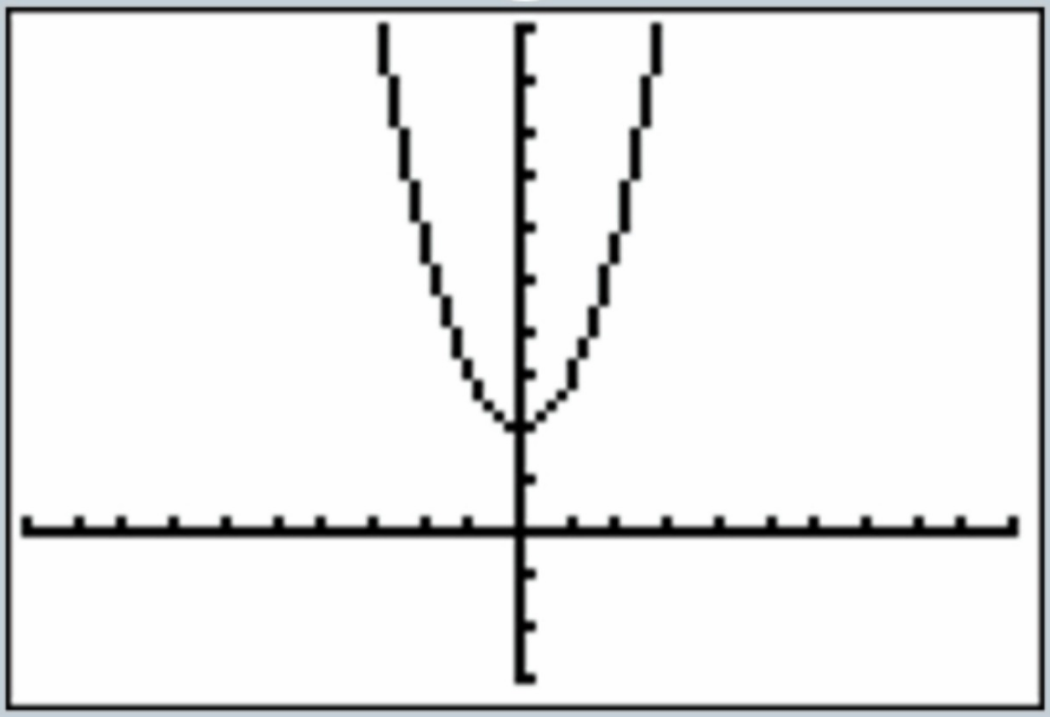
$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

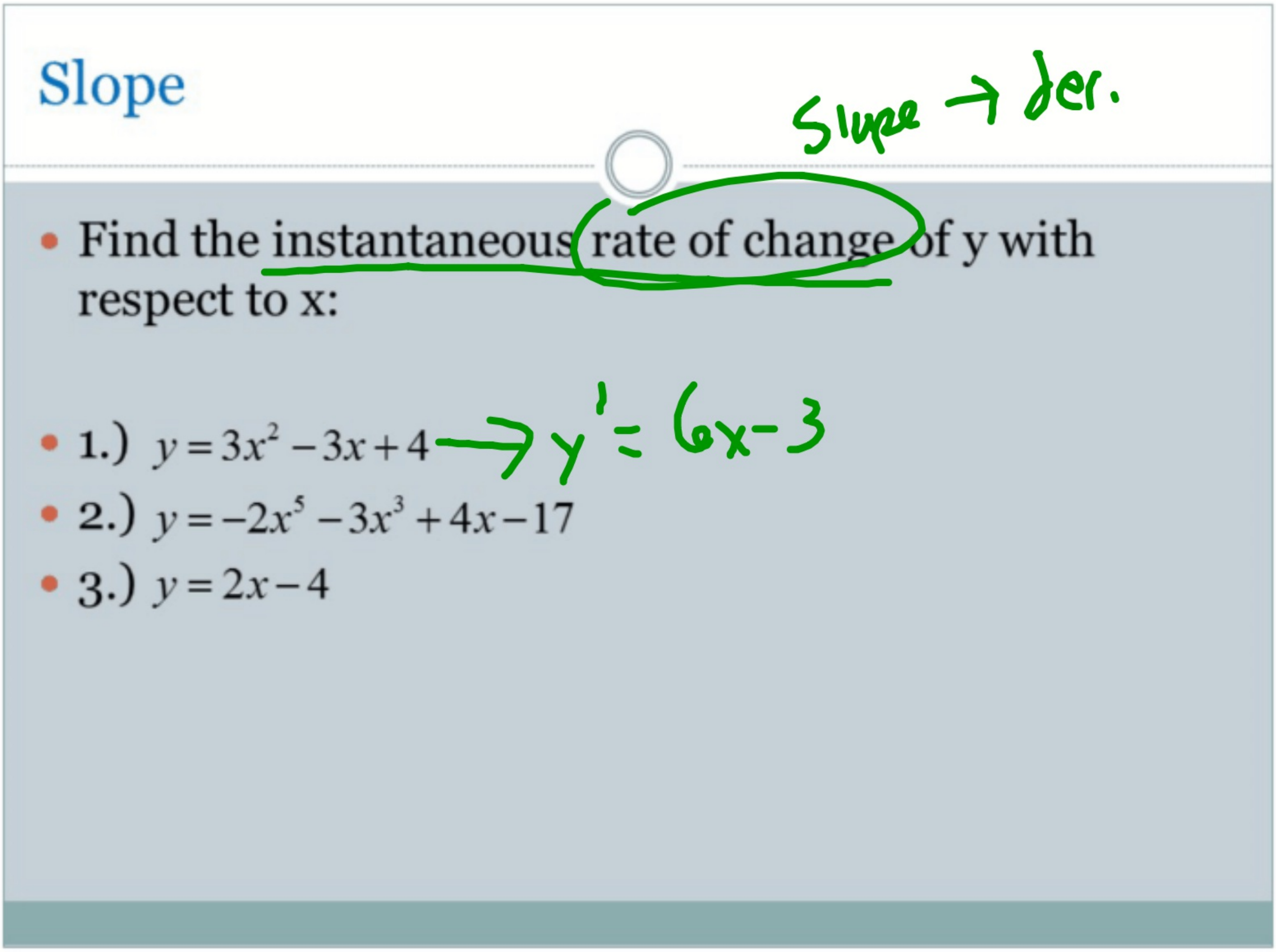
- 1.) $f(x)=3x+5$
- 2.) $g(x)=3x^2-5$
- 3.) $h(x)=3x^2-4x$

$$\lim_{h \to 0} \frac{3(x+h)^2-5-(3x^2-5)}{h}$$

$$\lim_{h \to 0} \frac{3(x+h)^2-4(x+h)-(3x^2-4x)}{h}$$

Slope- describe the slope on the graph

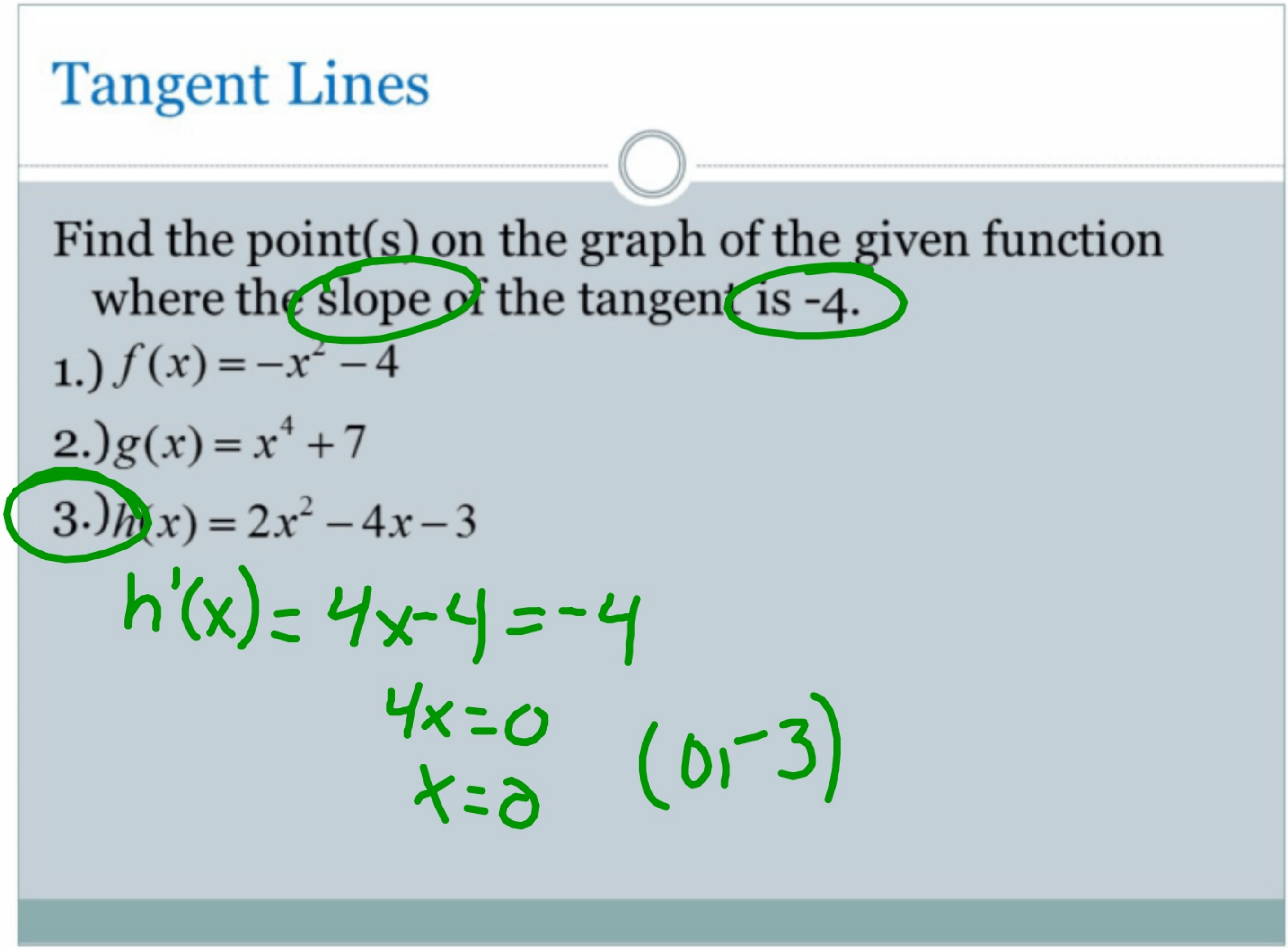




Slope

Slope → der.

