Unit 8: Factors, Fractions, Exponents

- Why is it necessary to understand number relationships?
- Why is there more than one way to represent a fraction?
- When is scientific notation useful?
- Why would you use an exponent?
- Are the rules of exponents necessary?

4.1 Factors and Prime Factorization

Goal: Write the prime factorization of a number.

Vocabulary

Prime number:

Composite number:

Prime factorization:

Factor tree:

Monomial:

Example 1 *Writing Factors*

A rectangle has an area of 18 square feet. Find all possible whole number dimensions of the rectangle.

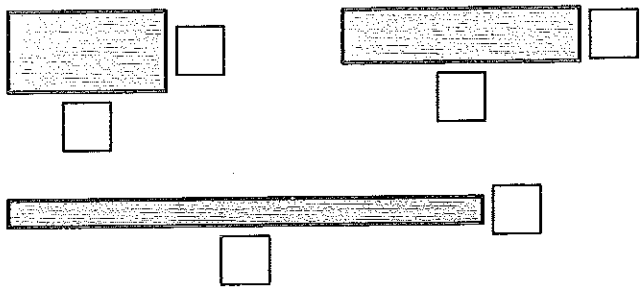
1. Write 18 as a product of two whole numbers in all possible ways.

$\square \cdot \square = 18$ $\square \cdot \square = 18$ $\square \cdot \square = 18$

The factors of 18 are _____.

The area of a rectangle can be found using the formula, Area = length × width.

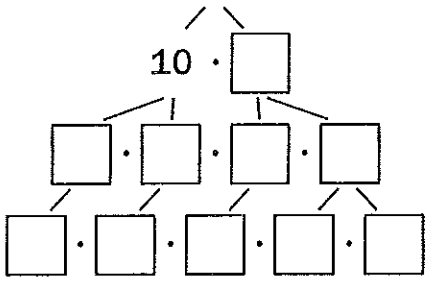
2. Use the factors to find all rectangles with an area of 18 square feet that have whole number dimensions. Then label the given rectangles.



Example 2 *Writing a Prime Factorization*

Write the prime factorization of 420.

One possible factor tree:



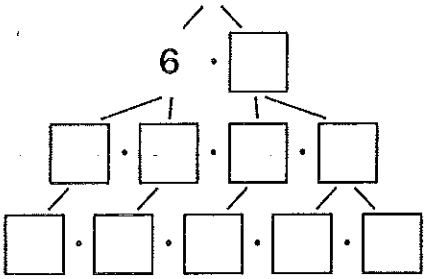
Write original number.

Write 420 as 10 • ☐.

Write 10 as ☐ • ☐. Write ☐ as ☐ • ☐.

Write 6 as ☐ • ☐.

Another possible factor tree:



Write original number.

Write 420 as 6 • ☐.

Write 6 as ☐ • ☐. Write ☐ as ☐ • ☐.

Write 10 as ☐ • ☐.

Both trees give the same result: 420 = ☐.

Answer: The prime factorization of 420 is ☐.

Example 3 *Factoring a Monomial*

Factor the monomial $24x^4y$.

$24x^4y =$ ☐ $\cdot\ x^4y$ Write 24 as ☐.

$=$ ☐ $\cdot\ y$ Write x^4 as ☐.

Checkpoint **Write all factors of the number.**

1. 28	**2.** 48

Tell whether the number is *prime* or *composite*. If it is composite, write its prime factorization.

3. 97	**4.** 117

Factor the monomial.

5. $21n^5$	**6.** $18x^2y^3$

4.2 Greatest Common Factor

Goal: Find the greatest common factor of two or more numbers.

Vocabulary

Common factor:

Greatest common factor (GCF):

Relatively prime:

Example 1 *Finding the Greatest Common Factor*

Volunteers A high school asks for volunteers to help clean up local highways on one Saturday each month. The group of volunteers has 27 freshman, 18 sophomores, 36 juniors, and 45 seniors. What is the greatest number of groups that can be formed if each group is to have the same number of each type of student? How many freshman, sophomores, juniors, and seniors will be in each group?

Solution

Method 1 List the factors of each number. Identify the greatest number that is on every list.

Factors of 27:

Factors of 18:

Factors of 36:

Factors of 45:

The common factors are _____.

The GCF is _____.

Method 2 Write the prime factorization of each number. The GCF is the product of the prime factors.

$27 =$ ☐
$18 =$ ☐
$36 =$ ☐
$45 =$ ☐

The common prime factors are ☐.

The GCF is ☐.

Answer: The greatest number of groups that can be formed is ☐. Each group will have $27 \div$ ☐ $=$ ☐ freshman, $18 \div$ ☐ $=$ ☐ sophomores, $36 \div$ ☐ $=$ ☐ juniors, and $45 \div$ ☐ $=$ ☐ seniors.

