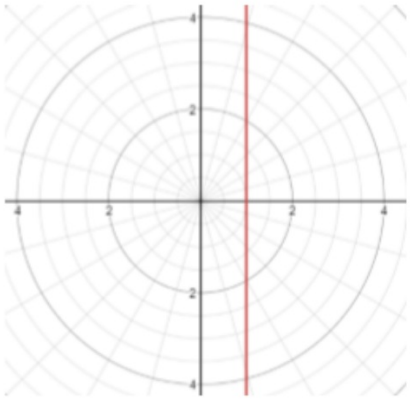
Polar Equations of Lines

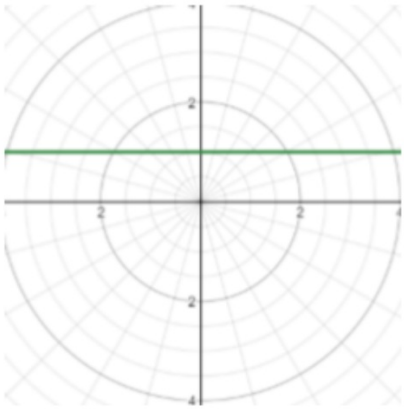
Vertical Line

$$r\cos(\theta) = a$$



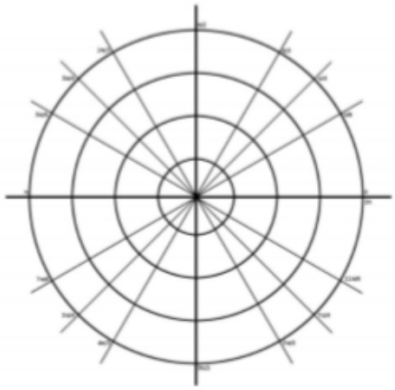
Horizontal Line

$$r\sin(\theta) = b$$



Sloped Line

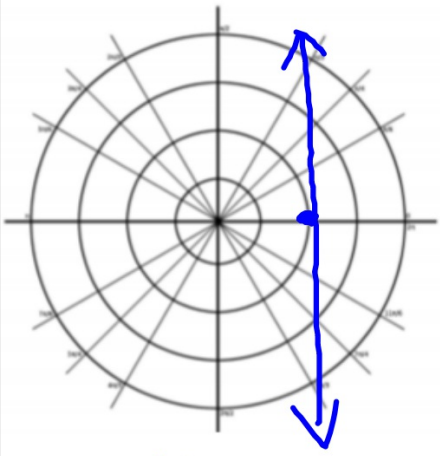
$$\theta = \beta$$



Polar Equations of Lines

Vertical Line

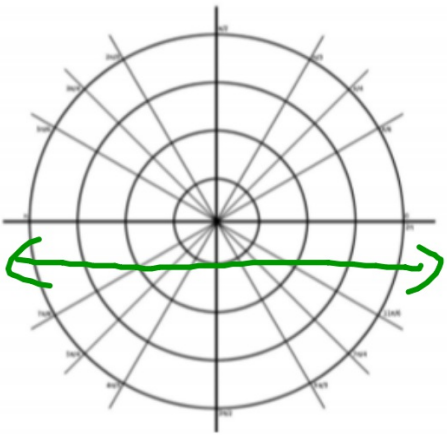
$$r\cos(\theta) = 2$$



$x = 2$

Horizontal Line

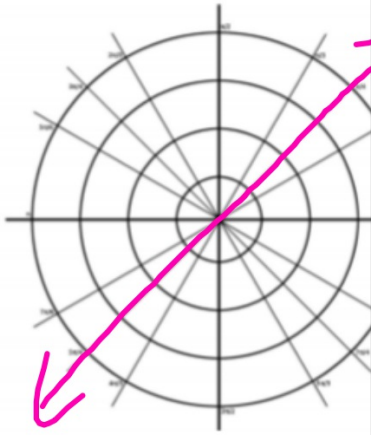
$$r\sin(\theta) = -1$$



$y = -1$

Sloped Line

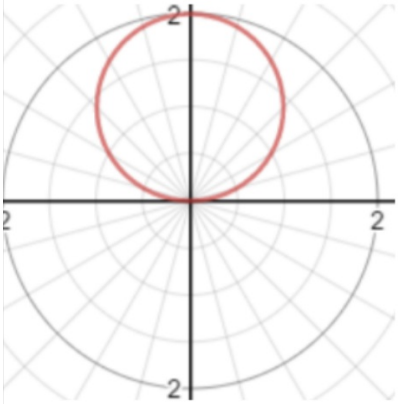
$$\theta = \frac{\pi}{4}$$



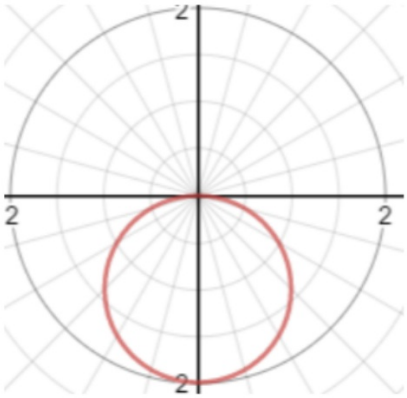
Polar Equations of Circles

