# Unit 9: Rational Numbers

- Why are common denominators needed to add or subtract fractions?
- Why is the product of a fraction and its reciprocal equal to 1?
- Why is the reciprocal used when dividing fractions?

# 5.1 Rational Numbers

**Goal:** Write fractions as decimals and vice versa.

**Vocabulary**

Rational number:

Terminating decimal:

Repeating decimal:

**Example 1** *Identifying Rational Numbers*

Show that the number is rational by writing it as a quotient of two integers.

**a.** 3 **b.** $-12$ **c.** $4\frac{2}{3}$ **d.** $-2\frac{1}{4}$

**Solution**

**a.** Write the integer 3 as ____.

**b.** Write the integer $-12$ as ____ or ____. These fractions are ____.

**c.** Write the mixed number $4\frac{2}{3}$ as the improper fraction ____.

**d.** Think of $-2\frac{1}{4}$ as the opposite of ____. First write ____ as ____. Then you can write $-2\frac{1}{4}$ as ____. To write ____ as a quotient of two integers, you can assign the negative sign to either the ____ or the ____. You can write ____ or ____.

## Example 2 *Writing Fractions as Decimals*

**a.** Write $\frac{5}{16}$ as a decimal.

**a.**
$$16\overline{)5.0000}$$
$$\begin{array}{r} 0.3125 \\ 16\overline{)5.0000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

**Answer:** The remainder is ☐, so the decimal is a ☐ decimal:
$\frac{5}{16} = 0.3125$.

**b.** Write $\frac{4}{9}$ as a decimal.

**b.**
$$\begin{array}{r} 0.44\ldots \\ 9\overline{)4.00} \\ \underline{36} \\ 40 \\ \underline{36} \end{array}$$

**Answer:** Use a bar to show the ☐ in the ☐ decimal:
$\frac{4}{9} = 0.\overline{4}$.

**Checkpoint** **Write the fraction or mixed number as a decimal.**

| | |
|---|---|
| **1.** $\frac{7}{20}$ | **2.** $-\frac{3}{5}$ |
| **3.** $2\frac{12}{25}$ | **4.** $\frac{23}{40}$ |

## Example 3 *Using Decimals to Compare Fractions*

**Compare $\frac{42}{48}$ and $\frac{27}{30}$.**

$\frac{42}{48} =$ ☐ and $\frac{27}{30} =$ ☐ Divide.

**Answer:** Compare the decimals. ☐ > ☐,
so $\frac{27}{30}$ ☐ $\frac{42}{48}$.

## Example 4 *Writing Terminating Decimals as Fractions*

**a.** $0.03 = \square$ 3 is in $\square$ place, so denominator is $\square$.

**b.** $-9.4 = \square$ 4 is in $\square$ place, so denominator is $\square$.

$= \square$ Simplify fraction.

## Example 5 *Writing a Repeating Decimal as a Fraction*

**To write $0.\overline{81}$ as a fraction, let $x = 0.\overline{81}$.**

**1.** Because $0.\overline{81}$ has 2 repeating digits, multiply each side of $x = 0.\overline{81}$ by $\square$, or $\square$. Then $\square x = \square$.

$\square x = \square$

**2.** Subtract $x$ from $\square x$.

$$- \quad (x = 0.\overline{81})$$

$$\square x = \square$$

**3.** Solve for $x$ and simplify.

$$\frac{\square x}{\square} = \frac{\square}{\square}$$

$$x = \square$$

**Answer:** The decimal $0.\overline{81}$ is equivalent to the fraction $\square$.

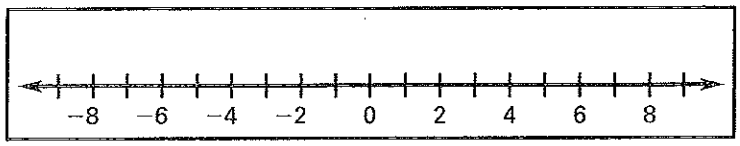
**Checkpoint** **Write the decimal as a fraction or mixed number.**

| | |
|---|---|
| **5.** 0.8 | **6.** $-3.75$ |
| **7.** $0.\overline{12}$ | **8.** $5.\overline{3}$ |

