One-to-one and Inverse Functions

Review:

A _______________is any set of ordered pairs.

A _______________ is a set of ordered pairs where x is not repeated.

A _______________ function does not have any y values repeated.

Only_____________ functions can have _______________ functions.

What is an Inverse?

An inverse relation is a relation that performs the opposite operation on *x* (the domain).

Examples:

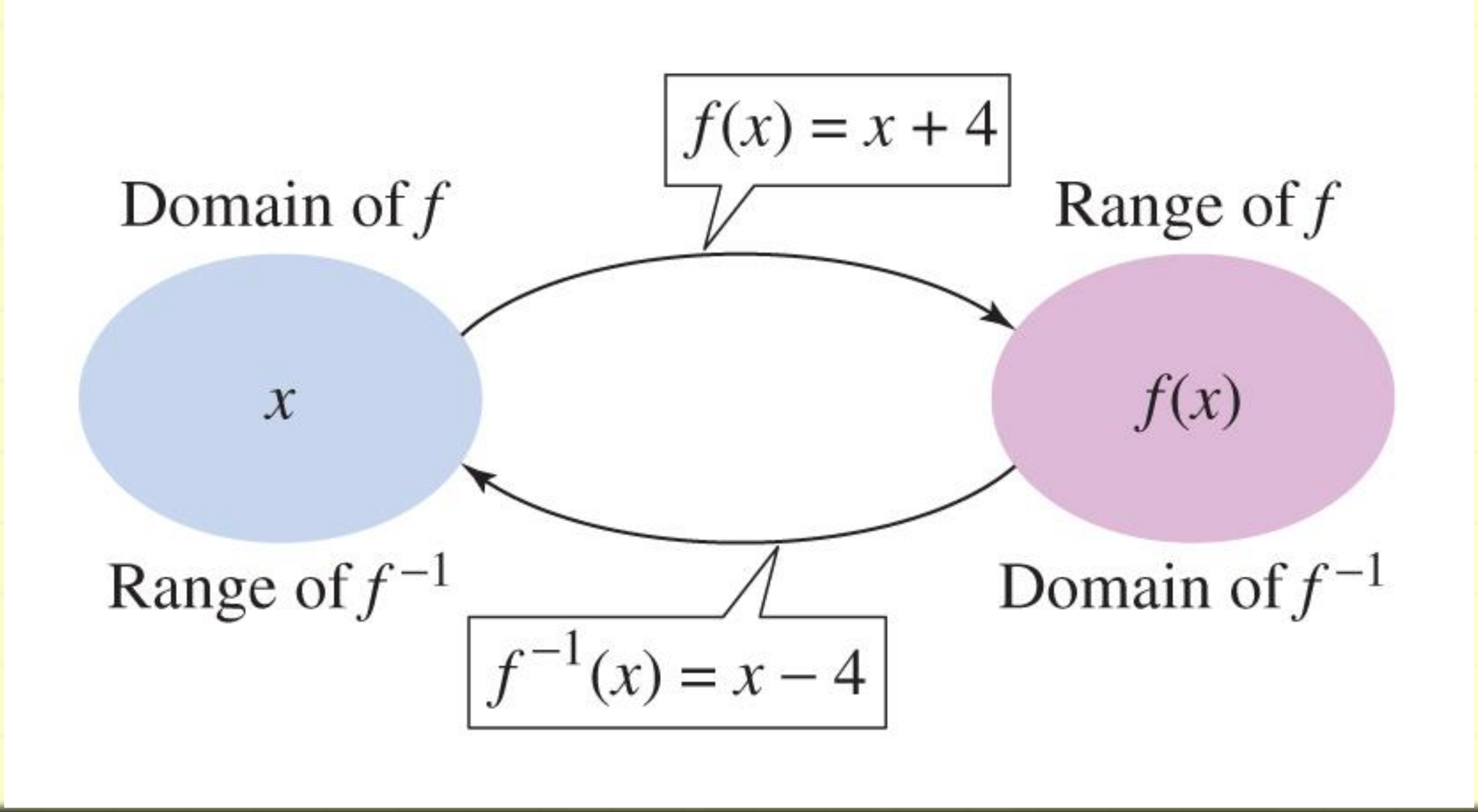
$f(x) = x - 3$ $\quad$ $f^{-1}(x) = x + 3$