Checkpoint **Find the greatest common factor of the numbers.**

1. 54, 81	2. 12, 48, 66

Example 2 ***Identifying Relatively Prime Numbers***

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

a. 28, 63 b. 42, 55

Solution

a. List the factors of each number. Identify the greatest number that the lists have in common.

Factors of 28: ☐

Factors of 63: ☐

The GCF is ☐. So, the numbers ☐ relatively prime.

b. Write the prime factorization of each number.

$42 =$ ☐ $55 =$ ☐

The GCF is ☐. So, the numbers ☐ relatively prime.

Checkpoint **Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.**

3. 30, 49	4. 52, 78

Example 3 ***Finding the GCF of Monomials***

Find the greatest common factor of $16x^2y$ and $26x^2y^3$.

Solution

Factor the monomials. The GCF is the product of the common factors.

$16x^2y =$ ☐

$26x^2y^3 =$ ☐

Answer: The GCF is ☐.

Checkpoint **Find the greatest common factor of the monomials.**

5. $12x^3, 18x^2$	6. $40xy^3, 24xy$

4.3 Equivalent Fractions

Goal: Write equivalent fractions.

Vocabulary

Equivalent fractions: _____

Simplest form: _____

Equivalent Fractions

Words To write equivalent fractions, multiply or divide the numerator and the denominator by the same nonzero number.

Algebra For all numbers a, b, and c, where $b \neq 0$ and $c \neq 0$, $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$.

Numbers $\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$ $\qquad$ $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

Example 1 *Writing Equivalent Fractions*

Write two fractions that are equivalent to $\frac{6}{18}$.

Multiply or divide the numerator and the denominator by the _____.

$\frac{6}{18} = \frac{6 \cdot 2}{18 \cdot 2} =$ _____ Multiply numerator and denominator by 2.

$\frac{6}{18} = \frac{6 \div 3}{18 \div 3} =$ _____ Divide numerator and denominator by 3.

Answer: The fractions _____ and _____ are equivalent to $\frac{6}{18}$.

Checkpoint Write two fractions that are equivalent to the given fraction.

1. $\frac{7}{14}$	2. $\frac{4}{16}$	3. $\frac{10}{25}$

Example 2 *Writing a Fraction in Simplest Form*

Write $\frac{8}{36}$ in simplest form.

Write the prime factorizations of the numerator and denominator.

$8 =$ ☐ $36 =$ ☐

The GCF of 8 and 36 is ☐.

$\frac{8}{36} = \frac{8 \div \square}{36 \div \square}$ Divide numerator and denominator by GCF.

$= \square$ Simplify.

Checkpoint Write the fraction in simplest form.

4. $\frac{3}{18}$	5. $\frac{12}{32}$	6. $\frac{24}{42}$

Example 3 *Simplifying a Variable Expression*

Write $\frac{14x^2y}{35x^3}$ in simplest form.

$\frac{14x^2y}{35x^3} = \frac{\square}{\square}$ Factor numerator and denominator.

$= \frac{\square\square\square}{\square}$ Divide out common factors.

$= \square$ Simplify.

Checkpoint **Write the variable expression in simplest form.**

7. $\frac{9a}{15a^2}$	**8.** $\frac{16mn^2}{28n}$	**9.** $\frac{39st^2}{3s^2t}$

4.4 Least Common Multiple

Goal: Find the least common multiple of two numbers.

Vocabulary

Multiple:

Common multiple:

Least common multiple (LCM):

Least common denominator (LCD):

Example 1 *Finding the Least Common Multiple*

Find the least common multiple of 6 and 14.

Solution

You can use one of two methods to find the LCM.

Method 1 List the multiples of each number. Identify the least number that is on both lists.

Multiples of 6:

Multiples of 14:

The LCM of 6 and 14 is ____.

Method 2 Find the common factors of the numbers.

6 = ____

14 = ____

The common factor is ____.

Multiply all of the factors, using each common factor only once.

LCM = ____ = ____

Answer: Both methods get the same result. The LCM is ____.

Example 2 *Finding the Least Common Multiple of Monomials*

Find the least common multiple $6xy$ and $16x^2$.

$6xy =$ ☐

$16x^2 =$ ☐

LCM = ☐ = ☐

Answer: The least common multiple of $6xy$ and $16x^2$ is ☐.

✔ ***Checkpoint*** **Find the least common multiple of the numbers or the monomials.**

1. 8, 18	**2.** 4, 5, 15
3. $12x$, $18x^2$	**4.** $4xy$, $10xz^2$

Example 3 *Comparing Fractions Using the LCD*

Summer Sports Last year, a summer resort had 165,000 visitors, including 44,000 water skiers. This year, the resort had 180,000 visitors, including 63,000 water skiers. In which year was the fraction of water skiers greater?