Vertical Circle

$$r = 2b\sin(\theta)$$



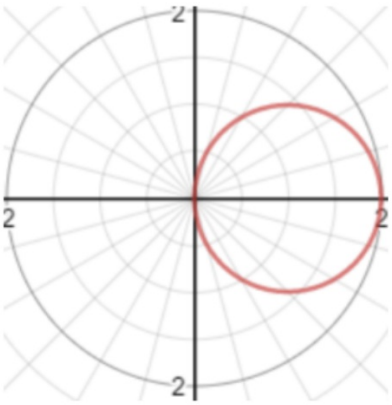
$$r = -2b\sin(\theta)$$



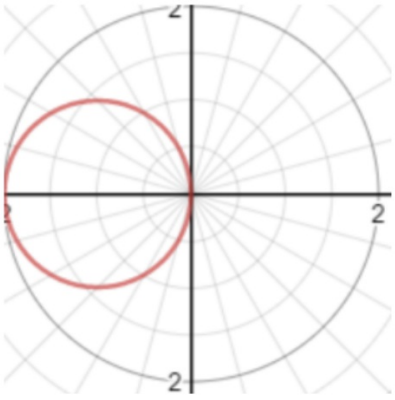
Polar Equations of Circles

Horizontal Circle

$$r = 2a\cos(\theta)$$

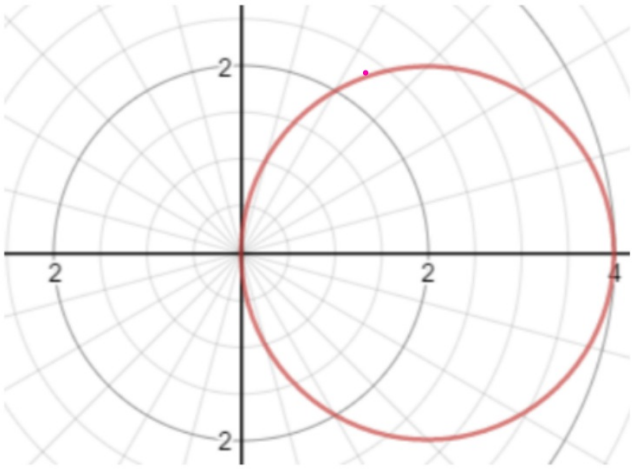


$$r = -2a\cos(\theta)$$



Circle Away from the Pole

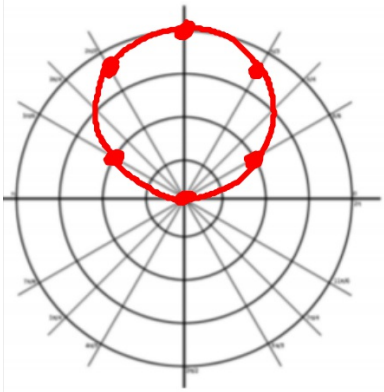
$$r = 2a\cos(\theta) + 2b\sin(\theta)$$



Polar Equations of Circles

Vertical Circle

$r = 4\sin(\theta)$



$(0, 0)$ $(0, 0)$

$(2, \frac{\pi}{6})$ $(-3, \frac{7\pi}{6})$

$(2\sqrt{3}, \frac{\pi}{3})$ $(-3\sqrt{3}, \frac{4\pi}{3})$

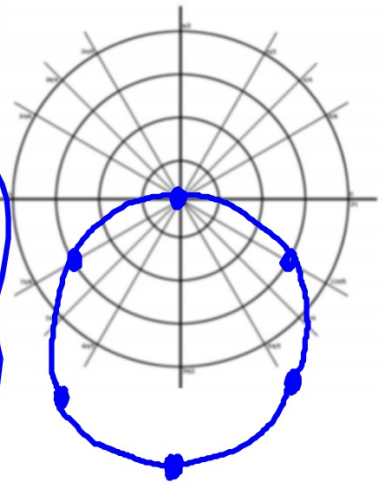
$(4, \frac{\pi}{2})$ $(-6, \frac{3\pi}{2})$

