Chapter 2
Section 5

Zeros of a Polynomial Function

Shortcuts?

Descartes's Rule of Signs

- The number of positive real zeros of a function is either equal to the number of variations in the sign of f(x) or less than that number by an even integer
- The number of negative real zeros of a function is either equal to the number of variations in sign of f(-x) or less than that number by an even integer

Examples: ~~find p and q~~

1 +1 +1 +3 −6

- 1. $x^4 - x^3 + x^2 - 3x - 6$

1 −1 1 −3 −6

Pos → 3 or 1

Neg → 1

−1 +2 +10 +8

- 2. $x^5 - 2x^3 + 10x^2 + 8$

1 −2 +10 +8

Pos: 2 or 0

Neg: 1

−2 +3 +8 +3

- 3.) $2x^3 + 3x^2 - 8x + 3$

2 3 −8 3

Pos → 2 or 0

Neg → 1

- 4.) $3x^3 + 5x - 10$

Upper and Lower Bound Rules-

- Pg 177

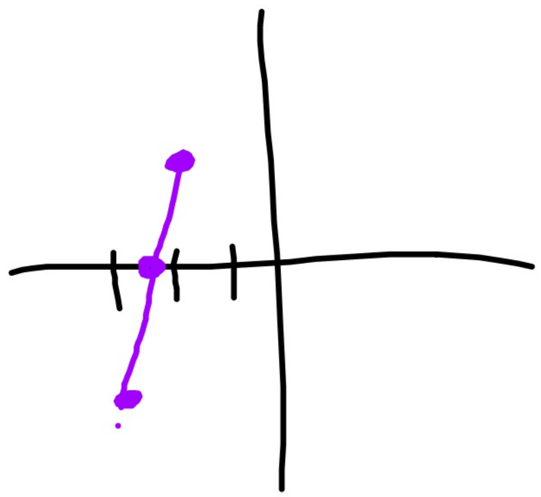
Upper and Lower Bound Rules

Let $f(x)$ be a polynomial with real coefficients and a positive leading coefficient. Suppose $f(x)$ is divided by $x - c$, using synthetic division.

1. If $c > 0$ and each number in the last row is either positive or zero, c is an **upper bound** for the real zeros of f.
2. If $c < 0$ and the numbers in the last row are alternately positive and negative (zero entries count as positive or negative), c is a **lower bound** for the real zeros of f.

Intermediate Value Theorem

- Read page 146



Classwork

- Page 181 # ~~79-86~~ 79-85 odd → pg 180 # 21-24
- Wkst 2.5 # 1-8

79 P: 0
N: 0

85 P: 3 or 1
N: 0

81 P: 0
N: 0

83 P: 1
N: 0

21) $f(x) = x^4 - x^3 - 2x - 4$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4$