**Example 6** ***Ordering Rational Numbers***

**Order the numbers $-\frac{17}{2}$, $-1.35$, $5.67$, $-6$, $\frac{11}{3}$, $-\frac{13}{4}$ from least to greatest.**

Graph the numbers on the number line. You may want to write improper fractions as mixed numbers.




**Answer:** Read the numbers graphed on the number line from left to right: ____, ____, ____, ____, ____, ____.

# 9.4 Real Numbers

**Goal:** Compare and order real numbers.

**Vocabulary**

Irrational number: _____

Real numbers: _____

## Example 1 *Classifying Real Numbers*

| Number | Decimal Form | Decimal Type | Type |
|---|---|---|---|
| a. $\frac{7}{10}$ | $\frac{7}{10} =$ ____ | _____ | _____ |
| b. $\frac{2}{9}$ | $\frac{2}{9} =$ _____ $=$ ____ | _____ | _____ |
| c. $\sqrt{5}$ | $\sqrt{5} =$ _____ | _____ _____ | _____ |

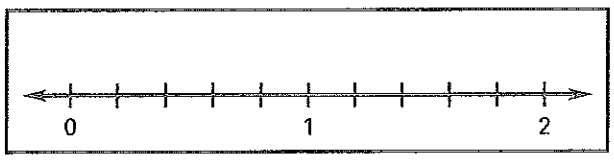
The square root of any whole number that is not a perfect square is irrational.

## Example 2 *Comparing Real Numbers*

Copy and complete $\sqrt{3}$ ? $\frac{6}{5}$ using $<$, $>$, or $=$.

**Solution**

Graph $\sqrt{3}$ and $\frac{6}{5}$ on a number line.



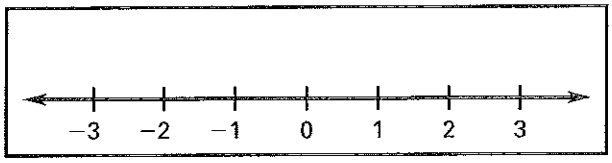

$\sqrt{3}$ is to the ____ of $\frac{6}{5}$.

**Answer:** $\sqrt{3}$ ____ $\frac{6}{5}$

## Example 3 *Ordering Real Numbers*

**Use a number line to order the numbers $\frac{\sqrt{6}}{2}$, $-2.2$, $\frac{5}{2}$, and $-2\sqrt{2}$ from least to greatest.**

Graph the numbers on a number line and read them from left to right.




**Answer:** From least to greatest, the numbers are _____.

**Checkpoint** **Tell whether the number is *rational* or *irrational*.**

| | | | |
|---|---|---|---|
| 1. $\frac{8}{11}$ | 2. $\sqrt{7}$ | 3. $\sqrt{49}$ | 4. $\sqrt{\frac{2}{5}}$ |

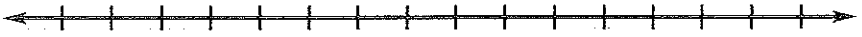
**Copy and complete the statement using $<$, $>$, or $=$.**

| | |
|---|---|
| 5. $-\frac{3}{2}$ ? $-\sqrt{3}$ | 6. $\sqrt{10}$ ? 3.5 |

**Checkpoint** **Use a number line to order the numbers from least to greatest.**

**7.** $5.9, 3\sqrt{5}, \frac{27}{5}, \sqrt{35}$

**8.** $\sqrt{21}, -4.6, -\frac{9}{2}, -2\sqrt{5}$



**Example 4** ***Using Irrational Numbers***

**Speed** After an accident, a police officer finds that the length of a car's skid marks is 98 feet. The car's speed $s$ (in miles per hour) and the length $\ell$ (in feet) of the skid marks are related by $s = \sqrt{27\ell}$. Find the car's speed to the nearest tenth of a mile per hour.