$(2\sqrt{3}, \frac{2\pi}{3})$ $(-3\sqrt{3}, \frac{5\pi}{3})$

$(2, \frac{5\pi}{6})$ $(-3, \frac{11\pi}{6})$

$(0, \pi)$ $(0, \pi)$

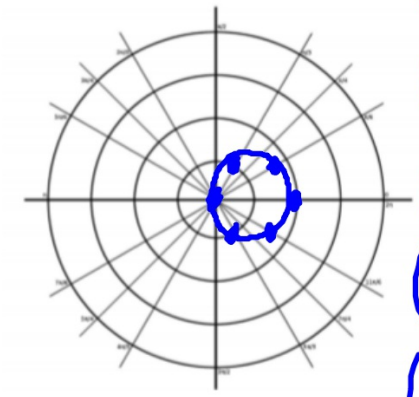
$r = -6\sin(\theta)$



Polar Equations of Circles

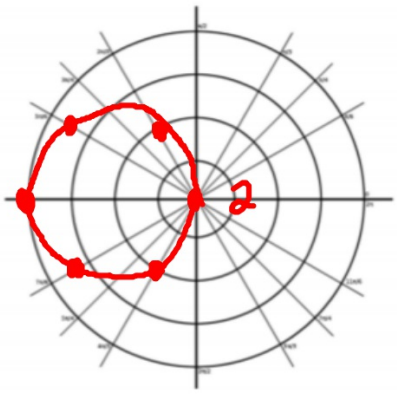
Horizontal Circle

$r = 2\cos(\theta)$



$(2, 0)$	$(-8, 0)$
$(\sqrt{3}, \pi/6)$	$(-4\sqrt{3}, \pi/6)$
$(1, \pi/3)$	$(-4, \pi/3)$
$(0, \pi/2)$	$(0, \pi/2)$
$(-1, 2\pi/3)$	$(4, 2\pi/3)$
$(-\sqrt{3}, 5\pi/6)$	$(4\sqrt{3}, 5\pi/6)$
$(-2, \pi)$	$(8, \pi)$

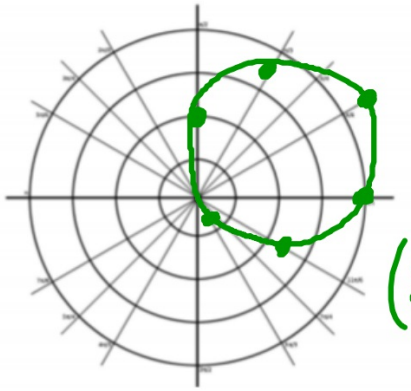
$r = -8\cos(\theta)$



Circle Away from the Pole

rcle with Sine and Cosine

$= 4\cos(\theta) + 2\sin(\theta)$



$(4, 0) \to 4 + 0$

$\left(2, \frac{\pi}{2}\right) \to 0 + 2$

$(-4, \pi) \to -4 + 0$

$\left(2\sqrt{3}+1, \frac{\pi}{6}\right) \to 2\sqrt{3} + 1$

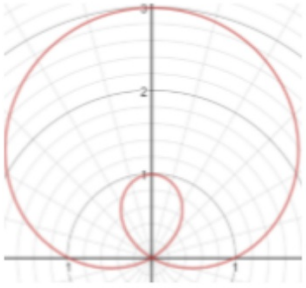
$\left(2+\sqrt{3}, \frac{\pi}{3}\right) \to 2 + \sqrt{3}$

$\left(2\sqrt{3}-1, \frac{4\pi}{6}\right) \to 2\sqrt{3} - 1$

$\left(2-\sqrt{3}, \frac{5\pi}{3}\right) \to 2 - \sqrt{3}$

Limacons - Inner Loop

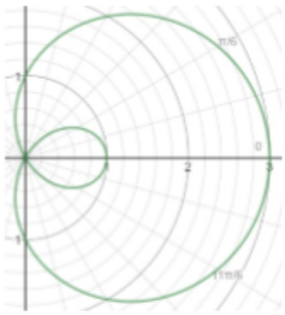
$r = a + b\sin(\theta)$



$\frac{a}{b} < 1$

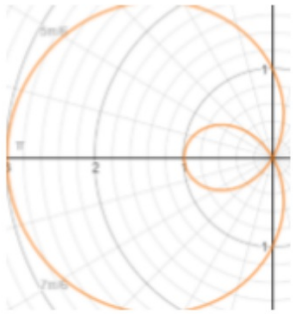
$r = a - b\sin(\theta)$

$r = a + b\cos(\theta)$



$\frac{a}{b} < 1$

$r = a - b\cos(\theta)$

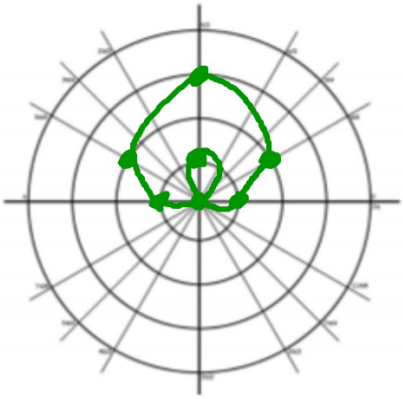


Quick Tips for Graphing Limacons

- $||a|-|b||$ distance for inner loop
- $|a|+|b|$ distance for outer loop
- a is x or y intercepts depending on orientation
- Choose four cardinal points and all multiples of $\frac{\pi}{6}$ for graphing sine equations
- Choose four cardinal points and all multiples of $\frac{\pi}{3}$ for graphing cos functions