- Find the instantaneous rate of change of y with respect to x:
- 1.) $y=3x^2-3x+4 \rightarrow y'=6x-3$
- 2.) $y=-2x^5-3x^3+4x-17$
- 3.) $y=2x-4$



Tangent Lines

Find the point(s) on the graph of the given function where the slope of the tangent is -4.

1.) $f(x) = -x^2 - 4$

2.) $g(x) = x^4 + 7$

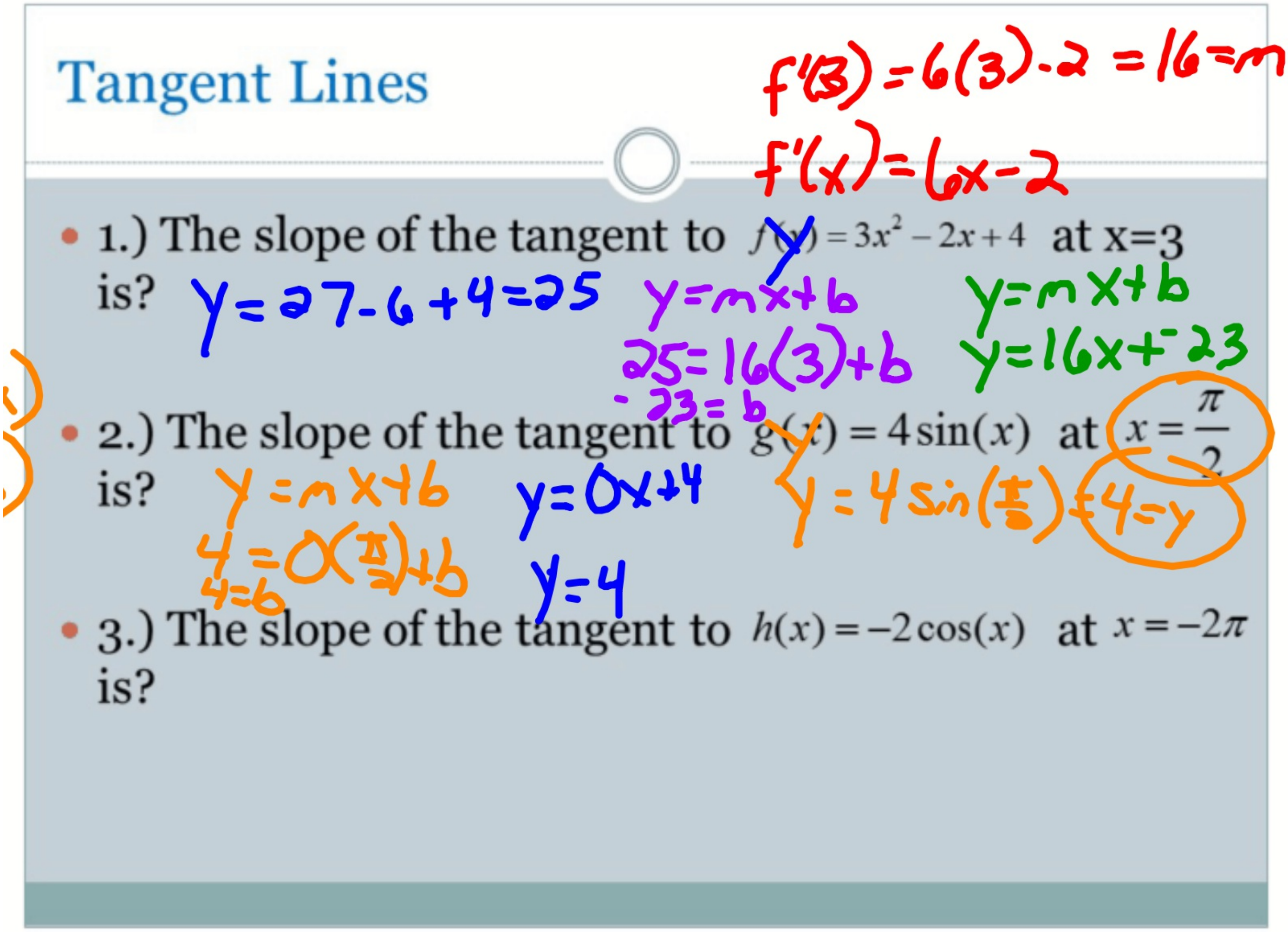
3.) $h(x) = 2x^2 - 4x - 3$

$h'(x) = 4x - 4 = -4$

$4x = 0$

$x = 0$

$(0, -3)$



Tangent Lines

$f'(3) = 6(3) - 2 = 16 = m$

$f'(x) = 6x - 2$

- 1.) The slope of the tangent to $f(x) = 3x^2 - 2x + 4$ at x=3 is?

  $y = 27 - 6 + 4 = 25$

  $y = mx + b$

  $25 = 16(3) + b$

  $-23 = b$

  $y = mx + b$

  $y = 16x + ^{-}23$

- 2.) The slope of the tangent to $g(x) = 4\sin(x)$ at $x = \frac{\pi}{2}$ is?

  $y = mx + b$

  $4 = 0(\frac{\pi}{2}) + b$

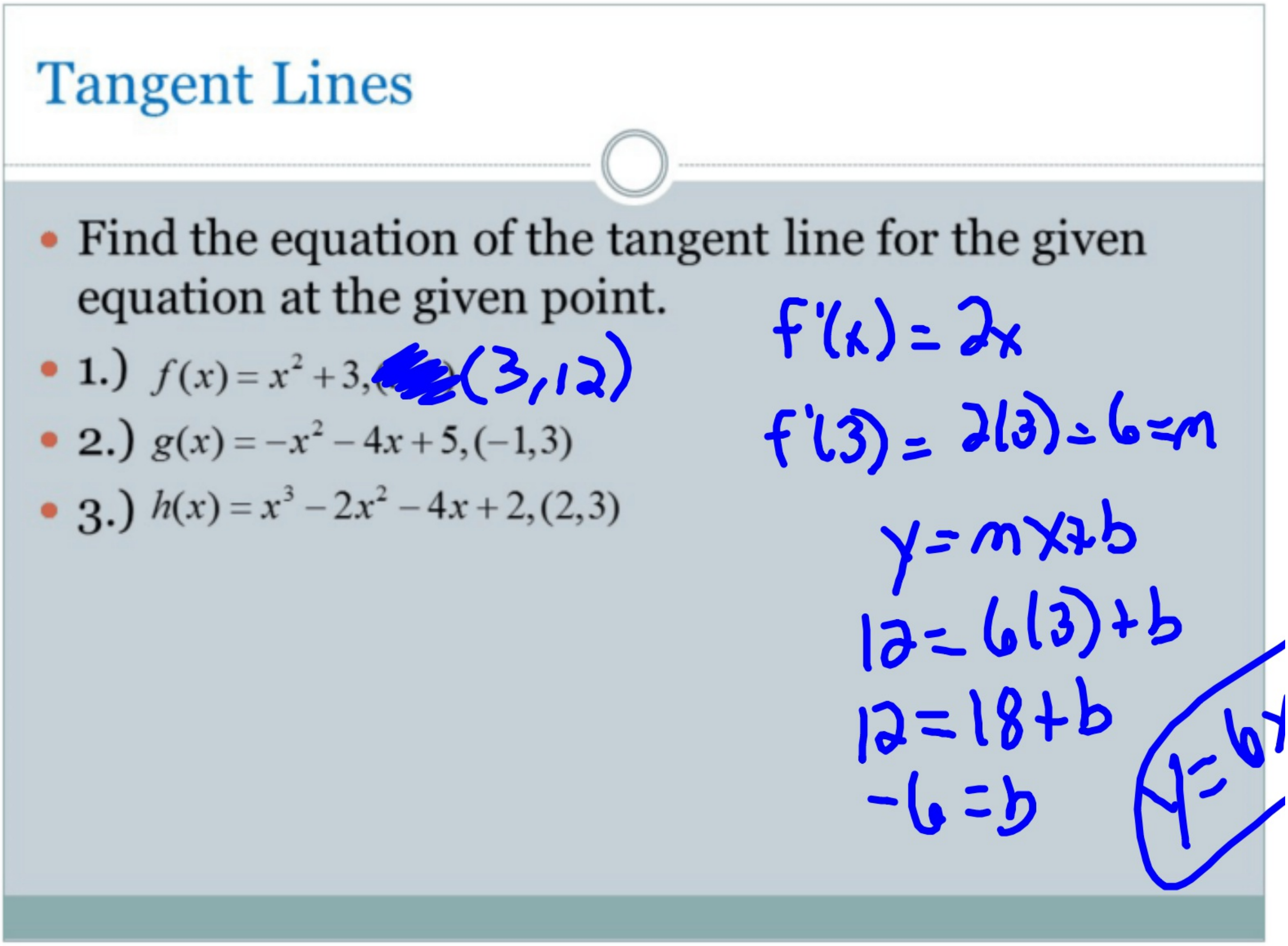
  $4 = b$

  $y = 0x + 4$

  $y = 4$

  $y = 4\sin(\frac{\pi}{2}) = 4 = y$

- 3.) The slope of the tangent to $h(x) = -2\cos(x)$ at $x = -2\pi$ is?



