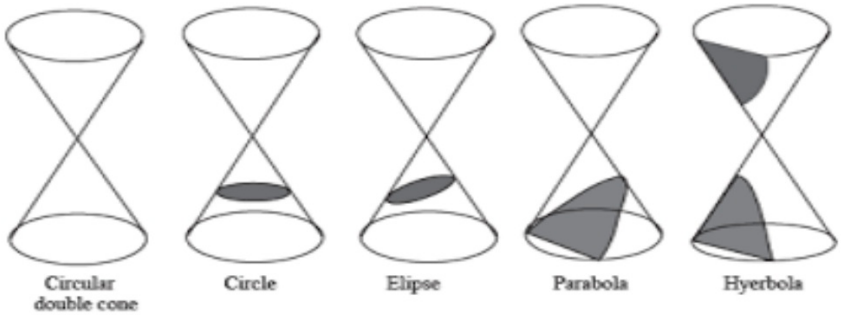
Sections

the intersection of a plane & a double napped cone



How to tell which conic you are working with based on its equation…

When the equation is equal to zero, ask yourself:

1. Is there more than one squared term?
   NO – Parabola YES – Go on to question 2

2. Do the squared terms both have the same sign?
   NO – Hyperbola YES – Go on to question 3

3. Are the coefficients in front of the squared terms the same?
   NO – Ellipse YES - Circle

Examples:
Classify each of the following equations as the equation of a parabola, ellipse, circle, or hyperbola.

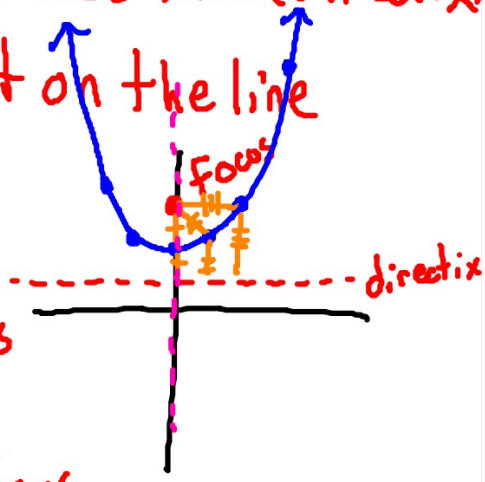
1. $3x^2 - 4x + 2 + 4y^2 - 6y + 7 = 0$

2. $5x^2 + 4x + 2 - 6y + 7 = 0$

3. $2y + 5y^2 + 2 + 5x^2 - 6x - 1 = 0$

4. $2 + 3x^2 + x + 8 - 7y^2 + 2y = 0$

Parabola: The set of all points (x, y) in a plane that are equidistant from a fixed line (directix) and a fixed point (focus) - not on the line



Vertex: the midpoint between the focus and the directix

Axis: the line passing through the focus and the vertex

Standard Form: opening up/down: $(x-h)^2 = 4p(y-k)$ $p \neq 0$

opening left/right: $(y-k)^2 = 4p(x-h)$ $p \neq 0$

Changing from the general form to standard form for a parabola

Completing the square:

Example:

1. $x^2 - 8x + 11 = 0$

$x^2 - 8x + 16 = -11 + 16$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2$

$(x-4)^2 = 5$

$(x-4)^2 - 5 = 0$

$V: (4, -5)$

Steps:

1. make sure the leading coeff is positive one
2. move constant to the other
3. Complete the square:
   add $\left(\frac{b}{2}\right)^2$ to both sides
4. simplify both sides factor left combine ri
   $(x \pm \frac{b}{2})^2$
5. move constant back

2. $\frac{2x^2}{2} - \frac{12x}{2} + \frac{23}{2} = \frac{0}{2}$

$x^2 - 6x + \frac{23}{2} = 0$

$x^2 - 6x + 9 = -\frac{23}{2} + 9$

$\left(\frac{-6}{2}\right)^2 = (-3)^2$ $\quad -\frac{23}{2} + \frac{18}{2}$

$(x-3)^2 = \frac{-5}{2}$

$(x-3)^2 + \frac{5}{2} = 0$

$V: (3, 5/2)$

Finding the Focus of a Parabola:

1. Covert to standard form by completing the square

2. Identify the vertex

3. Solve for "p"

4. Decide if the parabola opens up/down or left/right based on "p"

5. Add "p" to the appropriate coordinate

Examples:
Find the vertex, focus, and directrix of each parabola and sketch its graph.