Limacons - Inner Loop

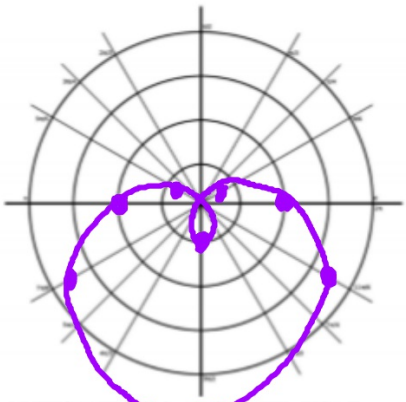
$r = 1 + 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
1	2	3	2	1	0	-1	0

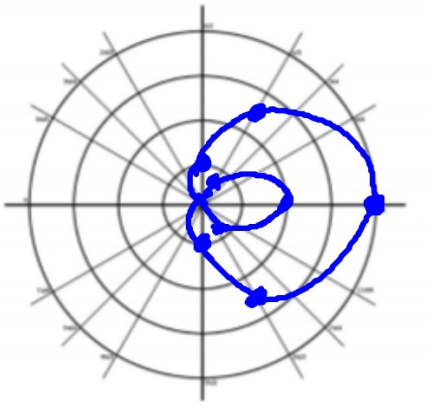
$\frac{a}{b} < 1$

$r = 2 - 3\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
2	$\frac{1}{2}$	-1	$\frac{1}{2}$	2	3.5	5	3.5

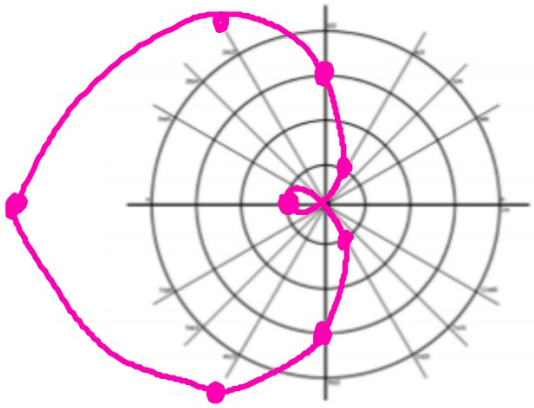
$r = 1 + 3\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
4	2.5	1	-.5	-2	-.5	1	2.5

$\frac{a}{b} < 1$

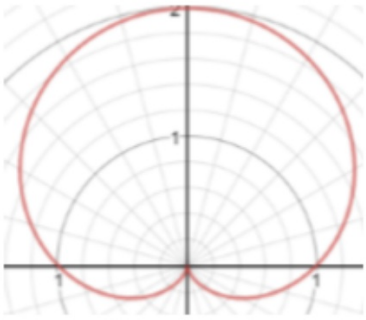
$r = 3 - 4\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
-1	1	3	5	7	5	3	1

