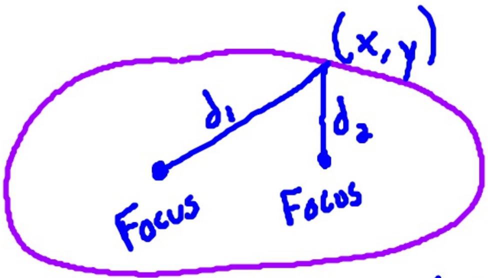
# Ellipse

The set of all points in a plane whose sum of the distances from 2 distinct points (foci) is constant.



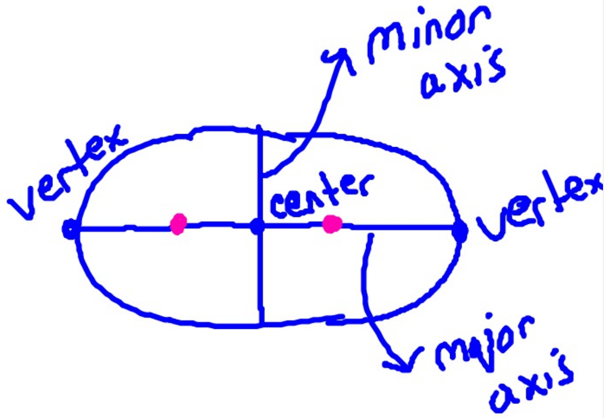

$d_1 + d_2$ is constant

Foci: 2 distinct points inside the ellipse

Vertices: endpoints of the ellipse

Major Axis: chord joining the vertices and its midpoint (center)

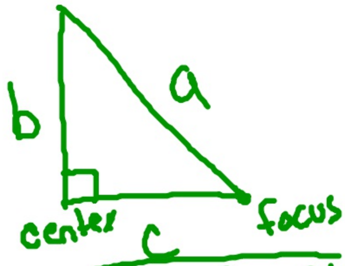
Minor Axis: chord perp. to the major axis at the center



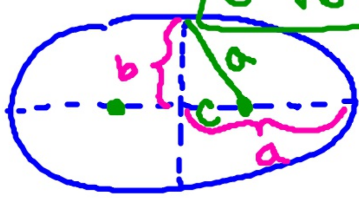
# Standard Equation of an Ellipse:

Major axis is horizontal:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$




$$\boxed{c^2+b^2=a^2}$$




Major axis is vertical:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

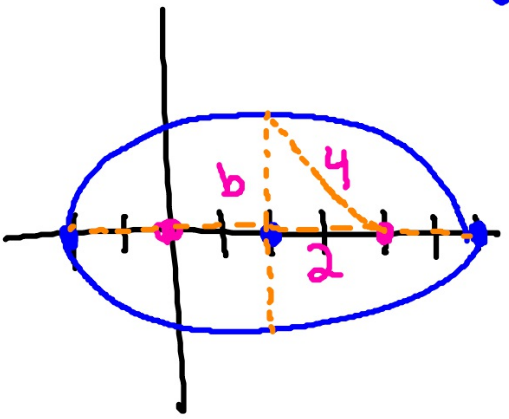

major axis $=2a$

minor axis $=2b$

Examples:

1. Find the standard form of the equation of an ellipse whose foci are at (0, 0) and (4, 0) and whose major axis has a length of 8.

Center: (2,0)




$b^2 + 2^2 = 4^2$

$b^2 + 4 = 16$

$b^2 = 12$

$b = \sqrt{12} = 2\sqrt{3}$

$2\sqrt{3} \cdot 2\sqrt{3}$

$4 \cdot 3$

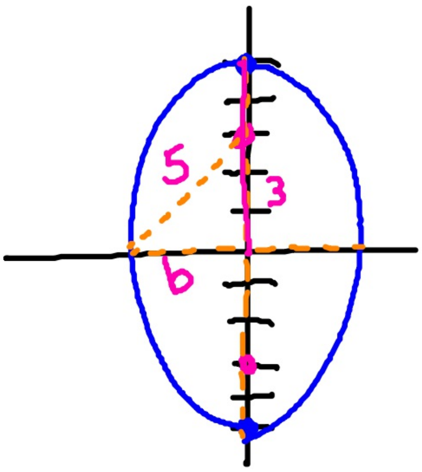
$8 = 2a$

$4 = a$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{16} + \frac{y^2}{12} = 1$$

2. Find the standard form of the equation of an ellipse whose vertices are at (0, 5) and (0, -5) and whose foci are at (0, 3) and (0,-3).




Center: $(0,0)$

$a = 5$

$b^2 + 3^2 = 5^2$

$b^2 + 9 = 25$

$b^2 = 16$

$b = 4$

$$\frac{(x-0)^2}{4^2} + \frac{(y-0)^2}{5^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

3. Write the equation of the given ellipse in standard form.

$$x^2 + 5y^2 + 4x - 70y + 209 = 0$$

$$x^2 + 4x \quad + 5y^2 - 70y \quad = -209 \qquad 5 \cdot 49$$

$$x^2 + 4x + 4 \quad + 5(y^2 - 14y + 49) = -209 + 4 + 245$$

$$\frac{(x+2)^2}{40} + \frac{5(y-7)^2}{40} = \frac{40}{40}$$

$$\frac{(x+2)^2}{40} + \frac{(y-7)^2}{8} = 1$$

4. Find the center, vertices, and foci of the given ellipse.

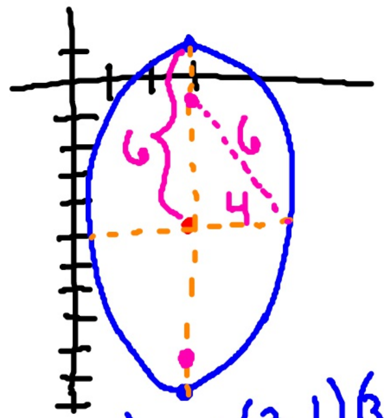
$$9x^2 + 4y^2 - 54x + 40y + 37 = 0$$

$$9x^2 - 54x \quad + 4y^2 + 40y \quad = -37$$

$$9(x^2 - 6x + 9) + 4(y^2 + 10y + 25) = -37 + \underbrace{81}_{9 \cdot 9} + \underbrace{100}_{4 \cdot 25}$$

$$\frac{9(x-3)^2}{144} + \frac{4(y+5)^2}{144} = \frac{144}{144}$$

$$\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1$$




Center: $(3, -5)$

$a = 6$
$b = 4$

$4^2 + c^2 = 6^2$
$16 + c^2 = 36$
$c^2 = 20$
$c = \sqrt{20} = 2\sqrt{5}$

Vertices: $(3, 1), (3, -11)$

Foci: $(3, -5 + 2\sqrt{5})$
$(3, -5 - 2\sqrt{5})$

Eccentricity

- measures the ovalness of an ellipse
- given by the ratio: $e = \frac{c}{a}$

$$0 < e < 1$$