Tangent Lines

- Find the equation of the tangent line for the given equation at the given point.
- 1.) $f(x) = x^2 + 3$, [redacted] $(3,12)$
- 2.) $g(x) = -x^2 - 4x + 5, (-1,3)$
- 3.) $h(x) = x^3 - 2x^2 - 4x + 2, (2,3)$

$f'(x) = 2x$

$f'(3) = 2(3) = 6 = m$

$y = mx+b$

$12 = 6(3) + b$

$12 = 18 + b$

$-6 = b$

$\boxed{y = 6x}$

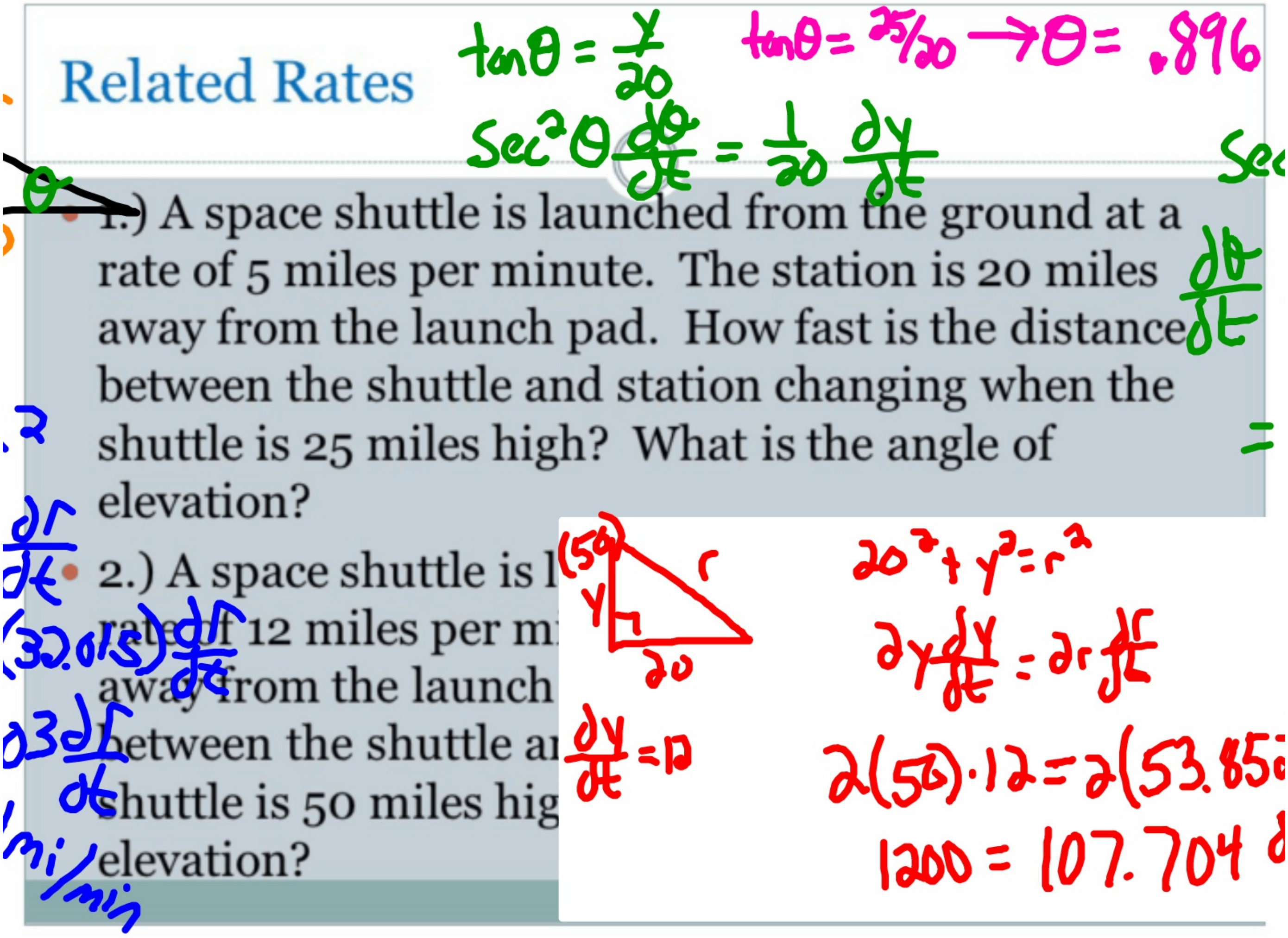
Tangent Lines

- Find all the points on the graph where there is a horizontal tangent line:
- 1.) $f(x) = x^2 - 6$
- 2.) $g(x) = x^3 - 4x^2 + 3$
- 3.) $h(x) = x^4 - x^3 - x^2 + 7$

0

$f'(x) = 2x = 0$

$x = 0 \quad (0, -6)$



Related Rates

$\tan\theta = \frac{y}{20}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{20}\frac{dy}{dt}$

$\tan\theta = 25/20 \rightarrow \theta = .896$

- 1.) A space shuttle is launched from the ground at a rate of 5 miles per minute. The station is 20 miles away from the launch pad. How fast is the distance between the shuttle and station changing when the shuttle is 25 miles high? What is the angle of elevation?
- 2.) A space shuttle is l
rate of 12 miles per mi
away from the launch
between the shuttle an
shuttle is 50 miles hig
elevation?

$\frac{dy}{dt} = 12$

$20^2 + y^2 = r^2$

$2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$2(50)\cdot 12 = 2(53.85$

$1200 = 107.704$

$\frac{dr}{dt}$

$(32.015)\frac{dr}{dt}$

$03\frac{dr}{dt}$

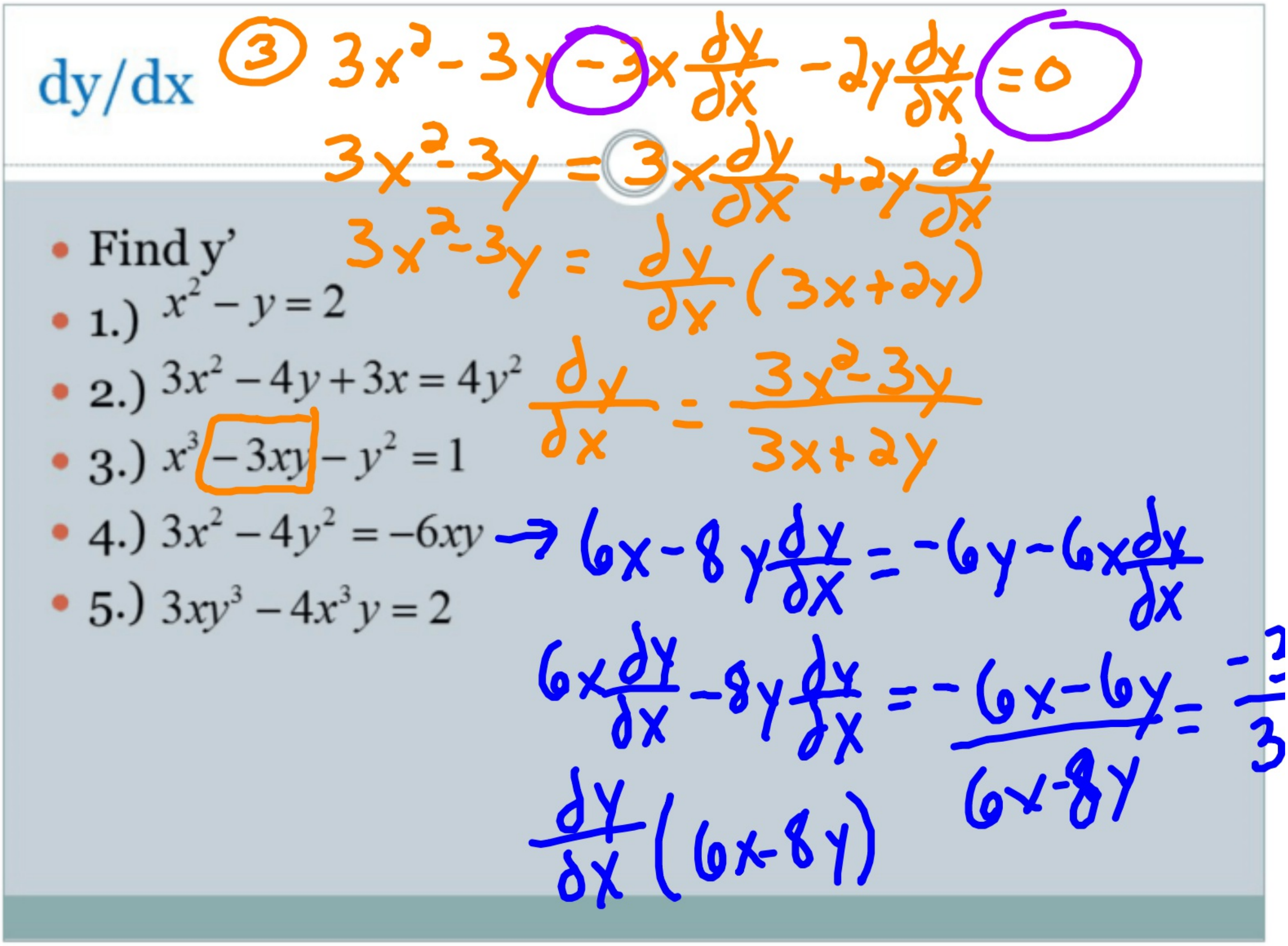
mi/min

$\frac{d\theta}{dt}$

Related Rates

- 1.) A snow ball rolling down a hill is increasing at a rate of 5 cubic cm/min. What is the rate of change of the radius when the radius is 13cm? What is the rate of change of the surface area at the same instant?