Solution

1. Write the fractions and simplify.

Last year: $\frac{\text{Number of water skiers}}{\text{Total number of visitors}} =$ ☐ $=$ ☐

This year: $\frac{\text{Number of water skiers}}{\text{Total number of visitors}} =$ ☐ $=$ ☐

2. Find the LCD of ☐ and ☐. The LCM of ☐ and ☐ is ☐. So, the LCD of the fractions is ☐.

3. Write equivalent fractions using the LCD.

Last year: $\square = \square = \square$

This year: $\square = \square = \square$

4. Compare the numerators: $\square < \square$, so $\square < \square$.

Answer: The fraction of water skiers was greater $\square$.

Example 4 ***Ordering Fractions and Mixed Numbers***

Order the numbers $4\frac{5}{12}$, $\frac{9}{2}$, and $\frac{33}{8}$ from least to greatest.

1. Write the mixed number as an improper fraction.

$$4\frac{5}{12} = \frac{\square}{12} = \frac{\square}{12}$$

2. Find the LCD of $\frac{\square}{12}$, $\frac{9}{2}$, and $\frac{33}{8}$. The LCM of 12, 2, and 8 is $\square$. So, the LCD is $\square$.

3. Write equivalent fractions using the LCD.

$$\frac{\square}{12} = \frac{\square \cdot \square}{12 \cdot \square} = \square \qquad \frac{9}{2} = \frac{9 \cdot \square}{2 \cdot \square} = \square$$

$$\frac{33}{8} = \frac{33 \cdot \square}{8 \cdot \square} = \square$$

4. Compare the numerators: $\square < \square$ and $\square < \square$, so $\square < \square$ and $\square < \square$.

Answer: From least to greatest, the numbers are $\square$, $\square$, and $\square$.

4.5 Rules of Exponents

Goal: Multiply and divide powers.

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Algebra $a^m \cdot a^n = a^{m+n}$

Numbers $4^3 \cdot 4^2 = 4^{\square} = 4^{\square}$

Example 1 *Using the Product of Powers Property*

a. $4^7 \cdot 4^{11} = 4^{\square}$ — Product of powers property

$= 4^{\square}$ — Add exponents.

b. $2x^2 \cdot 7x^6 = 2 \cdot 7 \cdot x^2 \cdot x^6$ — Commutative property of multiplication

$= 2 \cdot 7 \cdot x^{\square}$ — Product of powers property

$= 2 \cdot 7 \cdot x^{\square}$ — Add exponents.

$= \square$ — Multiply.

Checkpoint **Find the product. Write your answer using exponents.**

1. $2^5 \cdot 2^{12}$	2. $5^6 \cdot 5^2 \cdot 5^3$
3. $x^6 \cdot x^{13}$	4. $b^2 \cdot b^4 \cdot b$

Quotient of Powers Property

Words To divide powers with the same base, subtract the exponent of the denominator from the exponent of the numerator.

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Numbers $\frac{5^7}{5^4} = 5^{\square} = 5^{\square}$

Example 2 ***Using the Quotient of Powers Property***

a. $\frac{6^8}{6^3} = 6^{\square}$ Quotient of powers property

$= 6^{\square}$ Subtract exponents.

b. $\frac{3x^7}{12x^3} = \frac{3x^{\square}}{12}$ Quotient of powers property

$= \frac{3x^{\square}}{12}$ Subtract exponents.

$= \square$ Divide numerator and denominator by $\square$.

Checkpoint **Find the quotient. Write your answer using exponents.**

5. $\frac{5^9}{5^2}$	**6.** $\frac{12^7}{12^4}$
7. $\frac{4x^{13}}{24x^9}$	**8.** $\frac{14x^{16}}{6x^{11}}$

Example 3 *Using Both Properties of Powers*

Simplify $\dfrac{4m^3 \cdot m^4}{12m^2}$.

$\dfrac{4m^3 \cdot m^4}{12m^2} = \dfrac{4m^{\square}}{12m^2}$ Product of powers property

$= \dfrac{4m^{\square}}{12m^2}$ Add exponents.

$= \dfrac{4m^{\square}}{12}$ Quotient of powers property

$= \dfrac{4m^{\square}}{12}$ Subtract exponents.

$= \square$ Divide numerator and denominator by $\square$.

Checkpoint **Simplify.**

9. $\dfrac{6m^5 \cdot m}{15m^3}$	**10.** $\dfrac{n^2 \cdot 10n^6}{5n^3}$

4.6 Negative and Zero Exponents

Goal: Work with negative and zero exponents.

