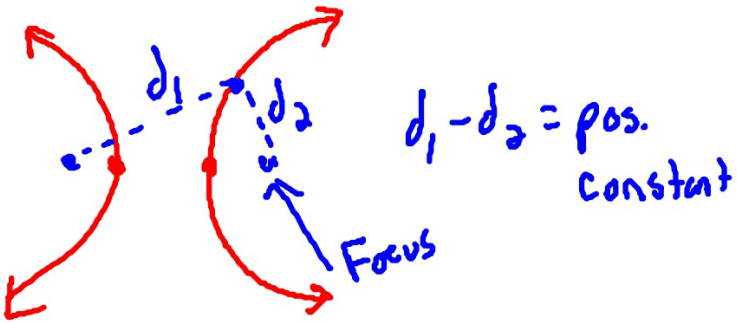
Hyperbola

The set of all points in a plane, the difference of whose distances from 2 distinct points (foci) is a positive constant.



Branches: the 2 disconnected parts of the hyperbola

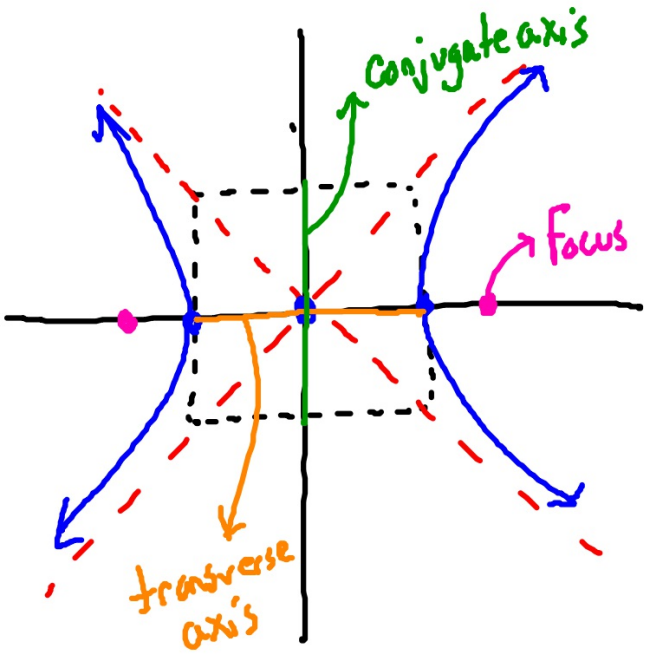
Vertices: one for each branch

Transverse Axis: line segment connecting the vertices

Conjugate Axis: perp. to the transverse axis at its center

Center: midpoint of the transverse axis

Foci: 2 fixed points on the inside of the branches



Transverse Axis = $2a$

Conjugate Axis = $2b$

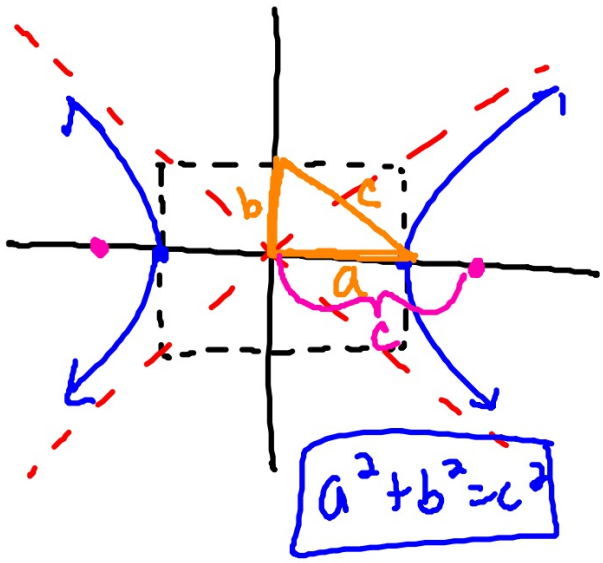
Standard Equation of a Hyperbola:

Transverse is horizontal (x)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse is vertical (y)

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



Asymptotes of a Hyperbola:

Transverse is horizontal

$$y = K \pm \frac{b}{a}(x-h)$$

Transverse is vertical

$$y = K \pm \frac{a}{b}(x-h)$$

Examples:

1. Given the hyperbola $\frac{x^2}{144} - \frac{y^2}{100} = 1$, find the vertices, foci, and state the asymptotes.

Transverse axis → X

$a = 12$

$b = 10$

$c = 2\sqrt{61}$

$a^2 + b^2 = c^2$

$144 + 100 = c^2$

$244 = c^2$

$c = \sqrt{244}$

Center: $(0, 0)$

V: $(12, 0)$ $(-12, 0)$

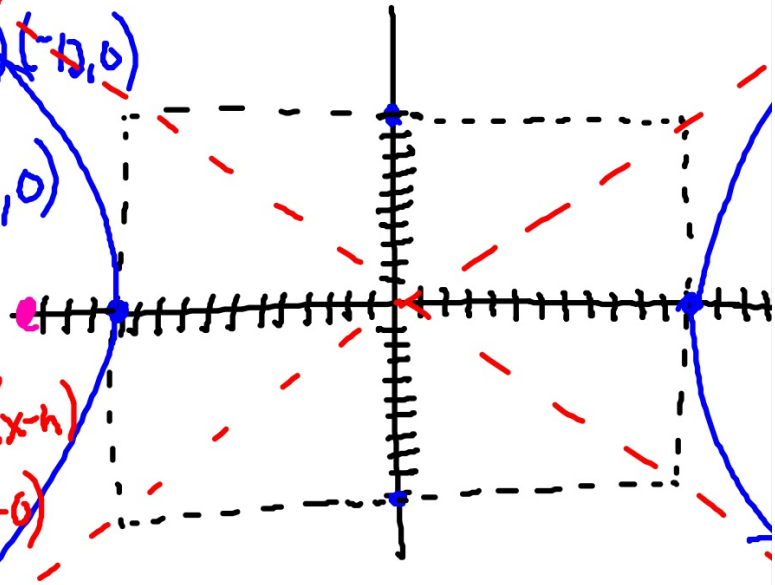
F: $(\pm 2\sqrt{61}, 0)$

Asm:

$y = k \pm \frac{b}{a}(x - h)$

$y = 0 \pm \frac{10}{12}(x - 0)$

$y = \pm \frac{5}{6}x$



2. Given the hyperbola $\frac{(y-2)^2}{64}-\frac{(x-3)^2}{25}=1$, find the vertices, foci, and state the asymptotes.

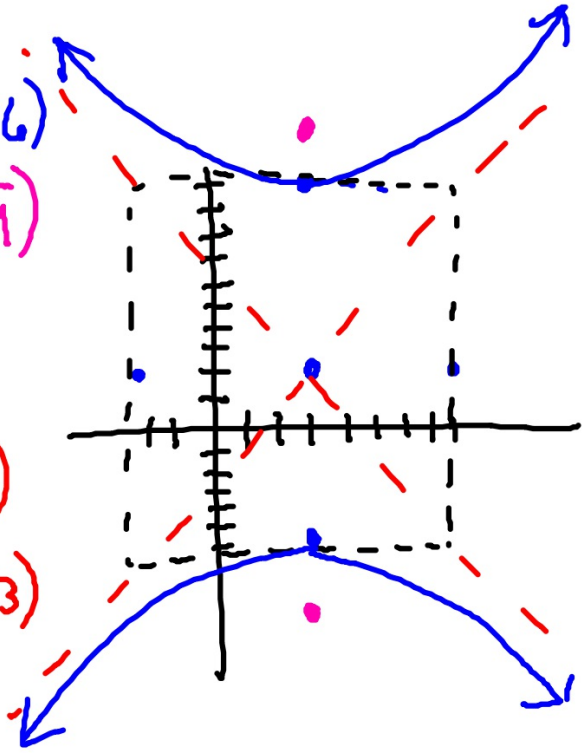
Transverse Axis
$\rightarrow y$

$a=8$
$b=5$
$c=\sqrt{89}$

$64+25=c^2$
$89=c^2$

Center: $(3,2)$
V: $(3,10)(3,-6)$
F: $(3, 2\pm\sqrt{89})$

Asm:
$y=k\pm\frac{a}{b}(x-h)$
$y=2\pm\frac{8}{5}(x-3)$



3. Sketch the hyperbola whose equation is $\frac{x^2}{9}-\frac{y^2}{25}=1$

Transverse Axis → x

$a=3$

$b=5$

$c=\sqrt{34}$

$9+25=c^2$

$34=c^2$

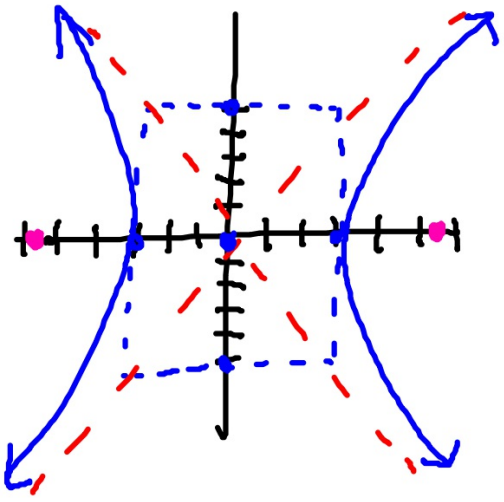
Center: $(0,0)$

V: $(-3,0)$ $(3,0)$

F: $(\pm\sqrt{34},0)$

Asm:

$y=\pm\frac{5}{3}x$



4. Sketch the hyperbola whose equation is $x^2 - 9y^2 + 36y - 72 = 0$

$x^2 - 9y^2 + 36y = 72$

$x^2 - 9(y^2 - 4y + 4) = 72 - 36$

$\frac{x^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$

$$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$$

Transverse → x

$a = 6$

$b = 2$

$c = 2\sqrt{10}$

$36 + 4 = c^2$

$40 = c^2$

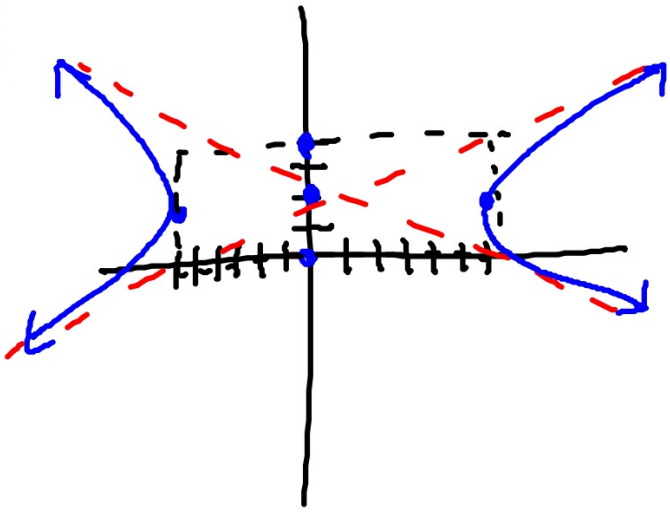
$\sqrt{40} = c$

Center: $(0, 2)$

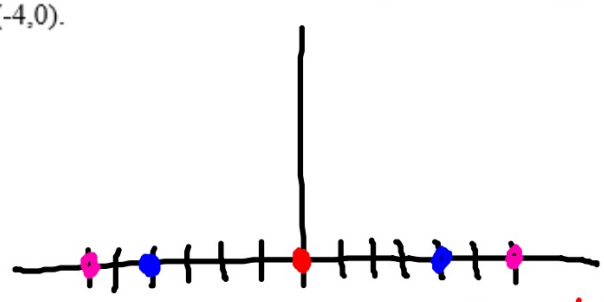
V: $(-6, 2)$ $(6, 2)$

F: $(\pm 2\sqrt{10}, 2)$

Asm: $y = 2 \pm \frac{1}{3}x$



5. Find the standard form of an equation of the hyperbola with foci at (6,0) and (-6, 0) and vertices at (4, 0) and (-4,0).



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center: (0,0)

$a = 4$

$c = 6$

$b =$

$4^2 + b^2 = 6^2$

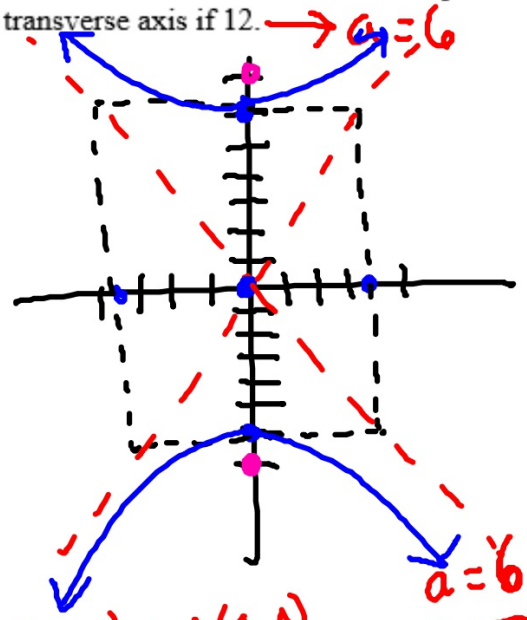
$16 + b^2 = 36$

$b^2 = 20$

$b = 2\sqrt{5}$

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

6. Write the standard form of an equation of the hyperbolas with foci (0, 7) and (0,-7) whose length of the transverse axis if 12.



$a = 6$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{36} - \frac{x^2}{13} = 1$$

$a = 6$

$c = 7$

Center: $(0,0)$

$V: (0,6)(0,-6)$

$6^2 + b^2 = 7^2$

$b^2 = 13$

$b = \sqrt{13}$

Asm: $y = \pm \frac{6}{\sqrt{13}} x$

$y = \pm \frac{6\sqrt{13}}{13} x$

7. Write the standard form of an equation of the hyperbolas with vertices (1, 2) and (3,2) whose asymptotes are $y = x$ and $y = 4 - x$.