- 2.) A snow ball is melting at a rate of 5 cubic cm/min. What is the rate of change of the radius when the radius is 13cm? What is the rate of change of the surface area at the same instant?



dy/dx

- Find y'
- 1.) $x^2 - y = 2$
- 2.) $3x^2 - 4y + 3x = 4y^2$
- 3.) $x^3 \boxed{-3xy} - y^2 = 1$
- 4.) $3x^2 - 4y^2 = -6xy$ →
- 5.) $3xy^3 - 4x^3y = 2$

③ $3x^2 - 3y - 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$3x^2 - 3y = 3x\frac{dy}{dx} + 2y\frac{dy}{dx}$

$3x^2 - 3y = \frac{dy}{dx}(3x+2y)$

$\frac{dy}{dx} = \frac{3x^2-3y}{3x+2y}$

$6x - 8y\frac{dy}{dx} = -6y - 6x\frac{dy}{dx}$

$6x\frac{dy}{dx} - 8y\frac{dy}{dx} = \frac{-6x-6y}{6x-8y} = \frac{-}{3}$

$\frac{dy}{dx}(6x-8y)$

PVA

- 1.) The position for an object is given by $s(t) = t^2 - 4t + 5$ Find the velocity and acceleration when t=3.

- 2.) The position for an object is given by $s(t) = -t^3 - 4t^2 + 7$ Find the velocity and acceleration when t=3.

- 3.) The position for an object is given by $s(t) = \frac{7}{2}t^2 - 4t + 1$ Find the velocity and acceleration when t=3.

PVA

- 1.) The position for an object is given by $s(t) = t^2 - 4t + 5$ When does the object change positions?

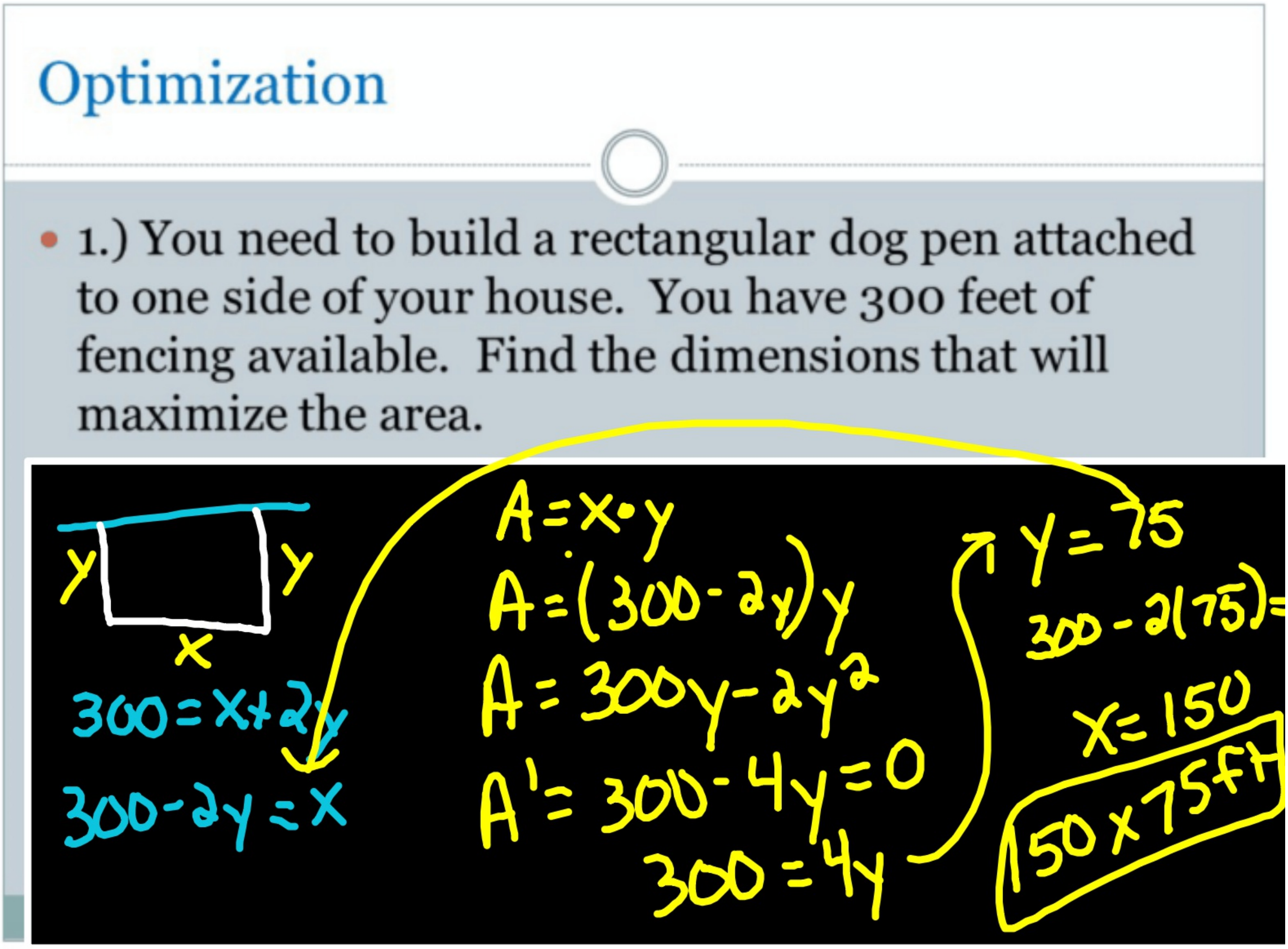
- 2.) The position for an object is given by $s(t) = -t^3 - 4t^2 + 7$ When does the object change positions?

- 3.) The position for an object is given by $s(t) = \frac{7}{2}t^2 - 4t + 1$ When does the object change positions?

PVA

- 1.) A projectile is thrown upward from a 310 foot building with an initial velocity of 15 m/sec. When does it reach its maximum? What is the velocity when it hits the ground?

- 2.) A projectile is dropped from a 310 foot building with an initial velocity of 15 m/sec. What is the velocity when it hits the ground? What is its velocity when it is half way to the ground?



Optimization

- 1.) You need to build a rectangular dog pen attached to one side of your house. You have 300 feet of fencing available. Find the dimensions that will maximize the area.