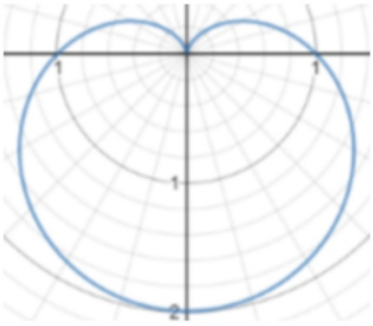
Cardioids

$r = a + b\sin(\theta)$

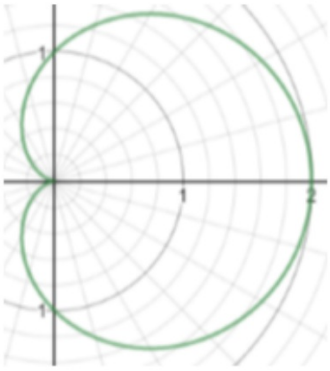


$\frac{a}{b} = 1$

$r = a - b\sin(\theta)$

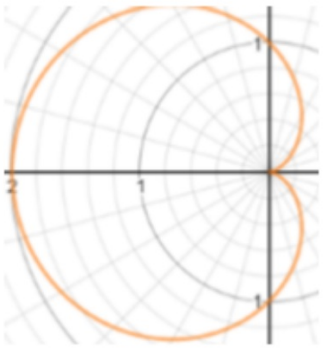


$r = a + b\cos(\theta)$



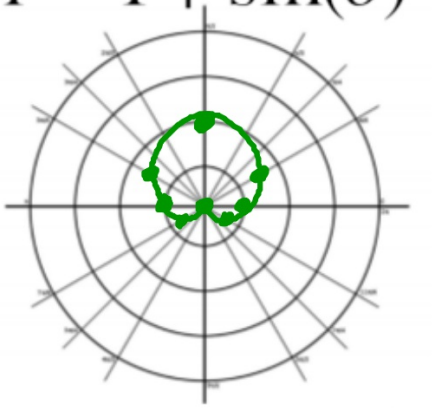
$\frac{a}{b} = 1$

$r = a - b\cos(\theta)$



Cardioids

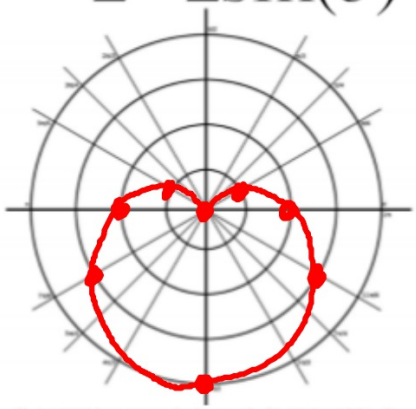
$r = 1 + \sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
1	1.5	2	1.5	1	.5	0	.5

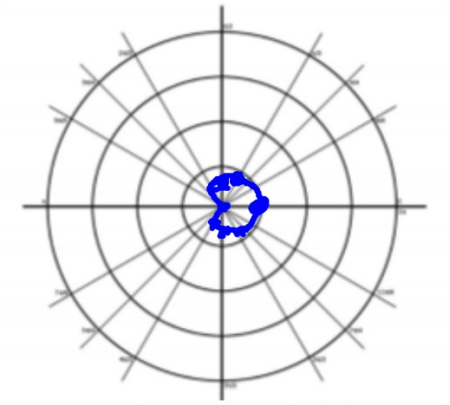
$\frac{a}{b} = 1$

$r = 2 - 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
2	1	0	1	2	3	4	3

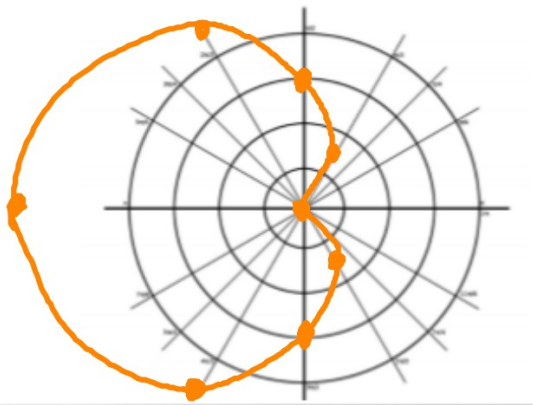
$$r = \frac{1}{2} + \frac{1}{2}\cos(\theta)$$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
1	.75	.5	.25	0	.25	.5	.75

$$\frac{a}{b} = 1$$

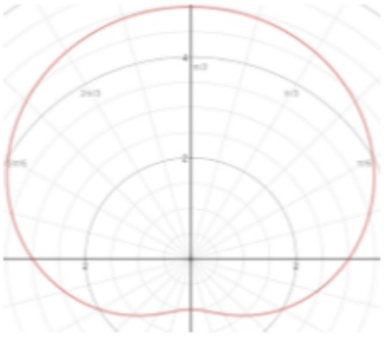
$$r = 3 - 3\cos(\theta)$$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
0	1.5	3	4.5	6	4.5	3	1.5

