Calculus I
Review: PVA and Area Between Two Curves

Name Answer Key
Block _____ Date _____

1) A ball is thrown vertically upward from the ground with an initial velocity of 60 feet per second.
a) What are the acceleration, velocity and position functions?
b) When does the ball reach the peak of its journey?
c) How high will the ball travel?
d) What is its velocity at 3.25 seconds?
e) What is the total distance traveled and the displacement of the ball?

a(t) = -32
v(t) = -32t + c
v(0) = 60
60 = -32(0) + c
v(t) = -32t + 60
$s(t) = -16t^2 + 60t + c$
s(0) = 0
$0 = -16(0)^2 + 60(0) + c$
$s(t) = -16t^2 + 60t$

b) When it is at the Peak Height (Max Height)
v(t) = 0
0 = -32t + 60
t = 1.875 seconds

c) What is the max height?
s(1.875) = 56.25 feet

d) v(3.25) = -44ft/sec

e) You must find the time when the ball hits the ground in order to do this problem.
s(t) = 0
t = 3.75 secs

Displacement
$\int_0^{3.75}(-32t+60)dt = 0ft$
This makes sense since it was thrown from the ground and ends on the ground.

Distance
$\int_0^{3.75}\left|(-32t+60)\right|dt = 112.5ft$

2) The function for the velocity of a particle is $v(t) = t^3 - 2t^2 + 1$ (feet per sec). At three seconds, its position is 6.25 feet.
a) What are the acceleration and position functions?
b) What are the particle's initial acceleration, velocity and position?
c) What is the total distance traveled and the displacement of the particle from 0.5 seconds to 3 seconds?

a(t) = derv of v(t)
$a(t) = 3t^2 - 4t$

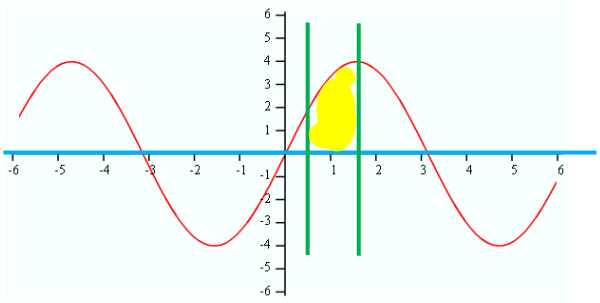
s(t) = integral of v(t)
$s(t) = \frac{1}{4}t^4 - \frac{2}{3}t^3 + t + c$
$s(3) = 6.25$
$6.25 = \frac{1}{4}(3)^4 - \frac{2}{3}(3)^3 + 3 + c$
$s(t) = \frac{1}{4}t^4 - \frac{2}{3}t^3 + t + 1$

b) Initial Values
$a(0) = 0\ ft/sec^2$
v(0) = 1 ft/sec
s(0) = 1 ft

c) Displacement
$\int_{.5}^{3}(t^3 - 2t^2 + 1)dt = 4.818ft$

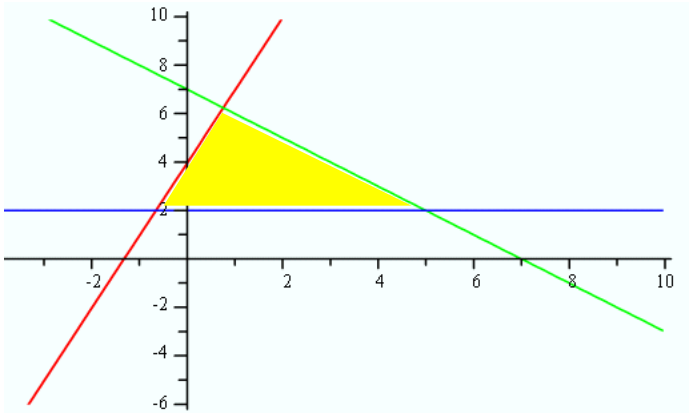
Distance
$\int_{.5}^{3}\left|(t^3 - 2t^2 + 1)\right|dt = 4.969ft$