1. $(x+3)^2+8(y-2)=0$

$(x+3)^2=-8(y-2)$

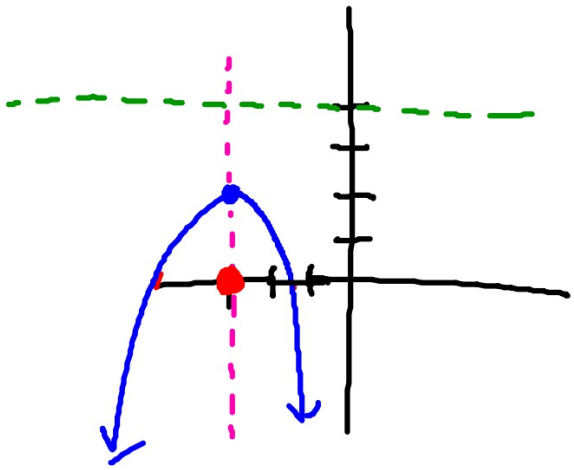
V: $(-3, 2)$

Focus: $4p=-8$

$p=-2$

$(-3, 0)$

Directrix: $y=4$



2. $x^2 - 2x + 8y + 9 = 0$

$x^2 - 2x + 1 = -8y - 9 + 1$

$\left(\frac{-2}{2}\right)^2$

$(x-1)^2 = -8y - 8$

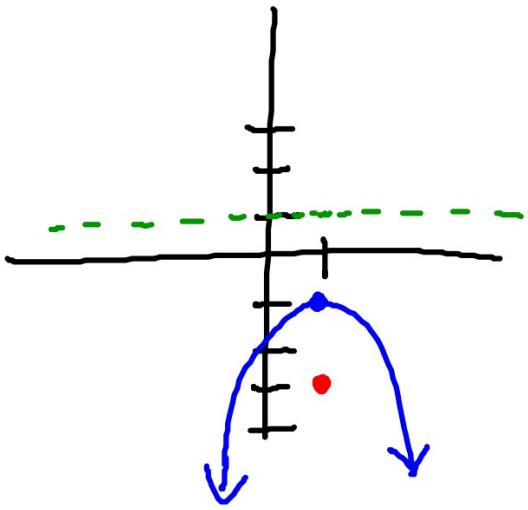
$(x-1)^2 = -8(y+1)$

$V = (1, -1)$

Focus: $4p = -8$ $(1, -3)$

$p = -2$

Directix: $y = 1$



3. $y^2 - 4y - 4x = 0$

$y^2 - 4y + 4 = 4x + 4$

$\left(\frac{-4}{2}\right)^2$

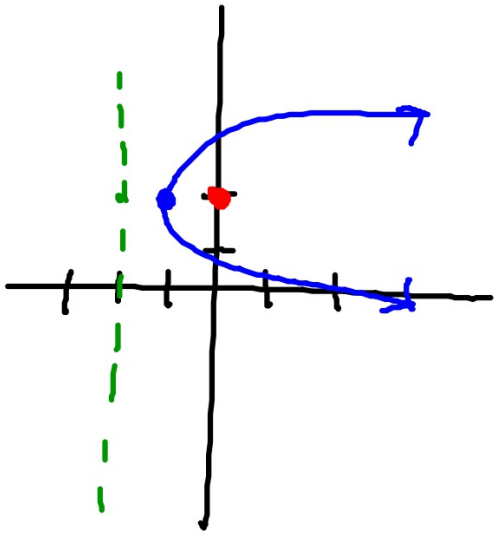
$(y-2)^2 = 4(x+1)$

V: $(-1, 2)$

Focus: $4p = 4$ $(0,2)$

$p = 1$

Directix: $x = -2$



Find the standard form of the equation of a parabola with the given characteristics.

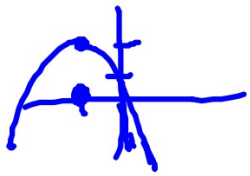
1. Vertex: (-1, 2) Focus: (-1, 0)

h k

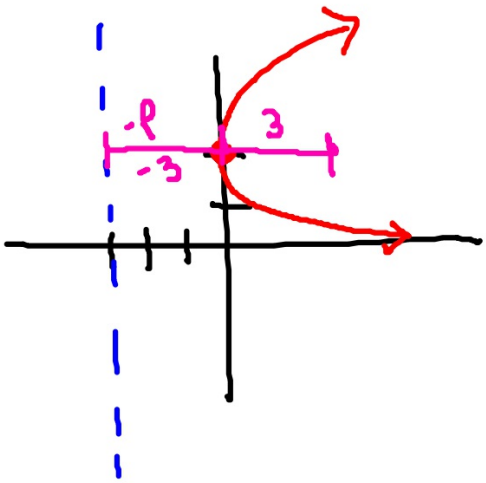
$(x-h)^2 = 4p(y-k)$

$(x+1)^2 = 4(-2)(y-2)$

$(x+1)^2 = -8(y-2)$



2. Vertex: (0, 2) Directrix: x = -3



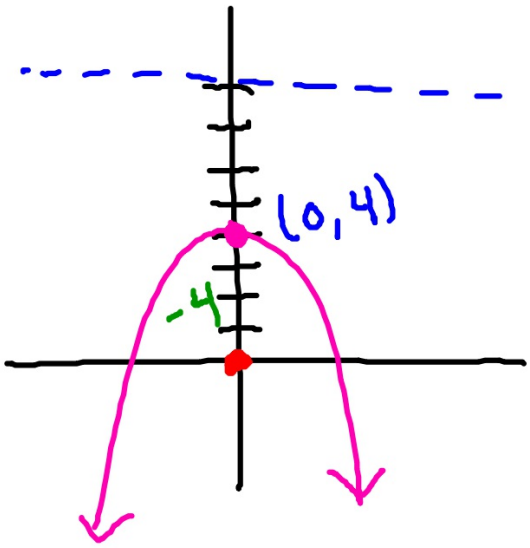
$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 4(3)(x-0)$

$(y-2)^2 = 12(x)$

$(y-2)^2 = 12x$

3. Focus: (0, 0) Directrix: y = 8



$(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4(-4)(y-4)$

$x^2 = -16(y-4)$

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