$300 = x + 2y$

$300 - 2y = x$

$A = x \cdot y$

$A = (300 - 2y)y$

$A = 300y - 2y^2$

$A' = 300 - 4y = 0$

$300 = 4y$

$y = 75$

$300 - 2(75) =$

$x = 150$

$\boxed{150 \times 75 \text{ ft}}$

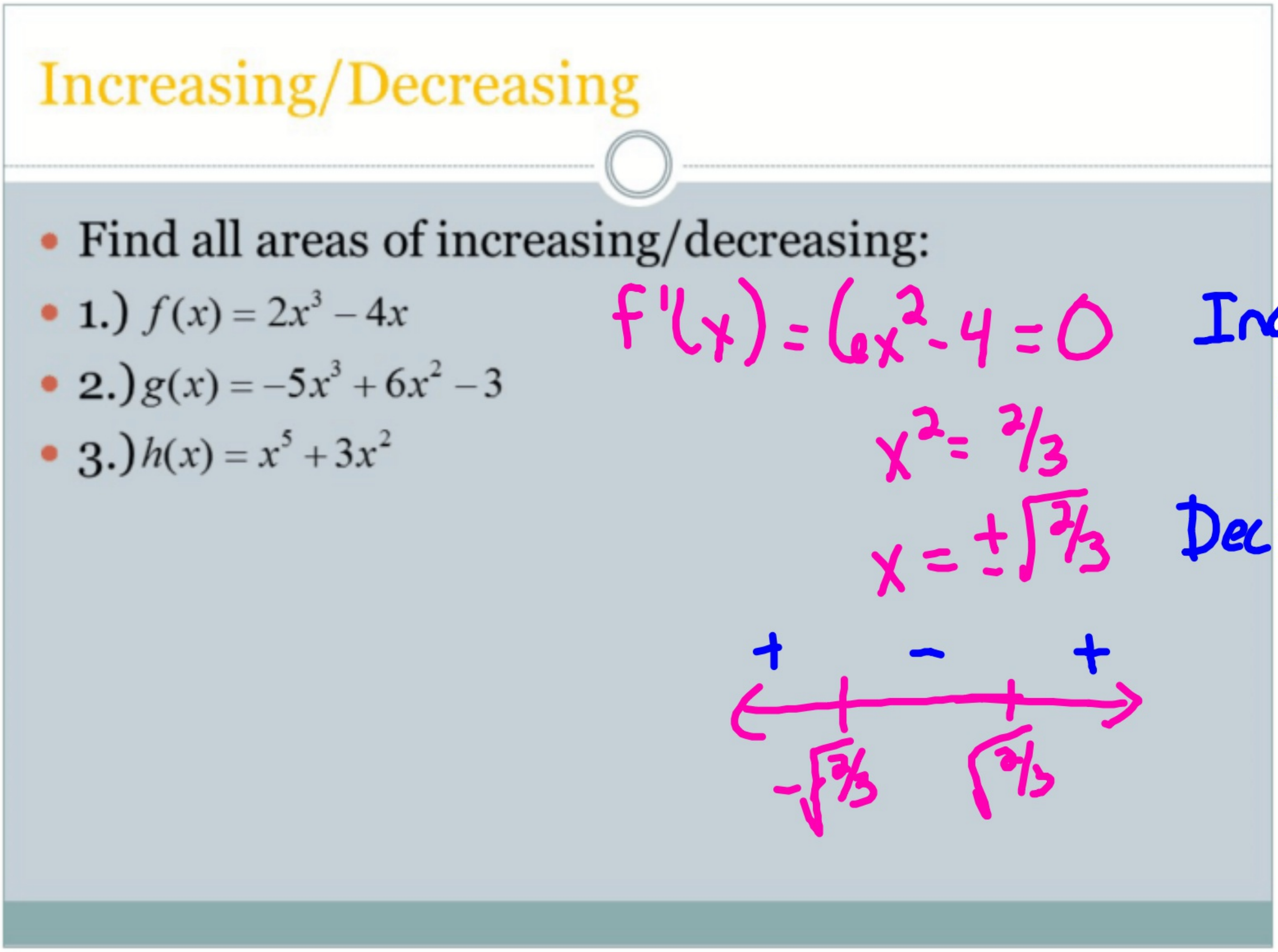
Min/Max

- Find the values of x that give the relative extrema:
- 1.) $f(x) = 2x^3 - 4x$
- 2.) $g(x) = -5x^3 + 6x^2 - 3$
- 3.) $h(x) = x^5 + 3x^2$

$f'(x) = 6x^2 - 4 = 0$

$x^2 = 2/3$

$x = \pm\sqrt{2/3}$



Increasing/Decreasing

- Find all areas of increasing/decreasing:
- 1.) $f(x) = 2x^3 - 4x$
- 2.) $g(x) = -5x^3 + 6x^2 - 3$
- 3.) $h(x) = x^5 + 3x^2$

$f'(x) = 6x^2 - 4 = 0$

$x^2 = 2/3$

$x = \pm\sqrt{2/3}$

$$\begin{array}{ccccc} + & & - & & + \\ \longleftarrow & \vert & \longrightarrow & \vert & \longrightarrow \\ & -\sqrt{2/3} & & \sqrt{2/3} & \end{array}$$

Inc

(

Dec:

Concavity

- Determine the intervals of concavity:
- 1.) $f(x) = 2x^3 - 4x$
- 2.) $g(x) = -5x^3 + 6x^2 - 3$
- 3.) $h(x) = x^5 + 3x^2$

$f'(x) = 6x^2 - 4$

$f''(x) = 12x = 0$ → POI

$x = 0$

− +

0

CU: (0,

CD: (-∞

Points of Inflection

- Find the points of inflection:
- 1.) $f(x) = 2x^3 - 4x$
- 2.) $g(x) = -5x^3 + 6x^2 - 3$
- 3.) $h(x) = x^5 + 3x^2$

Mean Value Theorem

- Determine if the mean value theorem applies in the given range.

1.) $3x^2 - 4x + 1 \ldots\ldots\ldots\ldots [-3,1]$

2.) $\dfrac{3}{x+2} \ldots\ldots\ldots\ldots\ldots\ldots [-3,1]$

3.) $\dfrac{3}{x-2} \ldots\ldots\ldots\ldots\ldots\ldots [2,4]$

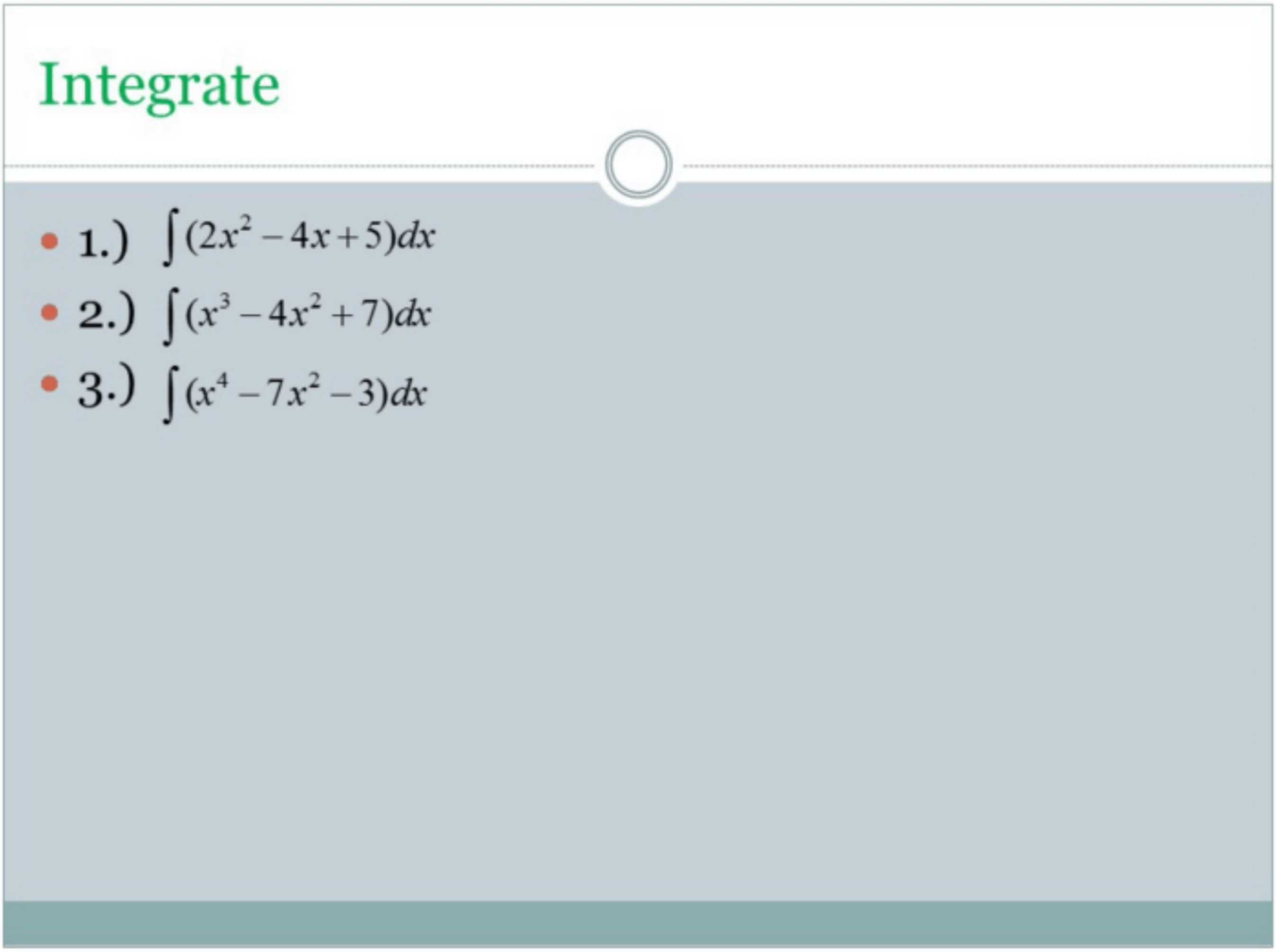
Min/Max Existence Theorem

- Use the Min/max existence theorem to find the min and max in the given range.

$$1.)3x^2-4x+1\ldots\ldots\ldots\ldots\ldots[-3,1]$$

$$2.)\frac{3}{x+2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots[-3,1]$$

$$3.)\frac{3}{x+2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots[2,4]$$



Integrate

- 1.) $\int (2x^2 - 4x + 5)dx$
- 2.) $\int (x^3 - 4x^2 + 7)dx$
- 3.) $\int (x^4 - 7x^2 - 3)dx$