**Solution**

$s = \sqrt{27\ell}$ Write formula.

$= \sqrt{27 \cdot \square}$ Substitute value.

$= \sqrt{\square}$ Multiply.

$\approx \square$ Approximate using a calculator.

**Answer:** The car's speed was about $\square$ miles per hour.

# 5.2 Adding and Subtracting Like Fractions

**Goal:** Add and subtract like fractions.

**Adding and Subtracting Like Fractions**

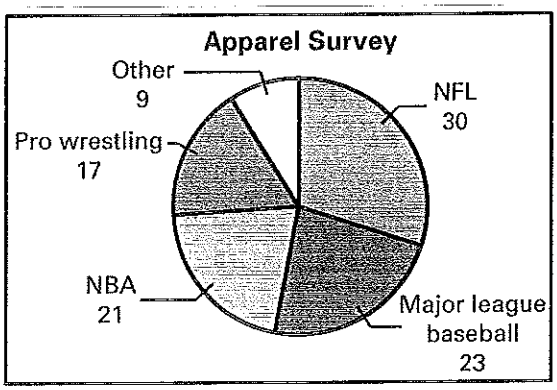
**Words** To add or subtract fractions with the same denominator, write the sum or difference of the numerators over the denominator.

**Numbers** $\frac{4}{9} + \frac{1}{9} = \frac{\square}{9}$ $\qquad \frac{9}{11} - \frac{2}{11} = \frac{\square}{11}$

**Algebra** $\frac{a}{c} + \frac{b}{c} = \frac{\square}{c}, c \neq 0$ $\qquad \frac{a}{c} - \frac{b}{c} = \frac{\square}{c}, c \neq 0$

**Example 1** *Adding Like Fractions*

A survey asked 100 students ages 7 to 11 what sports apparel they prefer to wear. The circle graph at the right summarizes their responses. What fraction of the students responded either major league baseball or NBA?




**Solution**

To find the fraction of the students who responded either major league baseball or NBA, find the sum of $\square$ and $\square$.

$\square + \square = \square$ Write sum of numerators over denominator.

$= \square = \square$ Add. Then simplify.

**Answer:** The fraction of the students who prefer either major league baseball or NBA apparel is $\square$.

## Example 2 *Subtracting Like Fractions*

When you perform operations with negative fractions, be sure to assign a negative sign in front of a fraction to the numerator of the fraction.

**a.** $-\frac{4}{9} - \frac{3}{9} = \square$ — Write difference of numerators over denominator.

$= \square$ — Subtract.

**b.** $\frac{2}{11} - \left(-\frac{4}{11}\right) = \square + \square$ — To subtract $-\frac{4}{11}$, add $\square$.

$= \square$ — Write sum of numerators over denominator.

$= \square$ — Add.

**Checkpoint** **Find the sum or difference.**

| | |
|---|---|
| **1.** $\frac{2}{7} + \frac{4}{7}$ | **2.** $\frac{3}{13} - \frac{8}{13}$ |

## Example 3 *Adding and Subtracting Mixed Numbers*

**a.** $4\frac{3}{7} + 3\frac{6}{7} = \square + \square$ — Write mixed numbers as improper fractions.

$= \square$ — Write sum of numerators over denominator.

$= \square = \square$ — Add. Then write fraction as mixed number.

**b.** $11\frac{3}{10} - 8\frac{9}{10} = \square - \square$ — Write mixed numbers as improper fractions.

$= \square$ — Write difference of numerators over denominator.

$= \square = \square$ — Subtract. Then write fraction as mixed number.

## Example 4 *Simplifying Variable Expressions*

**a.** $\frac{4a}{21} + \frac{10a}{21} = \square$ Write sum of numerators over denominator.

$= \square$ Add.