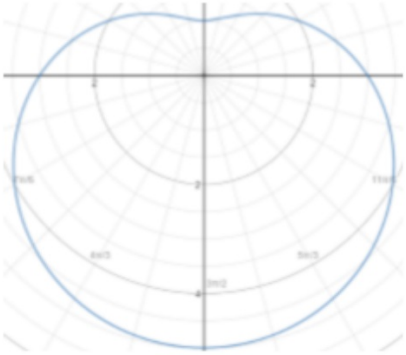
Limacons - Dimpled

$r = a + b\sin(\theta)$

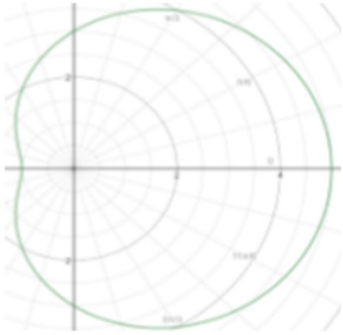


$1 < \frac{a}{b} < 2$

$r = a - b\sin(\theta)$



$r = a + b\cos(\theta)$

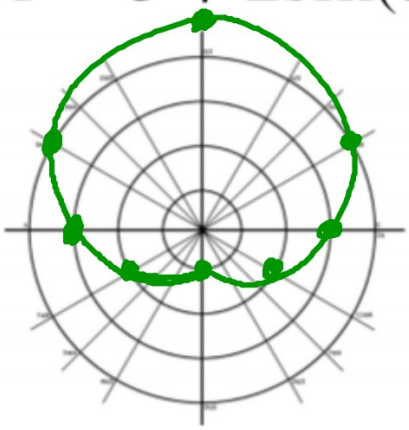


$1 < \frac{a}{b} < 2$

$r = a - b\cos(\theta)$

Limacons - Dimpled

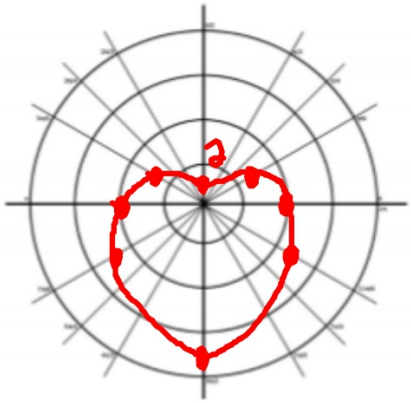
$r = 3 + 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
3	4	5	4	3	2	1	2

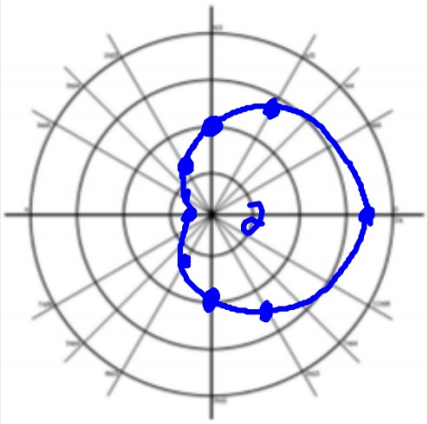
$1 < \frac{a}{b} < 2$

$r = 4 - 3\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
4	2.5	1	2.5	4	5.5	7	5.5

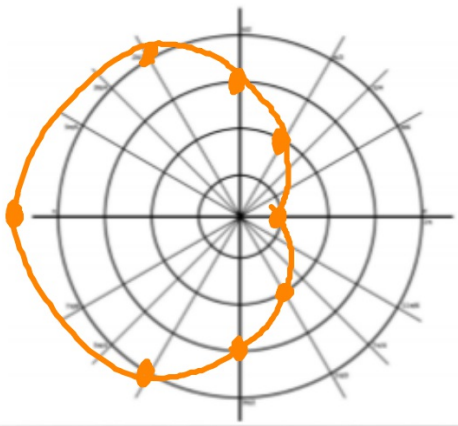
$r = 4 + 3\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
7	5.5	4	2.5	1	2.5	4	5.5

$1 < \frac{a}{b} < 2$

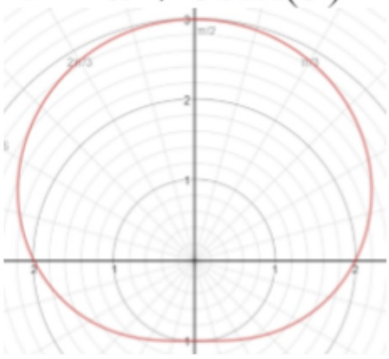
$r = 3 - 2\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
1	2	3	4	5	4	3	2

Limacons - Convex

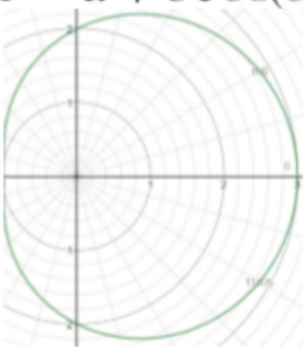
$r = a + b\sin(\theta)$



$r = a - b\sin(\theta)$

$\frac{a}{b} \geq 2$

$r = a + b\cos(\theta)$

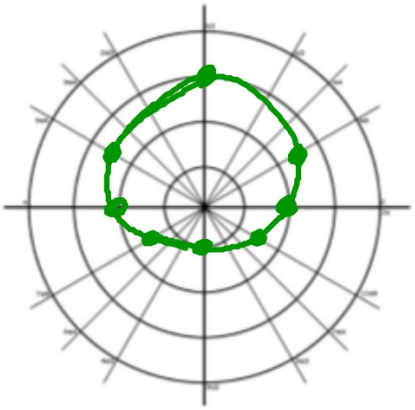


$r = a - b\cos(\theta)$

$\frac{a}{b} \geq 2$

Limacons - Convex

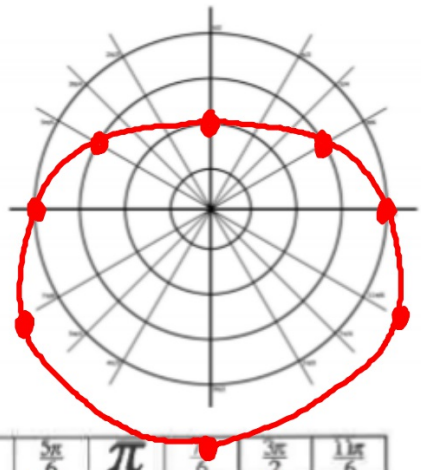
$r = 2 + \sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
2	2.5	3	2.5	2	1.5	1	1.5

