

Verify the following trigonometric identities.

$$1. \frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 = 1$$

$$2. \frac{\tan \theta \cos \theta}{\sin \theta} = 1$$

$$\frac{\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}{\sin \theta} =$$

$$\frac{\sin \theta}{\sin \theta} =$$

$$1 = 1$$

$$3. \frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = 1$$

$$\frac{\sin \theta}{\frac{1}{\sin \theta}} + \frac{\cos \theta}{\frac{1}{\cos \theta}} =$$

$$\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta =$$

$$\sin^2 \theta + \cos^2 \theta$$

$$1 = 1$$

$$4. \cos^2 \theta + \tan^2 \theta \cos^2 \theta = 1$$

$$\cos^2 \theta (1 + \tan^2 \theta) =$$

$$\cos^2 \theta (\sec^2 \theta) =$$

$$1 = 1$$

$$5. \sin \theta + \cos \theta = \frac{1 + \tan \theta}{\sec \theta}$$

$$= \frac{1 + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}}$$

$$= \left(1 + \frac{\sin \theta}{\cos \theta}\right) \cos \theta$$

$$= \cos \theta + \frac{\sin \theta}{\cos \theta} \cdot \cos \theta$$

$$\sin \theta + \cos \theta = \cos \theta + \sin \theta$$

$$6. \frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$$

$$\frac{\cos \theta \cdot \frac{1 - \sin \theta}{1 - \sin \theta}}{1 - \sin^2 \theta} + \frac{\cos \theta \cdot \frac{1 + \sin \theta}{1 + \sin \theta}}{1 - \sin^2 \theta} =$$

$$\frac{\cos \theta - \cos \theta \sin \theta + \cos \theta + \cos \theta \sin \theta}{\cos^2 \theta} =$$

$$\frac{2 \cos \theta}{\cos^2 \theta} =$$

$$\frac{2}{\cos \theta} =$$

$$2 \sec \theta = 2 \sec \theta$$

Solve the following trigonometric equation for the principal values (between 0° and 360°).

7. $2\cos^2\theta - 5\cos\theta + 2 = 0$

$2\cos^2\theta - 5\cos\theta + 2 = 0$

$(2\cos\theta - 1)(\cos\theta - 2) = 0$

$\cos\theta = \frac{1}{2}$

$\cos\theta = 2$

$\theta = 60^\circ$

\emptyset

S/A
T/C

$\theta = 60^\circ, 300^\circ$

8. $\sin^2\theta = 2\sin\theta + 3$

$\sin^2\theta - 2\sin\theta - 3 = 0$

$(\sin\theta - 3)(\sin\theta + 1) = 0$

$\sin\theta = 3$

$\sin\theta = -1$

\emptyset

270°

~~h~~

9. $3\tan^2\theta + 4\sec\theta = -4$

$3\left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{4}{\cos\theta} = -4\right) \quad * \cos\theta \neq 0$

$3\sin^2\theta + 4\cos\theta = -4\cos^2\theta$

$3(1 - \cos^2\theta) + 4\cos\theta + 4\cos^2\theta = 0$

$3 - 3\cos^2\theta + 4\cos\theta + 4\cos^2\theta = 0$

$\cos^2\theta + 4\cos\theta + 3 = 0$

$(\cos\theta + 1)(\cos\theta + 3) = 0$

11. $\csc^2\theta - 2 = 0$

$\csc\theta = -1$

$\csc\theta = 3$

$\theta = 180^\circ$

$\csc^2\theta = 2$

$\csc\theta = \pm\sqrt{2}$

$\sin\theta = \frac{1}{\pm\sqrt{2}}$

sin

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

S/A

$45^\circ = \theta$

T/C

$45^\circ, 135^\circ, 225^\circ, 315^\circ$

10. $2\sin\theta \cos\theta + 4\sin\theta = \cos\theta + 2$

$2\sin\theta(\cos\theta + 2) = \cos\theta + 2$

$2\sin\theta(\cos\theta + 2) - (\cos\theta + 2) = 0$

$(\cos\theta + 2)(2\sin\theta - 1) = 0$

$\cos\theta = -2$

$2\sin\theta - 1 = 0$

\emptyset

$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$

$150^\circ, 30^\circ$

12. $-\tan^2\theta + 2\tan\theta + 3 = 0$

$\tan^2\theta - 2\tan\theta - 3 = 0$

$(\tan\theta + 1)(\tan\theta - 3) = 0$

$\tan\theta = -1$

$\tan\theta = 3$

$\theta = 45^\circ$

$\theta = 75.57^\circ$

S/A
T/C

$135^\circ, 315^\circ$

S/A
T/C

75.57°

255.17°

$\theta = 75.57^\circ, 135^\circ, 255.17^\circ, 315^\circ$