$= \square$ Simplify.

**b.** $-\frac{9}{5b} - \left(-\frac{4}{5b}\right) = \square + \square$ To subtract $-\frac{4}{5b}$, add $\square$.

$= \square$ Write sum of numerators over denominator.

$= \square$ Add.

$= \square$ Simplify.

**Checkpoint** **Find the sum or difference.**

| | |
|---|---|
| **3.** $3\frac{2}{11} + 5\frac{4}{11}$ | **4.** $-4\frac{5}{13} - 3\frac{6}{13}$ |
| **5.** $\frac{2a}{25} + \frac{8a}{25}$ | **6.** $-\frac{17}{3c} - \left(-\frac{5}{3c}\right)$ |

# 5.3 Adding and Subtracting Unlike Fractions

**Goal:** Add and subtract unlike fractions.

**Example 1** *Adding and Subtracting Fractions*

**a.** $\frac{7}{15} + \frac{1}{5} = \frac{7}{15} + \square$ Write $\frac{1}{5}$ using LCD.

$= \square$ Write sum of numerators over denominator.

$= \square$ Add.

$= \square$ Simplify.

**b.** $-\frac{2}{3} - \frac{3}{4} = \square - \square$ Write fractions using LCD.

$= \square$ Write difference of numerators over denominator.

$= \square$ Subtract.

$= \square$ Write fraction as a mixed number.

**Checkpoint** **Find the sum or difference.**

| | |
|---|---|
| **1.** $\frac{3}{7} + \frac{5}{21}$ | **2.** $\frac{1}{4} - \frac{3}{10}$ |

## Example 2 *Adding Mixed Numbers*

$-5\frac{1}{6} + \left(-2\frac{3}{10}\right) = \square + \square$ Write mixed numbers as improper fractions.

$= \square + \square$ Write fractions using LCD.

$= \square$ Write sum of numerators over denominator.

$= \square = \square$ Add. Then write fraction as a mixed number.

## Example 3 *Subtracting Mixed Numbers*

You are hiking a $12\frac{1}{5}$-mile trail. You have already hiked $6\frac{1}{2}$ miles. How many more miles do you have to hike before reaching the end of the trail?

**Solution**

Your total hiking distance is $\square$. You have already hiked $\square$. To find the remaining distance, subtract.

$12\frac{1}{5} - 6\frac{1}{2} = \square - \square$ Write mixed numbers as improper fractions.

$= \square - \square$ Write fractions using LCD.

$= \square$ Write difference of numerators over denominator.

$= \square = \square$ Subtract. Then write fraction as a mixed number.

**Answer:** You need to hike $\square$ miles.

**Checkpoint** **Find the sum or difference.**

| | |
|---|---|
| **3.** $4\frac{5}{6} + 2\frac{4}{9}$ | **4.** $-2\frac{1}{3} - 3\frac{3}{7}$ |

**Example 4** ***Simplifying an Expression***

**Simplify the expression $\frac{a}{4} - \frac{a}{8}$.**

$\frac{a}{4} - \frac{a}{8} = \left(\frac{a}{4} \cdot \square\right) - \frac{a}{8}$ Write $\frac{a}{4}$ using LCD.

$= \square - \frac{a}{8}$ Multiply.

$= \square$ Write difference of numerators over denominator.

$= \square$ Subtract.

**Checkpoint** **Find the sum or difference.**

| | |
|---|---|
| **5.** $\frac{b}{3} + \frac{b}{8}$ | **6.** $\frac{c}{5} - \frac{c}{7}$ |

# 5.4 Multiplying Fractions

**Goal:** Multiply fractions and mixed numbers.

**Multiplying Fractions**

**Words** The product of two or more fractions is equal to the product of the numerators over the product of the denominators.