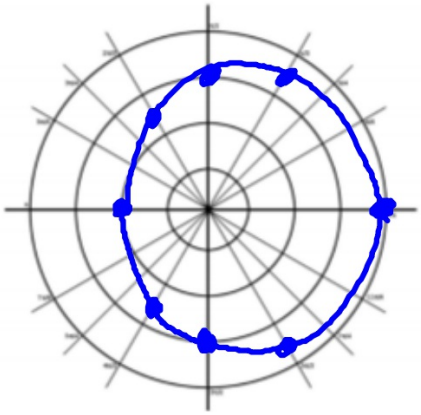
$\frac{a}{b} \geq 2$

$r = 4 - 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
4	3	2	3	4	5	6	5

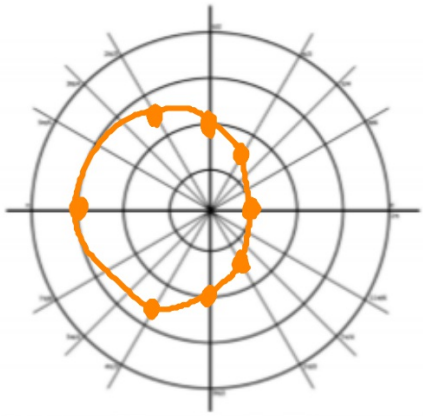
$r = 3 + \cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
4	3.5	3	2.5	2	2.5	3	3.5

$\frac{a}{b} \geq 2$

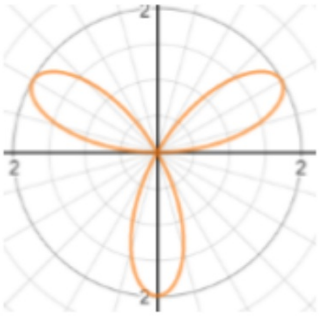
$r = 2 - \cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
1	1.5	2	2.5	3	2.5	2	1.5

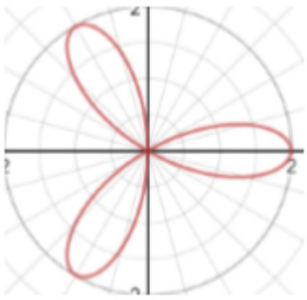
Rose Petals

$r = a\sin(n\theta)$

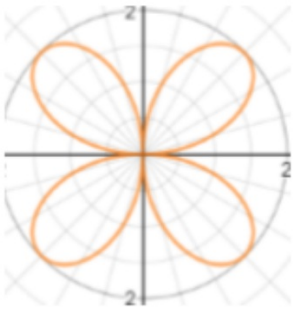


n is odd

$r = a\cos(n\theta)$

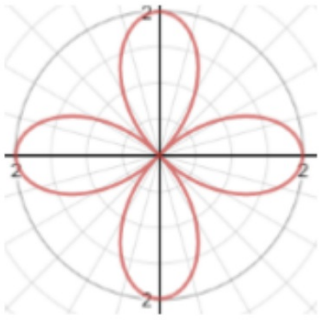


$r = a\sin(n\theta)$



n is even

$r = a\cos(n\theta)$



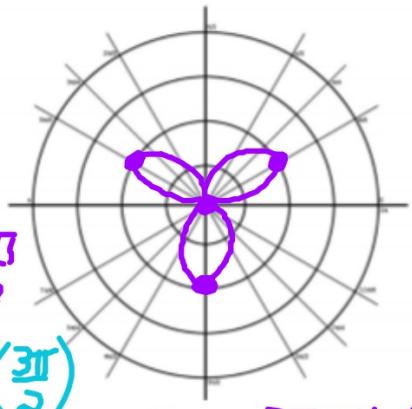
Quick Tips for Graphing Rose Petal Curves

- If n is odd same number of petals
- If n is even double the number of petals
- a is the length of a petal
- Choose four cardinal points and determine the period to aid in choosing points

- If n is odd $\rightarrow$ add 2π, then divide by n
- If n is even $\rightarrow$ add π, then divide by n