P: $1 - 1 - 2 - 4 = \cancel{-6}$ | 1

N: $1 + 1 + 2 - 4 = \boxed{0}$ | 1

1	1	−1	0	−2	−4
		−1	2	−2	4
	1	−2	2	−4	$\boxed{0}$

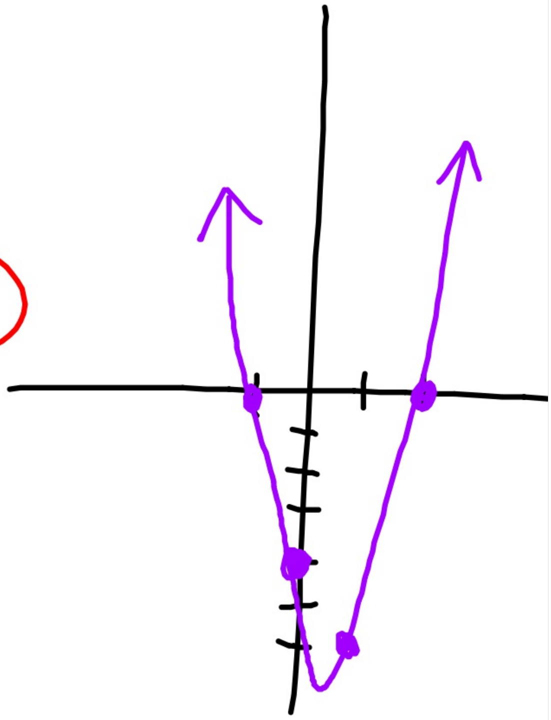
$(x^3 - 2x^2) + (2x - 4) = 0$

$x^2(x-2) + 2(x-2) = 0$

$(x-2)(x^2+2) = 0$

$x = -1, 2, \pm i\sqrt{2}$

y-int $(0, -4)$



22) $x^4 - 13x^2 - 12x = 0$

$x(x^3 - 13x - 12) = 0$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

P: 1 −13 −12	~~2~~	1
N: −1 +13 −12	0	2 or 0

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$x^2 - x - 12 = 0$

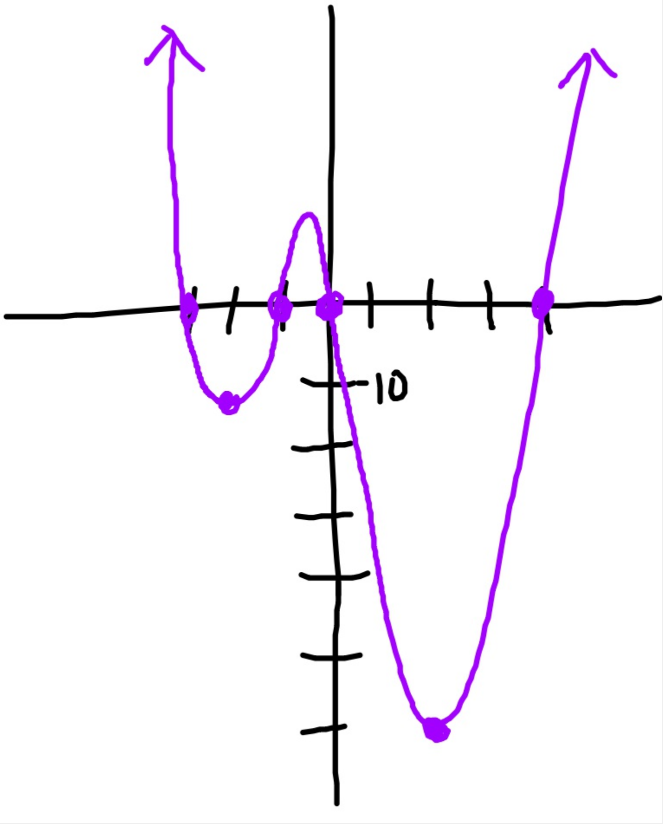
$(x-4)(x+3) = 0$

$x = 0, -1, -3, 4$

$y: (0,0)$

$f(-2) = 16 - 52$

$f(2) = 16 - 52$



23. $f(x) = 2x^4 + 7x^3 - 26x^2 + 23x - 6 = 0$

$f(-3) = 162 - 189 - 234$
$= -336$

$p/q: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

P: $2 + 7 - 26 + 23 - 6$	0	4 or 2 or 0
N: $2 - 7 - 26 - 23 - 6$	$-\cancel{36}$	1

$$\begin{array}{r|rrrrr} 1 & 2 & 7 & -26 & 23 & -6 \\ & & 2 & 9 & -17 & 6 \\ \hline & 2 & 9 & -17 & 6 & \boxed{0} \end{array}$$

$2x^3 + 9x^2 - 17x + 6$

$$\begin{array}{r|rrrr} 1 & 2 & 9 & -17 & 6 \\ & & 2 & 11 & -6 \\ \hline & 2 & 11 & -6 & 0 \end{array}$$

$2x^2 + 11x - 6 = 0$

$(2x - 1)(x + 6) = 0$

$\boxed{x = \frac{1}{2}, -6, 1 \text{ (DR)} \quad \text{y-int: } (0, -6)}$