Integrate

1.) $\int \frac{x^2 + 4x}{x} dx$

2.) $\int \frac{x^5 - x^3}{x^2} dx$

3.) $\int \frac{x^3 - 5x^2}{x^{\frac{1}{2}}} dx$

Integrate

1.) $\int \sqrt[3]{x^2}\,dx$

2.) $\int \sqrt{x}\,dx$

3.) $\int (\sqrt[4]{x^3} + 2x)dx$

4.) $\int (\sqrt[3]{x^5} + 2x^2)dx$

Integrate

$1.)\int \sin(x)dx$

$2.)\int -\cos^2(x)dx$

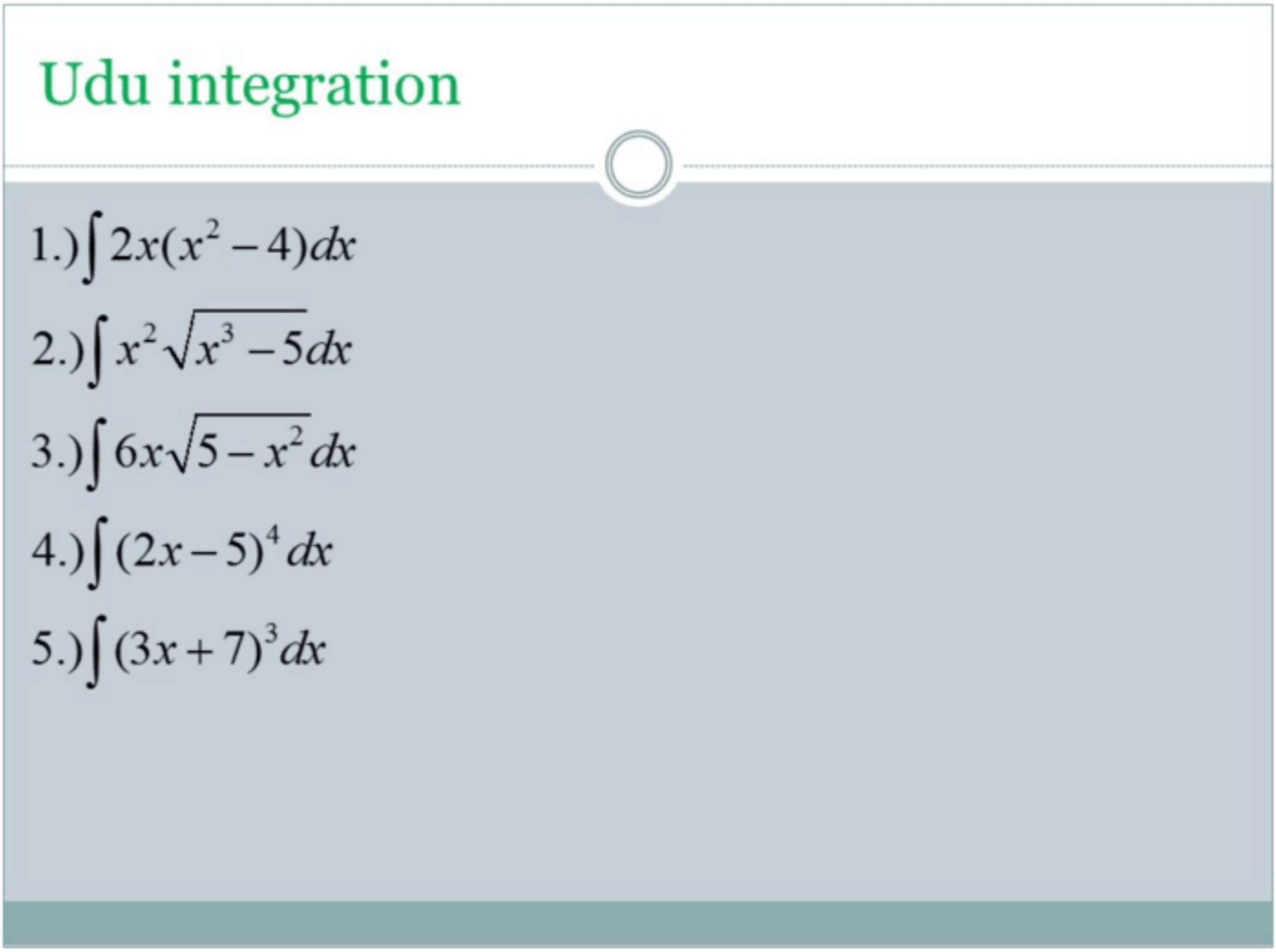
$3.)\int 5\sec(x)\tan(x)dx$

Solve for the Constant of Integration

$1.)\, f'(x) = \int (2x - 7)dx.......(1,3)$

$2.)\, g'(x) = \int \sqrt[3]{x}dx..........(2,-3)$

$3.)\, h'(x) = \int x(x^2 + 1)dx.......(1,1)$



Udu integration

$1.)\int 2x(x^2-4)dx$

$2.)\int x^2\sqrt{x^3-5}dx$

$3.)\int 6x\sqrt{5-x^2}dx$

$4.)\int (2x-5)^4 dx$

$5.)\int (3x+7)^3 dx$

Udu Integration

1.) $\int \sin(2x)dx$ → $u = 2x$

$\frac{du}{dx} = 2$ → $dx = \frac{du}{2}$ ⇒ $\int \sin u \frac{d}{}$

$\int \frac{1}{2}\sin u \; d$

$-\frac{1}{2}\cos u$

$-\frac{1}{2}\cos(2$

2.) $\int x\cos(x^2 + 1)dx$

3.) $\int -3x^2(\sin(x^3))dx$

Udu Integration

1.) $\int \cos^2(x)\sin(x)dx$ $\longrightarrow$ $U = \cos(x)$

2.) $\int \sin^2(x)\cos(x)dx$

3.) $\int \cos(x)\sin(\sin(x))dx$

$$\frac{du}{dx} = -\sin(x)$$

$$\frac{du}{-\sin(x)} = dx$$

$$\int u^2 \sin(x) \frac{du}{-\sin(x)}$$

$$\int -u^2 du$$

$$-\frac{1}{3}u^3 + c$$

$$-\frac{1}{3}\cos^3(x) + c$$

Definite Integrals

1.) $\int_{1}^{5} (3x-7)dx$

2.) $\int_{-1}^{1} (2x+5)dx$

3.) $\int_{0}^{3} (3x^2-7x)dx$

Definite Integrals

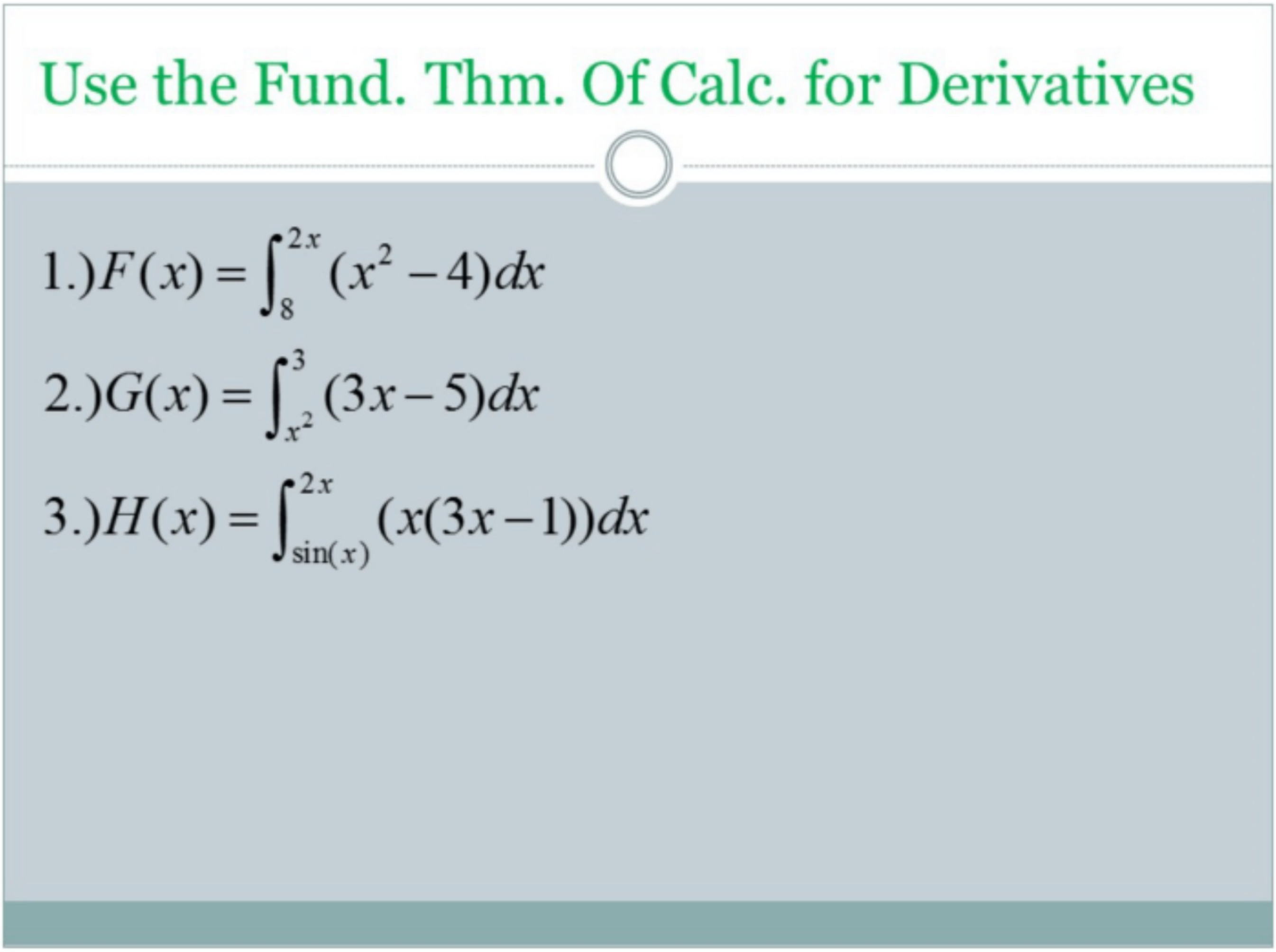
1.) $\int_{1}^{4} (3x-2)^2\,dx$

2.) $\int_{-1}^{1} (4x+2)^3\,dx$

3.) $\int_{1}^{2} x(2x^2-7)^2\,dx$

Find the area- then find total area & sketch it

1.) $\int_{-2}^{7}(x^2 - x - 6)dx$



Use the Fund. Thm. Of Calc. for Derivatives

$$1.)F(x)=\int_{8}^{2x}(x^2-4)dx$$

$$2.)G(x)=\int_{x^2}^{3}(3x-5)dx$$

$$3.)H(x)=\int_{\sin(x)}^{2x}(x(3x-1))dx$$

Find the mean value and where it occurs

1.) $f(x) = 3x - 4$……….[1,3]

2.) $g(x) = x^2 - 4x + 10$……….[2,6]

3.) $h(x) = \sin(x) - \cos(x)$………$[0, \pi]$

Given f ”(x), find f(x)

1.) $f''(x) = 3x, f'(0) = 3, f(0) = 5$

2.) $f''(x) = x^3 - 4, f'(0) = -2, f(0) = 7$

3.) $f''(x) = x^2 + 5, f'(1) = 2, f(2) = 4$

PVA

- 1.) If the velocity of an object is $v(t)=3t-6$, and the initial position is s =10, find the position function.