$g(x) = \sqrt{x}$, $x \geq 0$ $\quad$ $g^{-1}(x) = x^2$, $x \geq 0$

$h(x) = 2x$ $\quad$ $h^{-1}(x) = \frac{1}{2}x$

$k(x) = -x + 3$ $\quad$ $k^{-1}(x) = -(x - 3)$

Section 1.9 : Illustration of the Definition of Inverse Functions



The ordered pairs of the function f are *reversed* to produce the ordered pairs of the inverse relation.

Example: Given the function $f = \{(1, 1), (2, 3), (3, 1), (4, 2)\}$, its domain is $\{1, 2, 3, 4\}$ and its range is $\{1, 2, 3\}$.

The inverse _________ of f is $\{(1, 1), (3, 2), (1, 3), (2, 4)\}$.

The *domain* of the inverse relation is the *range* of the original function.

The *range* of the inverse relation is the *domain* of the original function.

How do we know if an inverse function exists?

- Inverse functions only exist if the original function is one to one. Otherwise it is an inverse relation and cannot be written as $f^{-1}(x)$.

- What does it mean to be one to one?

That there are no repeated y values.

Horizontal Line Test

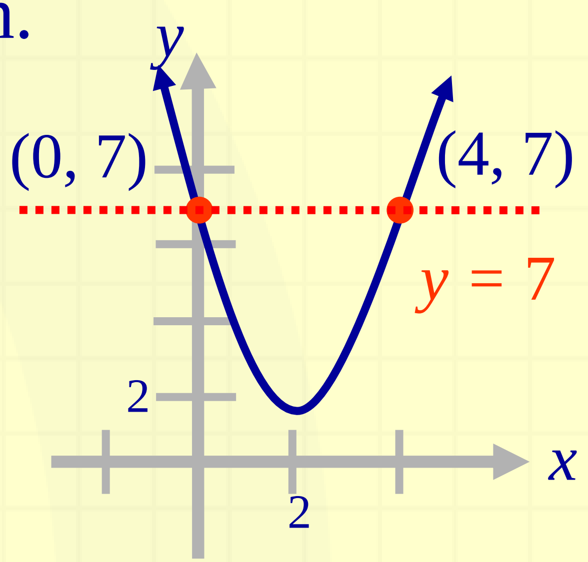
Used to test if a function is one-to one

If the line intersection more than once then it is not one to one.

Therefore there is not inverse function.

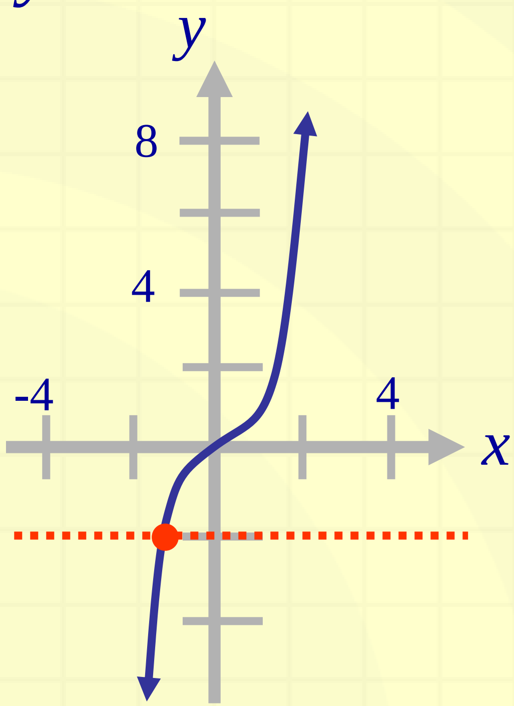
Example: The function $y = x^2 - 4x + 7$ is **not one-to-one** because a horizontal line can intersect the graph twice.

Examples points: (0, 7) & (4, 7).