Negative and Zero Exponents

For any nonzero number a, $a^0 = 1$.

For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

Example 1 *Powers with Negative and Zero Exponents*

Write the expression using only positive exponents.

a. $4^{-3} =$ ☐ Definition of negative exponent

b. $m^{-5}n^0 = m^{-5} \cdot$ ☐ Definition of zero exponent

$=$ ☐ Definition of negative exponent

c. $13xy^{-8} =$ ☐ Definition of negative exponent

✔ ***Checkpoint*** **Write the expression using only positive exponents.**

1. $33{,}333^0$	**2.** 7^{-3}	**3.** $2z^{-2}$	**4.** $3x^{-4}y$

Example 2 *Rewriting Fractions*

Write the expression without using a fraction bar.

a. $\frac{1}{15} =$ ☐ Definition of negative exponent

b. $\frac{a^3}{c^5} =$ ☐ Definition of negative exponent

Checkpoint **Write the expression without using a fraction bar.**

5. $\frac{1}{18}$	6. $\frac{1}{100}$	7. $\frac{3}{c^2}$	8. $\frac{x^5}{y^7}$

Example 3 ***Using Powers Properties with Negative Exponents***

Find the product or quotient. Write your answer using only positive exponents.

a. $6^{12} \cdot 6^{-4}$ **b.** $\frac{7n^{-4}}{n}$

Solution

a. $6^{12} \cdot 6^{-4} = 6^{\square}$ Product of powers property

$= 6^{\square}$ Add exponents.

b. $\frac{7n^{-4}}{n} = 7n^{\square}$ Quotient of powers property

$= 7n^{\square}$ Subtract exponents.

$= \square$ Definition of negative exponent

Checkpoint **Find the product or quotient. Write your answer using only positive exponents.**

9. $3^{10} \cdot 3^{-7}$	10. $\frac{7d^{-4}}{d^2}$

4.7 Scientific Notation

Goal: Write numbers using scientific notation.

Using Scientific Notation

A number is written in **scientific notation** if it has the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer.

Standard form	Product form	Scientific notation
725,000	$7.25 \times 100{,}000$	7.25×10^5
0.006	6×0.001	6×10^{-3}

Example 1 *Writing Numbers in Scientific Notation*

a. The average distance Mars is from the sun is 141,600,000 miles. Write this number in scientific notation.

Standard form	Product form	Scientific notation

b. The diameter of a quarter-ounce gold American Eagle coin is 0.022 meter. Write this number in scientific notation.

Standard form	Product form	Scientific notation

Example 2 *Writing Numbers in Standard Form*

a. Write 4.1×10^4 in standard form.

Scientific notation	Product form	Standard form

b. Write 7.23×10^{-6} in standard form.

Scientific notation	Product form	Standard form

Checkpoint **Write the number in scientific notation.**

1. 3,050,000,000	2. 0.000082

Write the number in standard form.

3. 6.53×10^7	4. 9.2×10^{-4}

Example 3 ***Ordering Numbers Using Scientific Notation***

Order 5.3×10^5, 520,000, and 7.5×10^4 from least to greatest.

1. Write each number in scientific notation if necessary.

520,000 = ☐

2. Order the numbers with different powers of 10.

Because $10^{\square} < 10^{\square}$, ☐ < ☐ and ☐ < ☐.

3. Order the numbers with the same power of 10.

Because ☐ < ☐, ☐ < ☐.

4. Write the original numbers in order from least to greatest.

☐; ☐; ☐

Checkpoint **Order the numbers from least to greatest.**

5. 23,000; 3.4×10^3; 2.2×10^4
6. 4.5×10^{-4}; 0.000047; 4.8×10^{-5}

Example 4 ***Multiplying Numbers in Scientific Notation***

Oxygen Atoms The density of an oxygen atom is about 1.429×10^{-3} grams per milliliter. Find the density of 1.5×10^{4} oxygen atoms.

Solution

Total density	=	Density of one oxygen atom	×	Number of oxygen atoms

$= (\quad)(\quad)$ Substitute values.

$= (\quad)(\quad)$ Commutative and associative properties of multiplication

$= \quad \times (\quad)$ Multiply ☐ and ☐.

$= \quad \times (\quad)$ Product of powers property

$= \quad \times \quad$ Add exponents.

Answer: The density of 1.5×10^{4} oxygen atoms is about ☐ × ☐ grams per milliliter.

✔ ***Checkpoint*** **Find the product. Write your answer in scientific notation.**

7. $(2.5 \times 10^{3})(2 \times 10^{5})$	8. $(1.5 \times 10^{-2})(4 \times 10^{-4})$