3) y = 4sin(x), y = 0, $x = \frac{\pi}{8}$, $x = \frac{\pi}{2}$



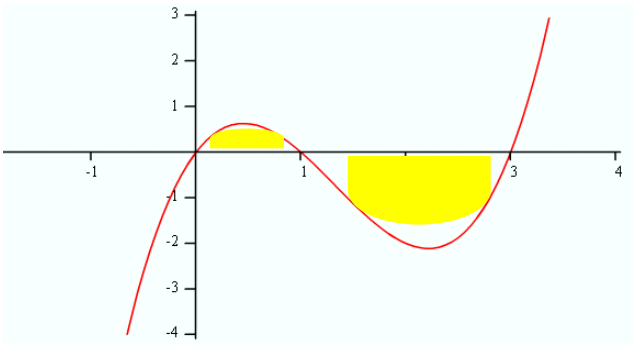
$\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} (4\sin x)dx = 3.696$

4) $y = 3x + 4$, $y = -x + 7$, *and* $y = 2$



$\int_{2}^{6.25} \left((-y + 7) - \left(\frac{1}{3}y - \frac{4}{3} \right) \right) dy = 12.042$

5) $y = x^3 - 4x^2 + 3x$ *and* $y = 0$

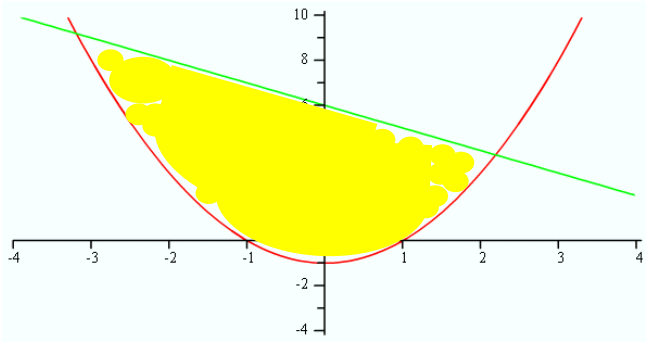


$\int_0^1 \left((x^3 - 4x^2 + 3x) - 0 \right) dx + \int_1^3 \left(0 - (x^3 - 4x^2 + 3x) \right) dx$

.417 + 2.667

3.084

6) $y = x^2 - 1$ *and* $x + y = 6$



$\int_{-3.193}^{2.1931} \left((-x + 6) - (x^2 - 1) \right) dx$

26.028

7) An object is fired straight up. At 3 seconds the velocity of the object is -51 feet per second. At 7 seconds its position is 731 feet.
a) What are the acceleration, velocity and position functions?
b) When does it hit the water and what is the impact velocity?
c) What is the maximum height of the object?
d) What is the displacement of the object?
e) What is the total distance traveled by the object?

a(t) = -32
v(t) = -32t + c
v(3) = -51
-51 = -32(3) + c
v(t) = -32t + 45
$s(t) = -16t^2 + 45t + c$
s(7) = 731
$731 = -16(7)^2 + 45(7) + c$
$s(t) = -16t^2 + 45t + 1200$

b) When it hits the water?
s(t) = 0
$0 = -16t^2 + 45t + 1200$
t = 10.18 seconds
Impact Velocity
v(10.18) = -280.76ft/sec

c) Max Height
v(t) = 0
0 = -32t + 45
t = 1.406
s(1.406) = 1231.641ft

d) Displacement
$\int_0^{10.18}(-32t+45)dt = 1200.018ft \text{ below}$

Distance
$\int_0^{10.18}\left|(-32t+45)\right|dt = 1263.200ft$

8) An object moves horizontally with an acceleration defined by $a(t) = 12t - 38$. At 2 seconds, the velocity is 5 feet per second. At 4 seconds, its position is -2 feet.
a) What are the velocity and position functions?
b) What are the acceleration, velocity and position of the object at 6 seconds?
c) What are the position and velocity when the acceleration is zero?
d) What are the values of the displacement and the distance traveled by the particle in the first 5.5 seconds?

a(t) = 12t – 38
$v(t) = 6t^2 - 38t + c$
v(2) = 5
$5 = 6(2)^2 - 38(2) + c$
$v(t) = 6t^2 - 38t + 57$

$s(t) = 2t^3 - 19t^2 + 57t + c$
s(4) = -2
$-2 = 2(4)^3 - 19(4)^2 + 57(4) + c$
$s(t) = 2t^3 - 19t^2 + 57t - 54$

