Unit 7: Percents

- Why is a percent a ratio out of 100?
- When is a percent higher than 100?
- How are percents, fractions, decimals, and proportions related?
- What situations require the use of percents?
- Why are percents so commonly utilized?

7.1 Percents and Fractions

Goal: Use a fraction to find the percent of a number.

Vocabulary

Percent:

Writing Percents

Words In the area model shown, 85 of the 100 squares are shaded. You can say that 85 percent of the squares are shaded.

Numbers $\frac{85}{100} = 85\%$

Algebra $\frac{p}{100} = p\%$

Example 1 *Writing Percents as Fractions, Fractions as Percents*

Write 31% and 60% as fractions in simplest form.

a. $31\% =$ ☐

b. $60\% =$ ☐ $=$ ☐

Write $\frac{3}{10}$ and $\frac{4}{5}$ as percents.

a. $\frac{3}{10} = \frac{3 \cdot \square}{10 \cdot \square} = \square = \square$

b. $\frac{4}{5} = \frac{4 \cdot \square}{5 \cdot \square} = \square = \square$

Checkpoint **Write the percent as a fraction in simplest form, or write the fraction as a percent.**

1. 73%	**2.** 40%	**3.** $\frac{9}{10}$	**4.** $\frac{4}{25}$

Example 2 ***Writing a Probability as a Percent***

Random Number A computer randomly generates an integer from 1 to 10. Find the probability that 8 is the integer generated.

Solution

There are ☐ possible outcomes, and ☐ outcome is favorable.

$P(8) =$ ☐ Write probability as a fraction.

$=$ ☐ Write fraction as percent.

Answer: The probability that 8 is the integer generated is ☐.

Checkpoint **A computer randomly generates an integer from 1 to 10. Find the probability of the given event. Write your answer as a percent.**

5. $P(2)$	**6.** P(prime number)

Example 3 ***Finding a Percent of a Number***

Crackers In a survey of 85 people, 20% of them said they usually eat crackers with soup. How many people in the survey said they usually eat crackers with soup?

Solution

To find 20% of 85, use the fact that 20% = $\square$. Then multiply.

20% of 85 = $\square \cdot \square$ Write percent as a fraction.

$= \square$ Simplify.

Answer: $\square$ people said they usually eat crackers with soup.

Checkpoint **Find the percent of the number.**

7. 30% of 80	**8.** 60% of 105

7.2 Percents and Proportions

Goal: Use proportions to solve percent problems.

Solving Percent Problems

You can represent "a is p percent of b" using the proportion

$$\frac{a}{b} = \frac{p}{100}$$

where a is a part of the base b and $p\%$, or $\frac{p}{100}$, is the percent.

Example 1 *Finding a Percent*

What percent of 9 is 5?

$\frac{a}{b} = \frac{p}{100}$ Write proportion.

$\frac{\square}{} = \frac{p}{100}$ Substitute for a and for b.

$\square \cdot \frac{5}{9} = \square \cdot \frac{p}{100}$ Multiply each side by $\square$.

$\square = p$ Simplify.

Answer: 5 is $\square$ % of 9.

Checkpoint **Use a proportion to answer the question.**

1. What percent of 28 is 4?	2. What percent of 80 is 30?

Example 2 *Finding a Part of a Base*

What number is 15% of 300?

$\frac{a}{b} = \frac{p}{100}$ Write proportion.

$\frac{a}{\square} = \frac{\square}{100}$ Substitute for b and for p.

$\square \cdot \frac{a}{\square} = \square \cdot \frac{\square}{100}$ Multiply each side by $\square$.

$a = \square$ Simplify.

Answer: $\square$ is 15% of 300.

Example 3 *Finding a Base*

Student Council Election You receive 189 votes, or 45%, of the votes in the student council election. How many students voted?

Solution

189 is a part of the total number of voters, which is the base.

$\frac{a}{b} = \frac{p}{100}$ Write proportion.

$\frac{\square}{b} = \frac{\square}{100}$ Substitute for a and for p.

$\square \cdot \square = \square \cdot \square$ Cross products property

$\square = \square$ Multiply.

$\square = b$ Divide each side by $\square$.

Answer: $\square$ students voted in the election.

Checkpoint **Use a proportion to answer the question.**

3. What number is 62% of 200?	4. 117 is 78% of what number?

7.3 Percents and Decimals

Goal: Use decimals to solve percent problems.

Percents and Decimals

- To write a decimal as a percent, move the decimal point two places to the [ ] and [ ].
- To write a percent as a decimal, move the decimal point two places to the [ ] and [ ].

Example 1 *Writing Decimals as Percents*

Write 0.17, 2, and 3.2 as percents.

a. $0.17 = 0.\underset{\smile}{17}$ $= \square$

b. $2 = 2.\underset{\smile}{00}$ $= \square$

c. $3.2 = 3.\underset{\smile}{20}$ $= \square$

Example 2 *Writing Percents as Decimals*

Write 63%, 0.7%, and 129% as decimals.

a. $63\% = \underset{\smile}{63}\%$ $= \square$

b. $0.7\% = \underset{\smile}{00}.7\%$ $= \square$

c. $129\% = \underset{\smile}{129}\%$ $= \square$

✔ ***Checkpoint*** **Write the decimal as a percent or the percent as a decimal.**

1. 0.54	2. 4	3. 1.75	4. 0.03
5. 41%	6. 147%	7. 9%	8. 12.5%

Example 3 *Writing Fractions as Percents*

Write $\frac{4}{9}$ and $\frac{7}{4}$ as percents.

a. $\frac{4}{9} =$ ☐ Write fraction as a decimal.