**Numbers** $\frac{3}{5} \cdot \frac{4}{7} = \frac{\square}{\square} = \frac{\square}{\square}$

**Algebra** $\frac{a}{b} \cdot \frac{c}{d} = \frac{\square}{\square}$, where $b \neq 0$ and $d \neq 0$

**Example 1** *Multiplying Fractions*

$\frac{5}{12} \cdot \left(-\frac{3}{20}\right) = \frac{5}{12} \cdot \square$ Assign negative sign to numerator.

$= \square$ Use rule for multiplying fractions.

$= \frac{\square\,\square}{\square\,\square}$ Divide out common factors.

$= \square = \square$ Multiply.

**Checkpoint** **Find the product.**

| | |
|---|---|
| **1.** $\frac{7}{16} \cdot \frac{5}{14}$ | **2.** $\frac{2}{15} \cdot \left(-\frac{5}{18}\right)$ |

## Example 2 *Multiplying a Mixed Number and an Integer*

**Water Use** The showerhead in your home uses $2\frac{1}{2}$ gallons of water per minute. If you take a 7-minute shower, how many gallons of water do you use?

**Solution**

| | | |
|---|---|---|
| Gallons used $=$ Gallons per minute $\cdot$ Number of minutes | | |
| $=$ $\square \cdot \square$ | | Substitute values. |
| $=$ $\square \cdot \square$ | | Write numbers as improper fractions. |
| $=$ $\square$ | | Use rule for multiplying fractions. |
| $=$ $\square$ | | Multiply. |
| $=$ $\square$ | | Write fraction as a mixed number. |

**Answer:** You use $\square$ gallons of water.

## Example 3 *Multiplying Mixed Numbers*

| | |
|---|---|
| $-3\frac{1}{5} \cdot 4\frac{1}{6} = \frac{\square}{\square} \cdot \frac{\square}{\square}$ | Write mixed numbers as improper fractions. |
| $= \frac{\square}{\square}$ | Use rule for multiplying fractions. Divide out common factors. |
| $= \square$ | Multiply. |
| $= \square$ | Write fraction as a mixed number. |

**Checkpoint** **Find the product.**

| | |
|---|---|
| 3. $5\frac{2}{9} \cdot 6$ | 4. $-2\frac{3}{4} \cdot 5\frac{1}{3}$ |

**Example 4** ***Simplifying Expressions***

**Simplify the expression.**

a. $\frac{m}{4} \cdot \left(-\frac{10}{7}\right) = \square$ Use rule for multiplying fractions. Divide out common factor.

$= \square = \square$ Multiply.

b. $\frac{n^4}{12} \cdot \frac{9n^2}{10} = \square$ Use rule for multiplying fractions. Divide out common factor.

$= \square$ Product of powers property

$= \square$ Add exponents.

**Checkpoint** **Simplify the expression.**

| | |
|---|---|
| 5. $\frac{2x}{5} \cdot \frac{3x^2}{8}$ | 6. $-\frac{4y^3}{15} \cdot \frac{5y^6}{16}$ |

# 5.5 Dividing Fractions

**Goal:** Divide fractions and mixed numbers.

**Vocabulary**

Reciprocals:

**Using Reciprocals to Divide**

**Words** To divide by any nonzero number, multiply by its reciprocal.

**Numbers** $\frac{2}{9} \div \frac{3}{7} = \frac{2}{9} \cdot \square = \square$

**Algebra** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \square = \square$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$

**Example 1** *Dividing a Fraction by a Fraction*

$-\frac{3}{7} \div \frac{6}{11} = \square \cdot \square$ Multiply by reciprocal.

$= \frac{\square}{\square}$ Use rule for multiplying fractions. Divide out common factor.

$= \square = \square$ Multiply.

**Check:** To check, multiply the quotient by the divisor to see if you get the dividend:

$\square \cdot \frac{6}{11} = \square$ .

**Example 2** *Dividing a Mixed Number by a Mixed Number*

$2\frac{1}{2} \div \left(-3\frac{3}{4}\right) = \square \div \square$ Write mixed numbers as improper fractions.