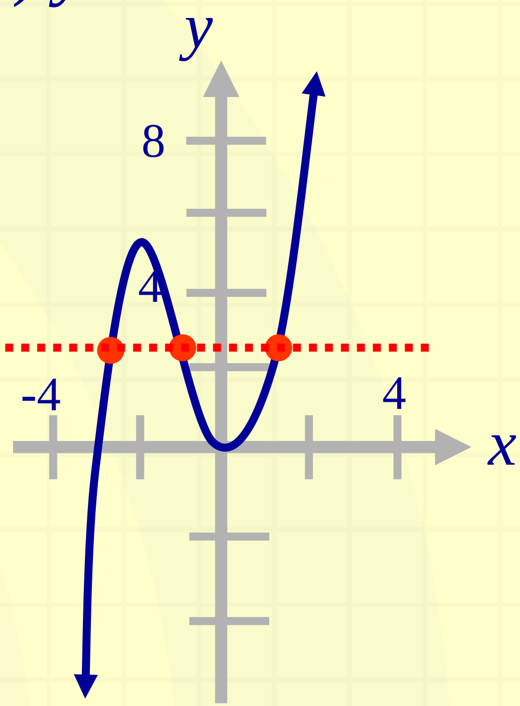


Example: Apply the *horizontal line test* to the graphs below to determine if the functions are one-to-one.

a) $y = x^3$



b) $y = x^3 + 3x^2 - x - 1$



one-to-one

The Inverse is a Function

not one-to-one

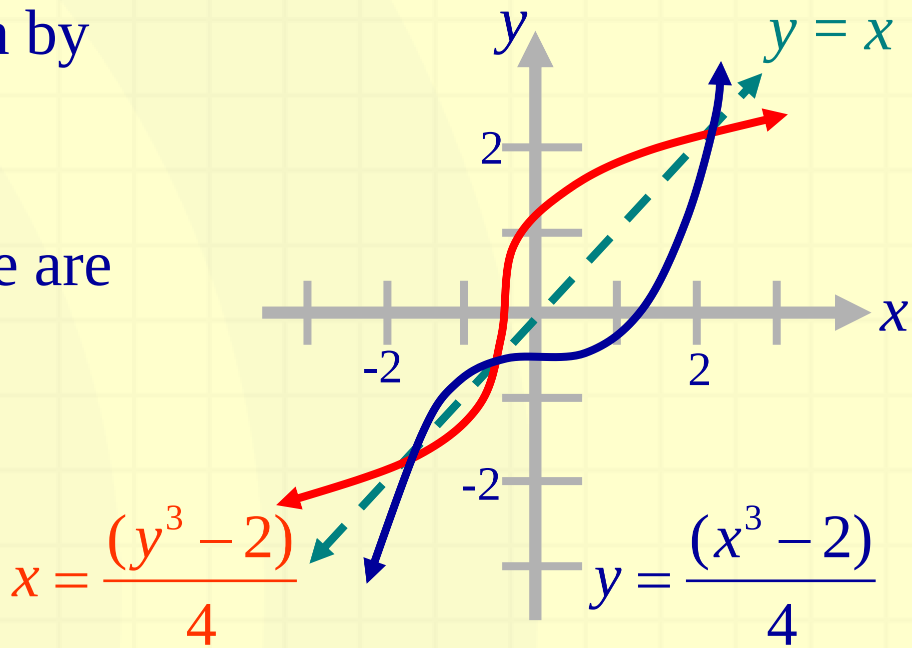
The Inverse is a Relation

The graphs of a relation and its inverse are reflections in the line $y = x$.