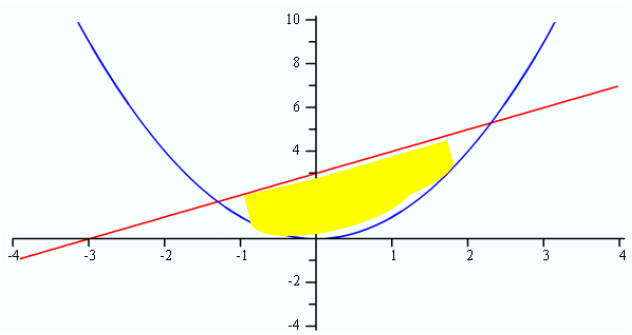
b) at 6 seconds
$a(6) = 34 \text{ ft/sec}^2$
v(6) = 45 ft/sec
s(6) = 36 ft

c) a(t) = 0
0 = 12t – 38
t = 3.167
v(3.167) = -3.167ft/sec
s(3.167) = -.52ft

d) Displacement
$\int_0^{5.5}\left(6t^2 - 38t + 57\right)dt = 71.5\text{ft to the right}$

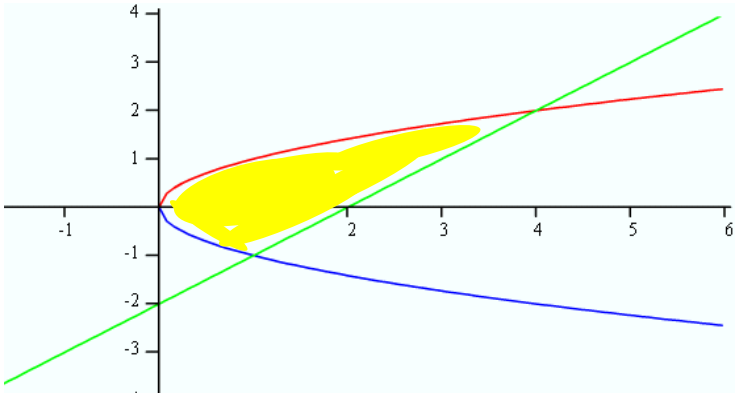
Distance
$\int_0^{5.5}\left|\left(6t^2 - 38t + 57\right)\right|dt = 77.635\text{ft}$

9) $x = y - 3$ *and* $y = x^2$



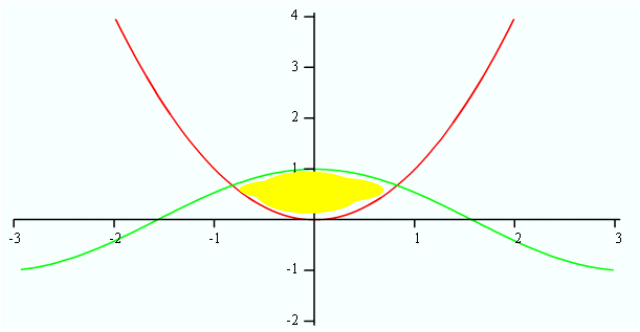
$\int_{-1.303}^{2.303}\left((x+3)-\left(x^2\right)\right)dx = 7.812$

10) $x = y^2$ and $y = x - 2$



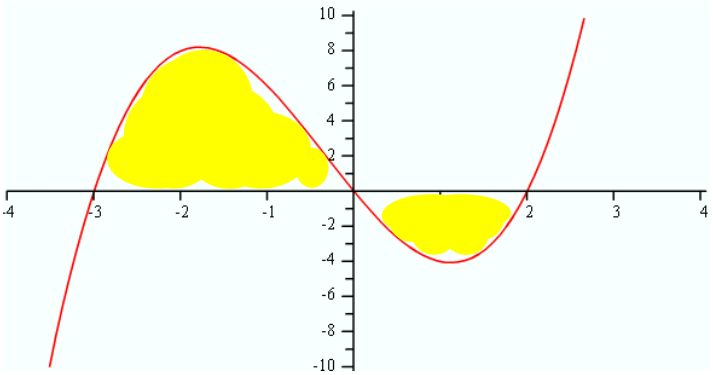
$\int_{-1}^{2}\left((y+2)-\left(y^2\right)\right)dy = 4.5$

11) $y = x^2$ and $y = \cos(x)$



$\int_{-.824}^{.824}\left((\cos x)-\left(x^2\right)\right)dx = 1.095$

12) $y = x^3 + x^2 - 6x$ and $y = 0$



$\int_{-3}^{0}\left(\left(x^3+x^2-6x\right)-(0)\right)dx + \int_{0}^{2}\left((0)-\left(x^3+x^2-6x\right)\right)dx$

15.75 + 5.333

21.083