Calculus 1 Name______________________

Review: Domain, Range, Piecewise and Limits (ans) Block ______Date____________

Fill in the missing information for the following functions.

1) $f(x) = \dfrac{3x^2 - 6x}{x - 2}$

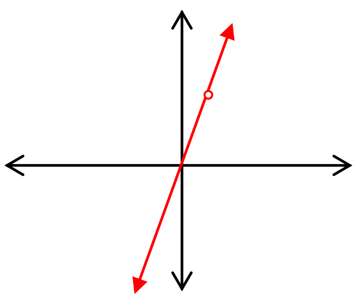
$\dfrac{3x(x-2)}{x-2}$

$3x$

Hole: $(2,6)$

VA: none

HA: none



Domain: $(-\infty,2)\cup(2,\infty)$

Range: $(-\infty,6)\cup(6,\infty)$

2) $f(x) = \dfrac{2x^2 - 3x + 1}{3x^2 + x - 4}$

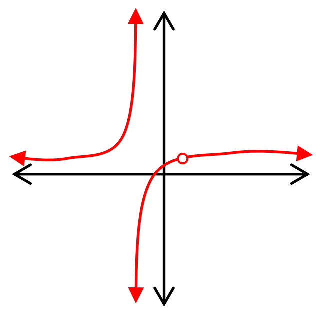
$\dfrac{(2x-1)(x-1)}{(3x+4)(x-1)}$

$\dfrac{(2x-1)}{(3x+4)}$

Hole: $\left(1,\dfrac{1}{7}\right)$

VA: $x = -\dfrac{4}{3}$

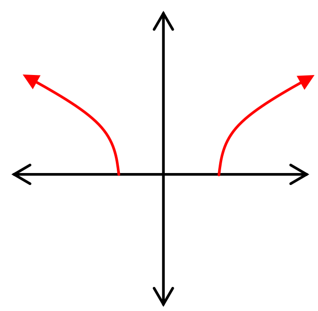
HA: $y = \dfrac{2}{3}$



Domain: $\left(-\infty,-\dfrac{4}{3}\right)\cup\left(-\dfrac{4}{3},1\right)\cup(1,\infty)$

Range: $\left(-\infty,\dfrac{1}{7}\right)\cup\left(\dfrac{1}{7},\dfrac{2}{3}\right)\cup\left(\dfrac{2}{3},\infty\right)$

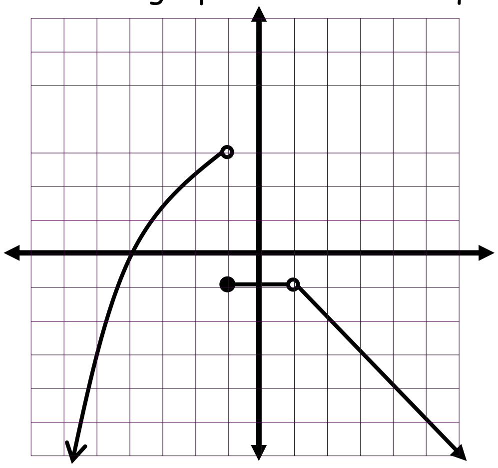
3) $g(x) = \sqrt{x^2 - 9}$



Domain: $(-\infty,-3]\cup[3,\infty)$

Range: $[0,\infty)$

Use the graph to find the requested values.



4) $\lim_{x\to -1^-} f(x) = 3$

5) $\lim_{x\to -4} f(x) = 0$

6) $\lim_{x\to -1} f(x) = \text{DNE}$

7) $\lim_{x\to \infty} f(x) = -\infty$

8) $\lim_{x\to -\infty} f(x) = -\infty$

9) $f(1) = \varnothing$

10) $\lim_{x\to 3^+} f(x) = -3$

11) $f(-4) = 0$

12) $f(-1) = -1$

Using the given piecewise function, find the requested values and justify your answers.

$$f(x) = \begin{cases} x^2 - 6x - 2 & \text{if} \quad x \le -1 \\ \dfrac{x^2+4}{x+2} & \text{if} \quad -1 < x \le 3 \\ -2x+5 & \text{if} \quad x > 3 \end{cases}$$

13) $\lim_{x\to -1^-} f(x) = 5$

$(-1)^2 - 6(-1) - 2$

14) $\lim_{x\to -1^+} f(x) = 5$

$\dfrac{(-1)^2+4}{-1+2} = 5$

15) $\lim_{x\to -1} f(x) = 5$

16) $\lim_{x\to 3} f(x) = \text{DNE}$

$\dfrac{(3)^2+4}{3+2} = \dfrac{13}{5}$

$-2(3)+5$

-1

17) $f(2) = 2$

$\dfrac{(2)^2+4}{2+2} = \dfrac{8}{4} = 2$

18) $f(4) = -3$

$-2(4)+5$

-3

19) $\lim_{x\to 4.5} f(x) = -4$

$-2(4.5)+5$

$-9+5$

-4

20) $f(-1) = 5$

$(-1)^2 - 6(-1) - 2$

21) $f(3) = \dfrac{13}{5}$

$\dfrac{(3)^2+4}{3+2} = \dfrac{13}{5}$

Find the following limits. If a graphing calculator was used, write how it was used.

