

Circles

The set of all points that are equidistant from a given point (center) in a plane.

Standard form of the equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

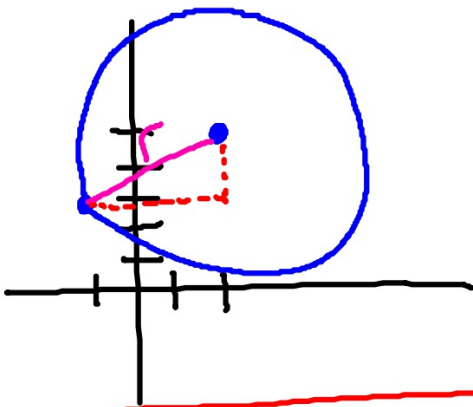
center: (h, k)

$r = \text{radius}$



Examples:

1. Find the equation of the circle in standard form whose center is at the point (2,5) and passes through the point (-1,3).



$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-5)^2 = \sqrt{13}^2$$

$$(x-2)^2 + (y-5)^2 = 13$$

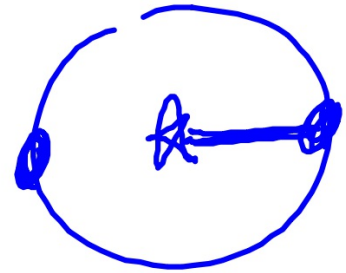
$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(2 - (-1))^2 + (5 - 3)^2}$$

$$r = \sqrt{9 + 4} = \sqrt{13}$$

2. Find the equation of the circle whose diameter has the endpoints $(-5, 4)$ and $(1, -4)$.

$$\text{Center: } \left(\frac{-5+1}{2}, \frac{4+(-4)}{2} \right)$$
$$(-2, 0)$$



$$\text{Diam: } \sqrt{(-5-1)^2 + (4-(-4))^2}$$
$$\sqrt{36 + 64}$$
$$\sqrt{100}$$

$$(x+2)^2 + y^2 = 25$$

$$\text{Diam} = 10$$

3. Find the center and radius of the following circle: $x^2 + y^2 + 12x - 16y - 10 = 0$

$$x^2 + y^2 + 12x - 16y - 10 = 0$$

$$x^2 + 12x + \underbrace{36}_{\left(\frac{12}{2}\right)^2} + y^2 - 16y + \underbrace{64}_{\left(-\frac{16}{2}\right)^2} = 10 + 36 + 64$$

$$(x+6)^2 + (y-8)^2 = 110$$

$$C: (-6, 8)$$

$$r = \sqrt{110}$$

$$\begin{array}{r} 2.55 \\ \sqrt{110} \end{array}$$