

Chapter 2
Section 5

Zeros of a Polynomial
Function

Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree n , where $n > 0$, the function has precisely n linear factors

Rational Zeros test

- If the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ has integer coefficients, every rational zero of $F(x)$ has the form:
$$\text{RationalZero} = \frac{p}{q}$$

Where p and q have no common factors other than 1, and

p = a factor of the constant term a_0

q = a factor of the leading coefficient a_n

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Examples: Find p and q

- 1. $x^4 - x^3 + x^2 - 3x - 6$
- 2. $x^5 - 2x^3 + 10x^2 + 8$

p: $\pm 1, \pm 2, \pm 3, \pm 6$

p: $\pm 1, \pm 2, \pm 4, \pm 8$

q: ± 1

q: ± 1

p/q: $\pm 1, \pm 2, \pm 3, \pm 6$

p/q: $\pm 1, \pm 2, \pm 4, \pm 8$

- 3.) $2x^3 + 3x^2 - 8x + 3$

- 4.) $3x^3 + 5x - 10$

p: $\pm 1, \pm 3$

p: $\pm 1, \pm 2, \pm 5, \pm 10$

q: $\pm 1, \pm 2$

q: $\pm 1, \pm 3$

p/q: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

p/q: $\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}$

Examples- solve and simplify

$$y = x^4 - x^3 + x^2 - 3x - 6$$

$$p: \pm 1, \pm 2, \pm 3, \pm 6$$

$$q: \pm 1$$

$$p/q: \pm 1, \pm 2, \pm 3, \pm 6$$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$$

$$x^3 - 2x^2 + 3x - 6$$

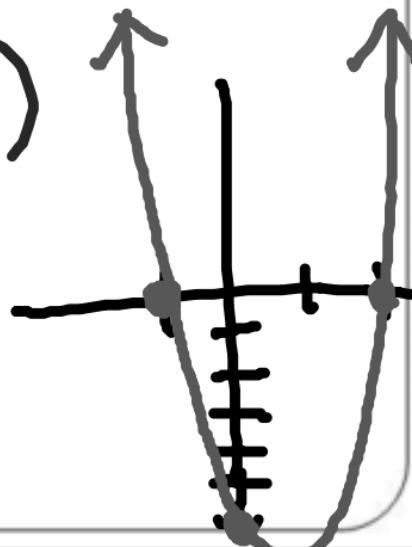
$$(x^3 - 2x^2) + (3x - 6)$$

$$x^2(x-2) + 3(x-2)$$

$$(x^2 + 3)(x-2)$$

$$x^2 + 3 = 0$$

$$x = \pm i\sqrt{3} \quad x = 2$$



Examples- solve and simplify

$$y = 2x^3 + 3x^2 - 8x + 3$$

$$p: \pm 1, \pm 3$$

$$q: \pm 1, \pm 2$$

$$p/q: \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$$

$$\begin{array}{r} \textcircled{-3} \\ 2 3 -8 3 \\ \underline{-6 9 -3} \\ 2 -3 1 \underline{0} \end{array}$$

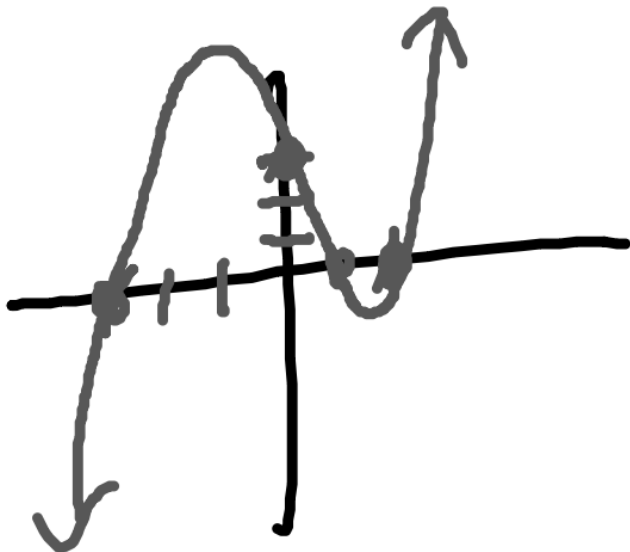
$$2x^2 - 3x + 1 = 0$$

$$x^2 - 3x + 2$$

$$\left(x - \frac{2}{2}\right) \left(x - \frac{1}{2}\right)$$

$$(x-1)(2x-1)$$

$$\textcircled{x=1} \quad \textcircled{x=\frac{1}{2}}$$



Homework

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