$= \square \cdot \square$ Multiply by reciprocal.

$= \frac{\square\square}{\square\square}$ Use rule for multiplying fractions. Divide out common factors.

$= \square = \square$ Multiply.

**Checkpoint** **Find the quotient.**

| | |
|---|---|
| 1. $\frac{8}{21} \div \frac{9}{14}$ | 2. $-\frac{5}{12} \div \frac{5}{24}$ |
| 3. $4\frac{3}{5} \div 1\frac{7}{10}$ | 4. $-3\frac{1}{4} \div 5\frac{1}{2}$ |

## Example 3 *Dividing a Whole Number by a Mixed Number*

**Dogs** You have two dogs that eat about $1\frac{1}{5}$ pounds of dog food per day. How many whole days will a 5-pound bag of dog food last?

**Solution**

Divide to find how long the bag of dog food will last.

| | | | | |
|---|---|---|---|---|
| Number of days | = | Number of pounds in bag | ÷ | Number of pounds eaten per day |

$=$ ☐ ÷ ☐ Substitute values.

$=$ ☐ ÷ ☐ Write numbers as improper fractions.

$=$ ☐ • ☐ Multiply by reciprocal.

$=$ ☐ Use rule for multiplying fractions.

$=$ ☐ Multiply.

$=$ ☐ Write fraction as a mixed number.

**Answer:** A 5-pound bag of dog food will last ☐.

# 5.6 Using Multiplicative Inverses to Solve Equations

**Goal:** Use multiplicative inverses to solve equations.

**Vocabulary**

Multiplicative inverse: ____

**Multiplicative Inverse Property**

**Words** The product of a number and its multiplicative inverse is 1.

**Numbers** $\frac{3}{5} \cdot \frac{5}{3} = 1$

**Algebra** $\frac{a}{b} \cdot \frac{b}{a} = 1$, where $a \neq 0, b \neq 0$

**Example 1** *Solving a One-Step Equation*

$\frac{3}{5}x = 15$ Original equation

$\square\left(\frac{3}{5}\right)x = \square(15)$ Multiply each side by multiplicative inverse of $\square$.

$\square x = \square(15)$ Multiplicative inverse property

$x = \square$ Multiply.

**Answer:** The solution is $\square$.

**Checkpoint** **Solve the equation. Check your solution.**

| | |
|---|---|
| 1. $\frac{6}{11}x = 18$ | 2. $-\frac{7}{13}x = 28$ |

**Example 2** *Solving a Two-Step Equation*

$-\frac{7}{12}x + \frac{3}{4} = \frac{1}{2}$ Original equation

$-\frac{7}{12}x + \frac{3}{4} - \square = \frac{1}{2} - \square$ Subtract $\square$ from each side.

$-\frac{7}{12}x = \square - \square$ Write fractions using LCD.

$-\frac{7}{12}x = \square$ Subtract.

$\square\left(-\frac{7}{12}\right)x = \square\left(\square\right)$ Multiply each side by multiplicative inverse of $\square$.

$x = \square$ Multiply.

**Example 3** *Writing and Solving a Two-Step Equation*

**Tree Growth** The height of a certain Norway Spruce is 10 feet. If the tree's height grows $2\frac{1}{2}$ feet per year, find how long it will take the tree to reach a height of 25 feet.

**Solution**

| Current height | + | Growth rate | · | Number of years | = | New height |
|---|---|---|---|---|---|---|

$10 + 2\frac{1}{2}x = 25$ Write equation.

$10 + 2\frac{1}{2}x - \square = 25 - \square$ Subtract $\square$ from each side.

$\square x = \square$ Simplify. Write mixed number as improper fraction.

$\square\left(\square\right)x = \square\left(\square\right)$ Multiply each side by multiplicative inverse of $\square$.

$x = \square$ Multiply.

**Answer:** The tree will be 25 feet tall after $\square$ years.