22) $\lim_{x\to -\infty} \dfrac{x^2-2x-8}{x-4} = -\infty$

23) $\lim_{x\to 4} \dfrac{x^2-2x-8}{x-4} = 6$

24) $\lim_{x\to 3} \dfrac{5x-9}{x^2-5} = \dfrac{3}{2}$

25) $\lim_{x\to \infty} \dfrac{2-6x-3x^2}{2x^3+8x-2} = 0$

26) $\lim_{x\to 2} \dfrac{5x}{x^2+3x-10} = \text{DNE}$

27) $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

28) $\lim_{x\to 1}\frac{x^2+9x-10}{x^3-1}=\frac{11}{3}$

29) $\lim_{x\to\infty}\frac{4x^3-6x^6-6}{5x^6+3x^3-5x^2}=-\frac{6}{5}$

30) $\lim_{x\to\infty}\frac{x-6}{\sqrt{2x^2+5}}=\frac{\sqrt{2}}{2}$

31) $\lim_{x\to 1}\frac{x}{|x-1|}=\infty$

32) $\lim_{x\to 3}9=9$

33) $\lim_{x\to-\infty}4x-6=-\infty$

Determine if the following functions are continuous or not. If not, state its type discontinuity and where it occurs. If it is removable, then create a new function that is continuous.

34) $f(x)=\frac{x^3+27}{x+3}$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & 0 & 27 \\ & & -3 & 9 & -27 \\ \hline & 1 & -3 & 9 & 0 \end{array}$$

$f(x)=\frac{(x+3)(x^2-3x+9)}{(x+3)}$

$f(x)=x^2-3x+9$

Discontinuous

Hole (-3, 27)

$$g(x)\begin{cases}\frac{x^3+27}{x+3}, & x\neq -3\\ 27, & x=-3\end{cases}$$

35) $f(x)=\frac{x-6}{x^2-x-6}$

$f(x)=\frac{x-6}{(x-3)(x+2)}$

Discontinuous

Asymptote at x = 3 and x = -2

36) $f(x)=\frac{2x-6}{|x-3|}$

Discontinuous

Jump at x = 3

Determine whether the following piecewise function is continuous. Show all work that leads to you decision. (A sketch may be helpful in determining your answer)

37)

$$f(x)=\begin{cases}-3 & x\leq -1\\ x^3 & -1<x<2\\ 2x+4 & x\geq 2\end{cases}$$

at $x=-1$

1st Eqn *2nd Eqn*

$(-1,-3)$ $(-1)^3$

$(-1,-1)$

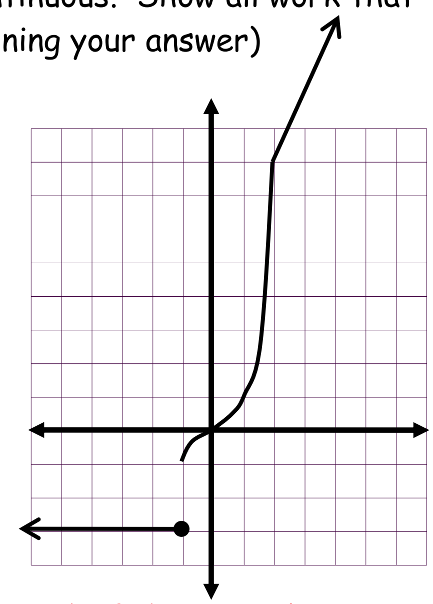
Discontinuous (jump) at x = -1

at $x=2$

2nd Eqn *3rd Eqn*

$(2)^3$ $2(2)+4$

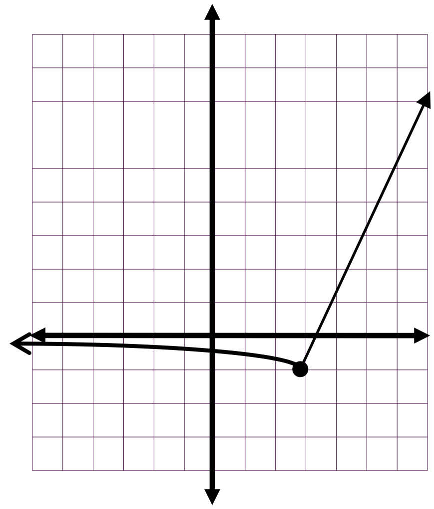
$(2,8)$ $(2,8)$



Note: In addition to the y-values being equal, one (and only one) of the inequalities must contain an equal sign in order for the graph to be continuous at both x = -1 and x = 2.

38)

$$f(x)=\begin{cases}\dfrac{1}{x-4} & x<3\\ 2x-7 & x\ge 3\end{cases}$$



The first equation is continuous everywhere but x=4, however since the domain is x<3, the graph is continuous everyone on its domain.

at x=3

1st eqn

$$\frac{1}{3-4}=\frac{1}{-1}=-1$$

2nd eqn

$$2(3)-7=6-7=-1$$

Note: In addition to the y-values being equal, one (and only one) of the inequalities must contain an equal sign in order for the graph to be continuous at x = 3.

For each of the following, find the value of 'a' that will make *f(x)* continuous for all values of x.

39) $f(x)=\begin{cases} ax+1 & x<2\\ a+\sqrt{x+14} & x\ge 2\end{cases}$

$a(2)+1$ $\quad$ $a+\sqrt{2+14}$

$2a+1$ $\quad$ $a+\sqrt{16}$

$a+4$

$2a+1=a+4$

$a=3$

40) $f(x)=\begin{cases} ax^2-2 & x\le -6\\ -5x-8 & x>-6\end{cases}$

$a(-6)^2-2$ $\quad$ $-5(-6)-8$

$36a-2$ $\quad$ $30-8$

$36a-2 = 22$

$36a=24$

$a=\dfrac{24}{36}$

$a=\dfrac{2}{3}$