- 2.) If the velocity of an object is $v(t) = -2t+3$, and the initial position is s =-15, find the position function.

- 3.) If the velocity of an object is $v(t) = 2t-3$, and the initial position is s =6, find the position function.

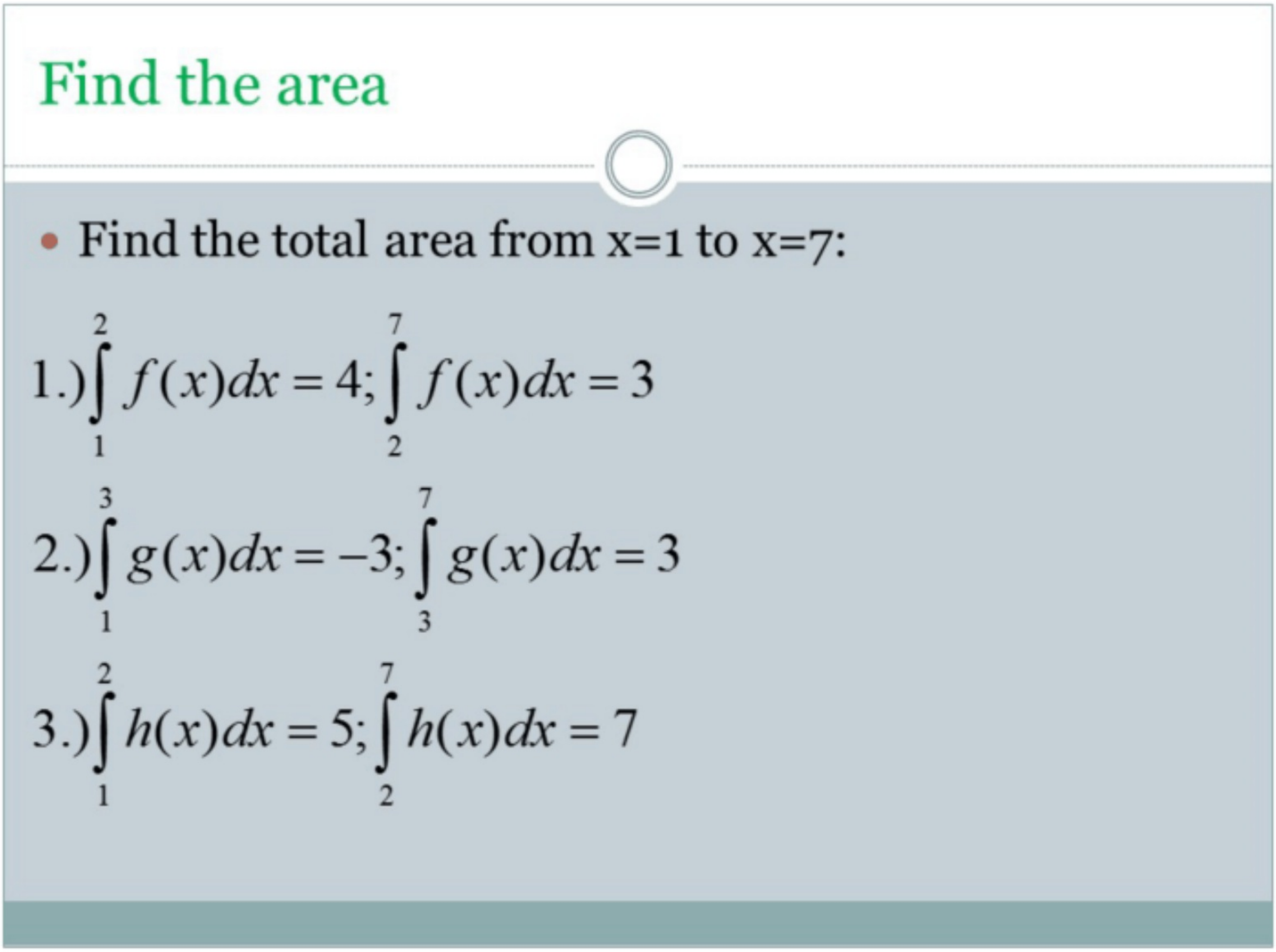
Find the area

- Write the formula that represents the region bounded by the given graphs:

1.) $y = x^2 + 3; y = -x^2 + 5$

2.) $y = x^2 + 1; y = 3x$

3.) $y = x^3; y = \sqrt{x}$



Find the area

- Find the total area from x=1 to x=7:

$$1.)\int_{1}^{2} f(x)dx = 4; \int_{2}^{7} f(x)dx = 3$$

$$2.)\int_{1}^{3} g(x)dx = -3; \int_{3}^{7} g(x)dx = 3$$

$$3.)\int_{1}^{2} h(x)dx = 5; \int_{2}^{7} h(x)dx = 7$$

Find the area: between the function, x-axis and y-axis

- Make up several graphs- find area both ways (TB; RL)

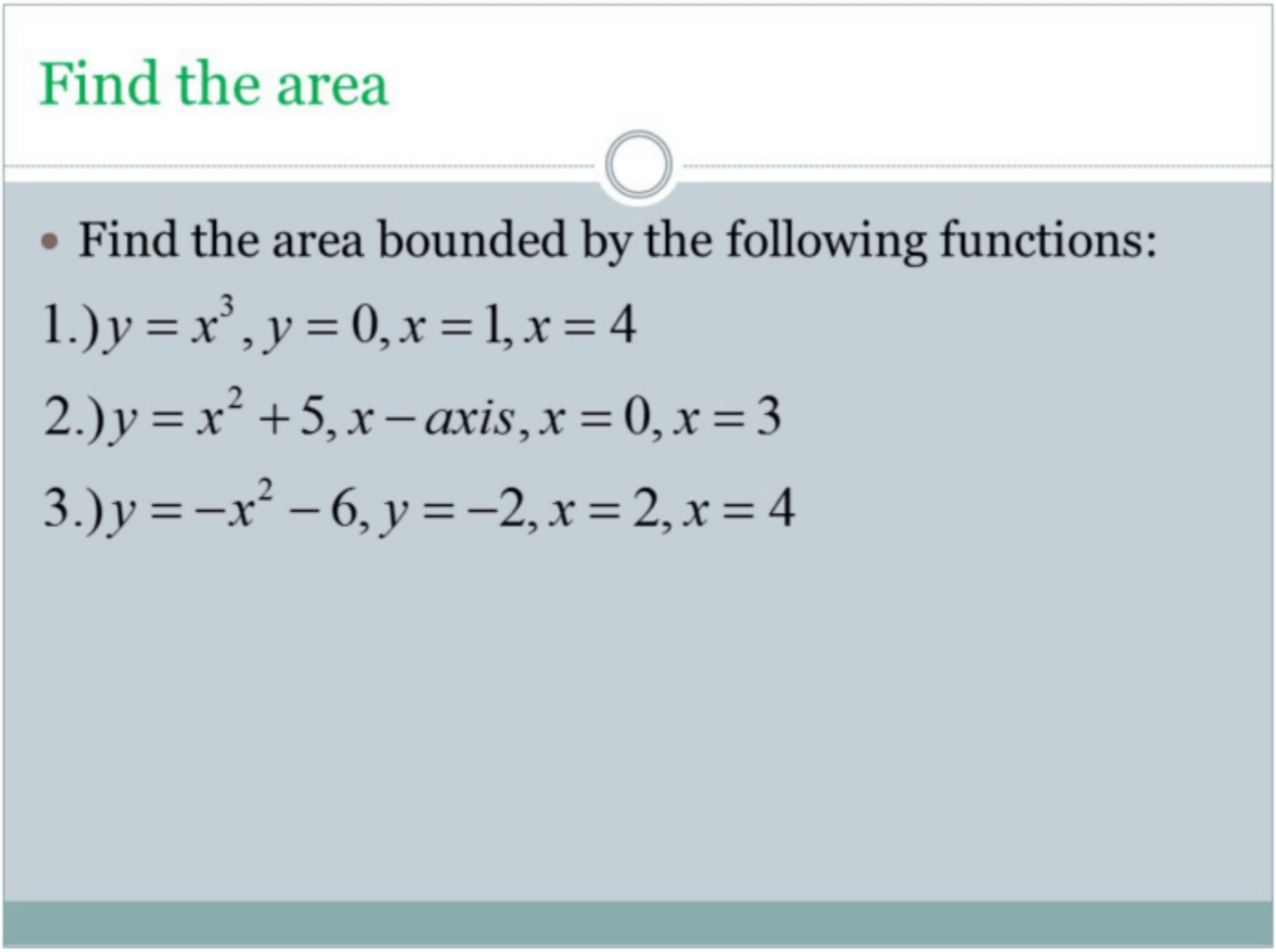
Find the area

- Write the formula that represents the area bounded by the following functions. Then find the area.