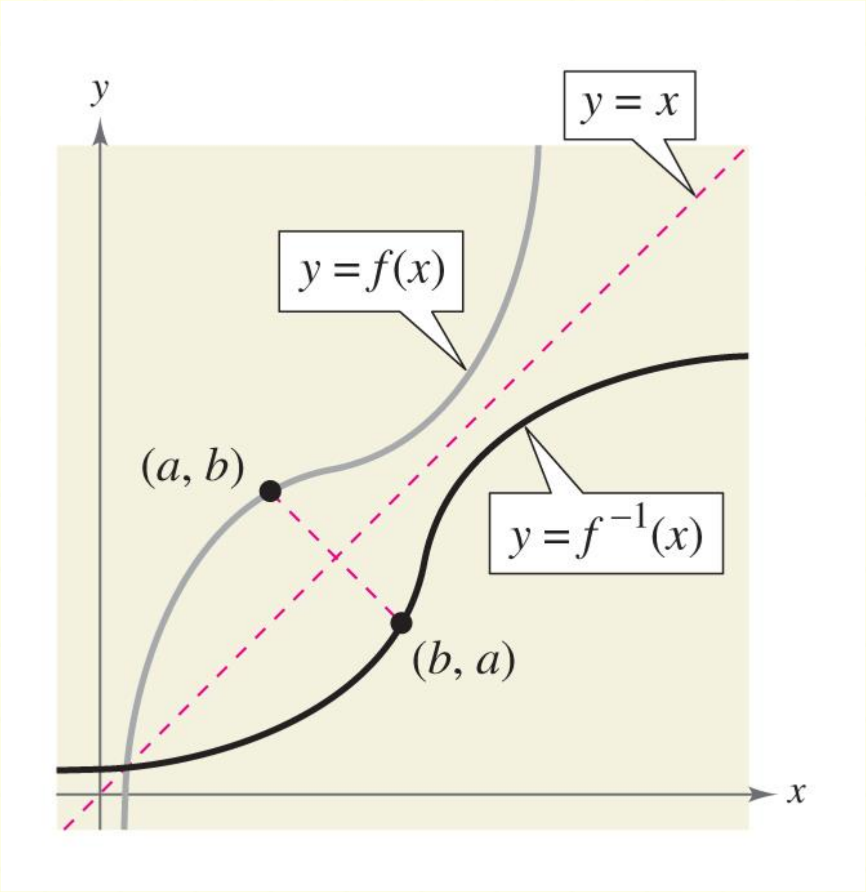
Example: Find the graph of the inverse relation *geometrically* from the graph of $f(x) = \frac{1}{4}(x^3 - 2)$

The ordered pairs of *f* are given by the equation $y = \frac{1}{4}(x^3 - 2)$.

The ordered pairs of the inverse are given by $x = \frac{1}{4}(y^3 - 2)$.



Section 1.9 : Figure 1.93, Graph of an Inverse Function



Functions and their inverses are symmetric over the line y =x

To find the inverse of a relation *algebraically*, interchange x and y and solve for y.

Example: Find the inverse relation *algebraically* for the function $f(x) = 3x + 2$.

DETERMINING IF 2 FUNCTIONS ARE INVERSES:

The inverse function "undoes" the original function, that is, $f^{-1}(f(x)) = x$.

The function is the *inverse* of its inverse function, that is, $f(f^{-1}(x)) = x$.

Example: The inverse of $f(x) = x^3$ is $f^{-1}(x) = \sqrt[3]{x}$.

$f^{-1}(f(x)) = \sqrt[3]{x^3} = x$ and $f(f^{-1}(x)) = (\sqrt[3]{x})^3 = x$.

Example: Verify that the function $g(x) = \dfrac{x+1}{2}$ is the *inverse* of $f(x) = 2x - 1$.

$$g(f(x)) = \frac{(f(x)+1)}{2} = \frac{((2x-1)+1)}{2} = \frac{2x}{2} = x$$

$$f(g(x)) = 2g(x) - 1 = 2\left(\frac{x+1}{2}\right) - 1 = (x + 1) - 1 = x$$

It follows that $g = f^{-1}$.

Now Try: Page 99 #13, 15, 23

pg 101 # 69, 71, 73

Review of Today's Material

- A function must be 1-1 (pass the horizontal line test) to have an inverse function (written $f^{-1}(x)$) otherwise the inverse is a relation (y =)

- To find an inverse: 1) Switch x and y

  2) Solve for y

- Original and Inverses are symmetric over y =x
- "   "   " have reverse domain & ranges

- Given two relations to test for inverses.

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ **both must be true**

Practice Problems and Homework

- Page 99-100

  # 16, 18, 20, 24, 39, 41, 43

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