Rose Petals

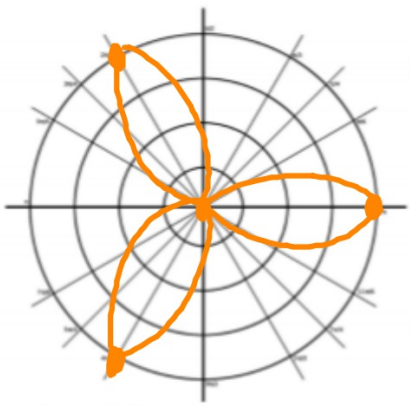
$r = 2\sin(3\theta)$



θ	0	$\pi/2$	π	$3\pi/2$	$\pi/6$	$5\pi/6$		
r	0	-2	0	2	2	2		

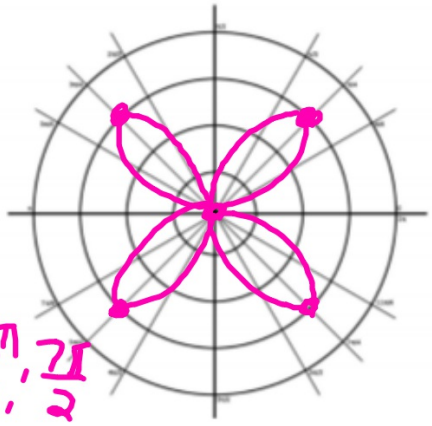
n is odd

$r = 4\cos(3\theta)$



θ	0	$\pi/2$	π	$3\pi/2$	$5\pi/3$	$4\pi/3$		
r	4	0	-4	0	4	4		

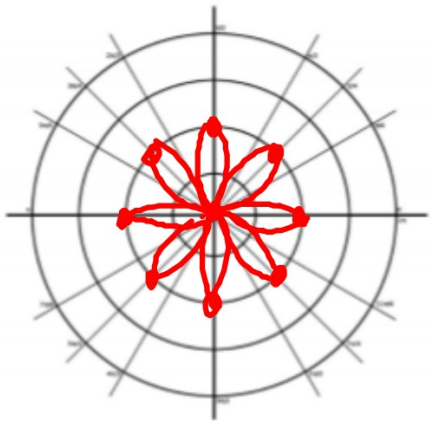
$r = 3\sin(2\theta)$



0	π/2	π	3π/2	7π/4	3π/4	5π/4	7π/4
0	0	0	0	3	-3	3	-3

n is even

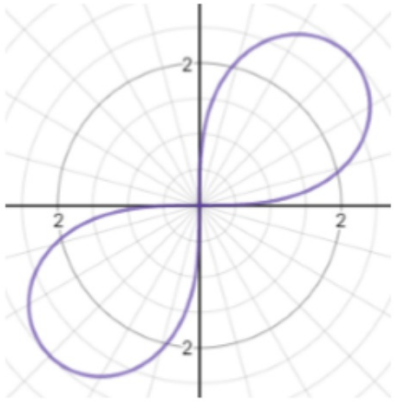
$r = 2\cos(4\theta)$



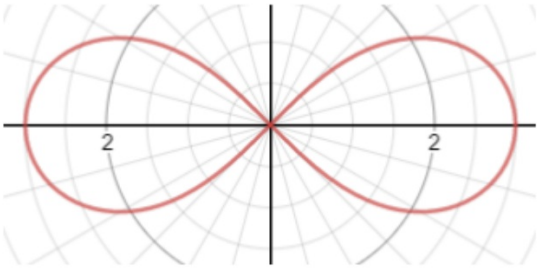
0	π/2	π	3π/2	π/4	3π/4	5π/4	7π/4
2	2	2	2	-2	-2	-2	-2

Lemniscates

$r^2 = a^2 \sin(2\theta)$



$r^2 = a^2 \cos(2\theta)$

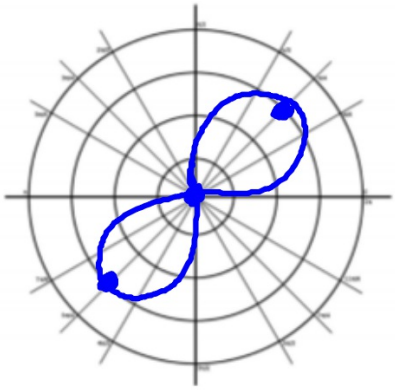


Quick Tips for Graphing Lemniscates

- Cosine graphs are longest along the x axis, this is equivalent to the a value
- Sine graphs are longest along the diagonal, 45 degrees, this is equivalent to the a value

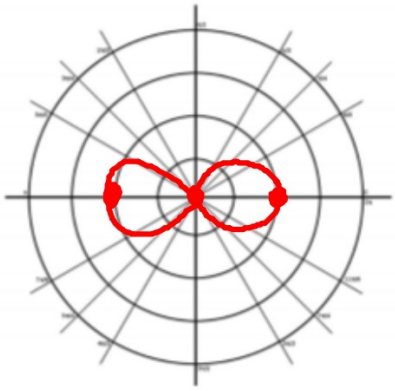
Lemniscates

$r^2 = 9\sin(2\theta)$



0	π/6	π	π/4	5π/4			
0	0	0	3	3			

$r^2 = 4\cos(2\theta)$



0	π/4	π					
2	X	2					

Spirals

$$r = \theta$$