1.)$y = 3x^2 - 5; y = 4, x = 0$

2.)$y = -x^2; y = -2, x = 0$

3.)$y = x^3; y = -x^2 + 3, x = 0$



Find the area

- Find the area bounded by the following functions:

1.) $y = x^3, y = 0, x = 1, x = 4$

2.) $y = x^2 + 5, x - axis, x = 0, x = 3$

3.) $y = -x^2 - 6, y = -2, x = 2, x = 4$

Find f(x)

1.) $\int f(x)dx = x^3 - 4x^2 + 7x + c$

2.) $\int g(x)dx = \sqrt{x} - 4x + c$

3.) $\int h(x)dx = 2\sec^2 x - \sin x + c$

4.) $\int j(x)dx = -\sin x - \csc x \cot x + c$

Find the volume

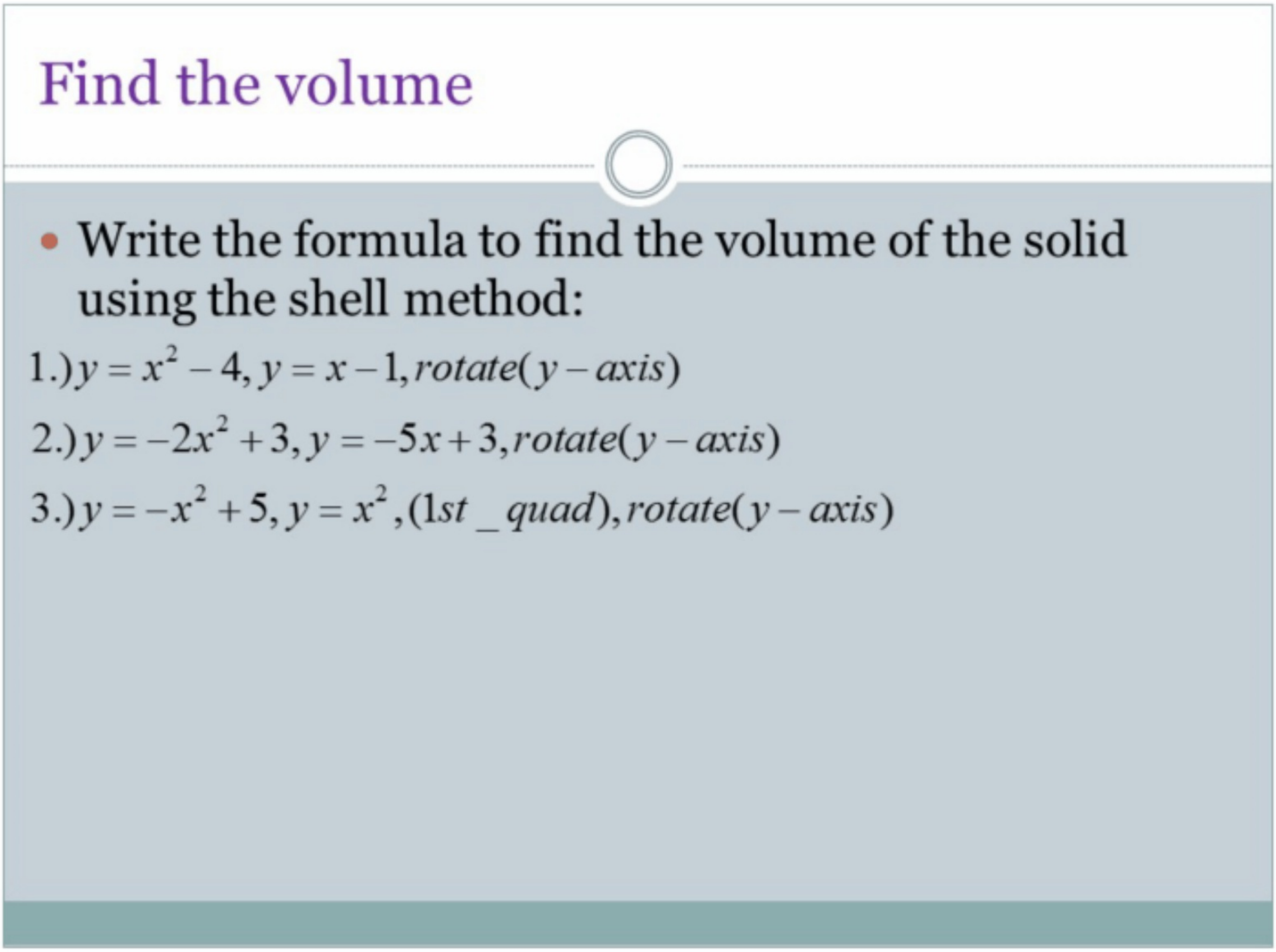
- Write the formula to find the volume of the solid:

1.)$y=-x^2+5, y=0, rotate(x-axis)$

2.)$y=x^3+1, y=0, x=3, rotate(x-axis)$

3.)$y=2x^2-2x-5, y=0, rotate(x-axis)$

4.)$y=-x^2+5, y=x^2, rotate)(x-axis)$



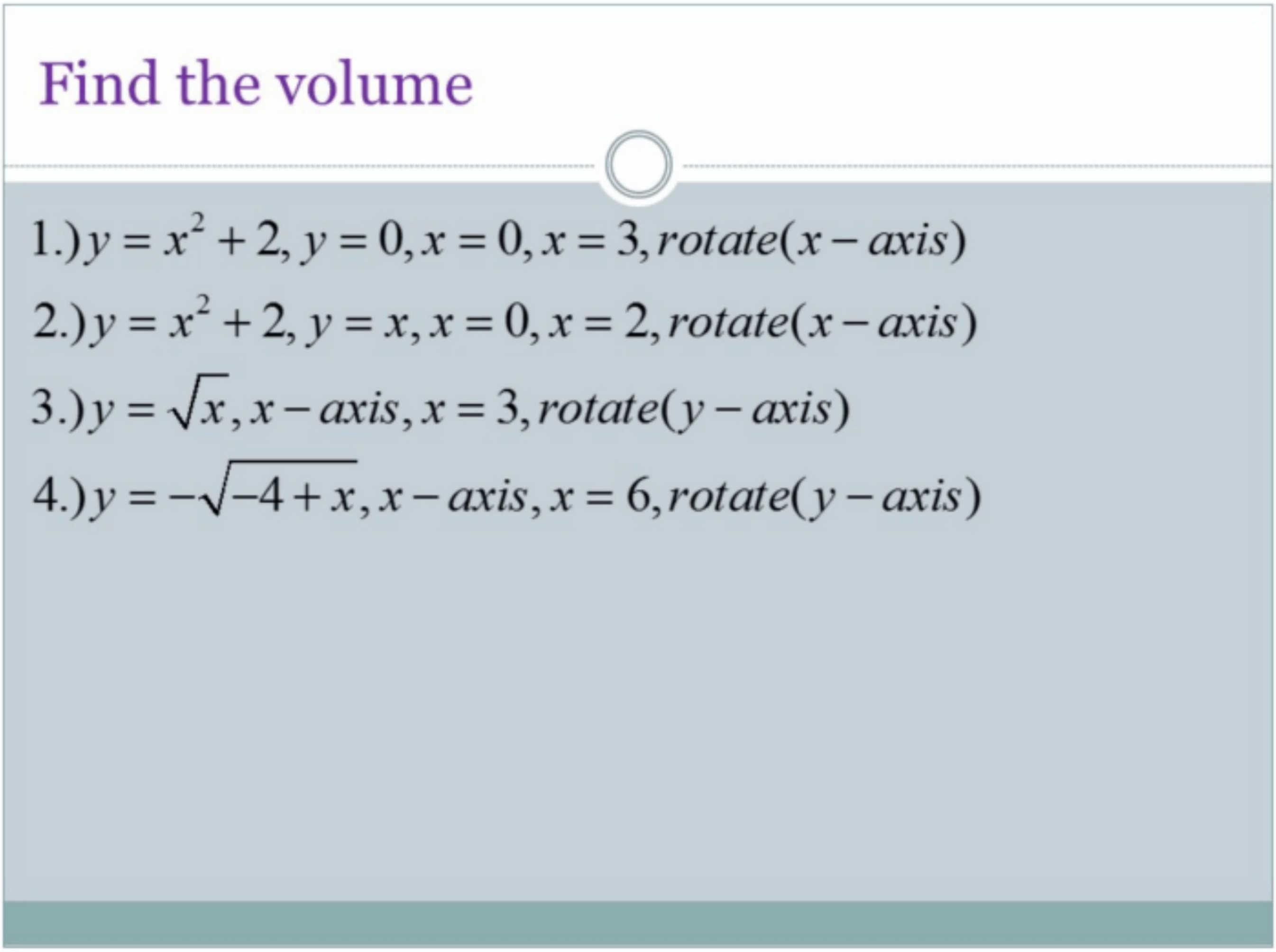
Find the volume

- Write the formula to find the volume of the solid using the shell method:

1.) $y = x^2 - 4, y = x - 1, rotate(y - axis)$

2.) $y = -2x^2 + 3, y = -5x + 3, rotate(y - axis)$

3.) $y = -x^2 + 5, y = x^2, (1st\_quad), rotate(y - axis)$



Find the volume

1.) $y = x^2 + 2, y = 0, x = 0, x = 3, rotate(x - axis)$

2.) $y = x^2 + 2, y = x, x = 0, x = 2, rotate(x - axis)$

3.) $y = \sqrt{x}, x - axis, x = 3, rotate(y - axis)$

4.) $y = -\sqrt{-4 + x}, x - axis, x = 6, rotate(y - axis)$