$=$ ☐ Write decimal as a percent.

b. $\frac{7}{4} =$ ☐ Write fraction as a decimal.

$=$ ☐ Write decimal as a percent.

Checkpoint **Write the fraction as a percent.**

9. $\frac{5}{8}$	**10.** $\frac{8}{9}$	**11.** $\frac{11}{5}$	**12.** $\frac{13}{6}$

Example 4 *Finding a Percent of a Number*

Day of Dread In a survey of 1300 adults, 18% said the day they dread the most is Monday. How many adults chose Monday?

Solution

Find 18% of 1300.

18% of 1300 $=$ ☐ $\cdot$ 1300 Write percent as decimal.

$=$ ☐ Multiply.

Answer: The number of adults that chose Monday is ☐.

Checkpoint **Find the percent of the number.**

13. 25% of 76	**14.** 110% of 65	**15.** 0.7% of 500

7.4 The Percent Equation

Goal: Use equations to solve percent problems.

The Percent Equation

You can represent "a is p percent of b" using the equation

$a = p\% \cdot b$

where a is a part of the base b and $p\%$ is the percent.

Example 1 *Finding a Part of a Base*

In a newspaper's survey, 1100 adults were asked to name their favorite condiment. The most frequent response was ketchup, which was given by 47% of the adults. How many adults chose ketchup?

Solution

To find how many adults chose ketchup as their favorite condiment, use the percent equation.

$a = p\% \cdot b$	Write percent equation.
$= \square \cdot \square$	Substitute for p and for b.
$= \square \cdot \square$	Write percent as a decimal.
$= \square$	Multiply.

Answer: The number of adults who chose ketchup as their favorite condiment was $\square$.

Checkpoint **Use the percent equation to answer the question.**

1. What number is 15% of 60?	2. What number is 78% of 105?

Example 2 *Finding a Commission*

Commission A sales person earns 5.5% commission on every car sold. The sales person sells a house for $41,200. What is the commission?

Solution

$a = p\% \cdot b$	Write percent equation.
$= \square \cdot \square$	Substitute for p and for b.
$= \square \cdot \square$	Write percent as decimal.
$= \square$	Multiply.

Answer: The commission is $\$\square$.

Checkpoint

3. In Example 2, find the commission if a car is sold for $45,000.

Example 3 *Finding a Percent*

What percent of 24 is 84?

$a = p\% \cdot b$	Write percent equation.
$\square = p\% \cdot \square$	Substitute for a and for b.
$\square = p\%$	Divide each side by $\square$.
$\square = p\%$	Write decimal as a percent.

Answer: 84 is $\square$% of 24.

Checkpoint **Use the percent equation to answer the question.**

4. What percent of 15 is 21?	5. What percent of 72 is 45?

Example 4 ***Finding a Base***

Football Your friend paid $48 for a ticket to a professional football game. This amount was 64% of the total amount your friend spent at the game. How much money did your friend spend?

Solution

$a = p\% \cdot b$ Write percent equation.

$\square = \square\% \cdot b$ Substitute for a and for p.

$\square = \square \cdot b$ Write percent as decimal.

$\square = b$ Divide each side by $\square$.

Answer: Your friend spent $\$\square$ at the game.

Checkpoint **Use the percent equation to answer the question.**

6. 33 is 30% of what number?	7. 90 is 37.5% of what number?

7.5 Percent of Change

Goal: Find a percent of change in a quantity.

Vocabulary

Percent of change:

Percent of increase:

Percent of decrease:

Percent of Change

The percent of change is the ratio of the amount of increase or decrease to the original amount.

$$\text{Percent of change, } p\% = \frac{\text{Amount of increase or decrease}}{\text{Original amount}}$$

Example 1 *Finding a Percent of Increase*

Enrollment A school had 720 students enrolled last year. This year, 745 students are enrolled. By about what percent did the number of students change from last year to this year?

$p\% = \frac{\text{Amount of increase}}{\text{Original amount}}$ Write formula for percent of increase.

$= \frac{\quad}{\quad}$ Substitute.

$= \quad$ Subtract.

$\approx \quad = \quad$ Divide. Then write decimal as a percent.

Answer: The number of students increased by about ____ %.

Example 2 *Finding a Percent of Decrease*

Find the percent of decrease from 576 to 216.

$p\% = \dfrac{\text{Amount of decrease}}{\text{Original amount}}$ — Write formula for percent of decrease.

$= \square$ — Substitute.

$= \square$ — Subtract.

$= \square = \square$ — Simplify fraction. Then write the fraction as a percent.

Answer: The percent of decrease is $\square$%.

Example 3 *Using a Percent of Increase*

Ticket Prices A professional baseball team announces that the average ticket price to one of their games will be 8% more than last year. If the average price of a ticket was $12 last year, how much will the average ticket cost this year?

Solution

To find the average ticket cost this year, you need to increase the average ticket cost last year by 8%.

$\boxed{\text{Ticket cost this year}} = \boxed{\text{Ticket cost last year}} + \boxed{\text{Amount of increase}}$

$= \square + \square \cdot \square$ — Substitute.

$= \square + \square \cdot \square$ — Write percent as a decimal.

$= \square$ — Evaluate.

Answer: This year, the average ticket will cost $\$\square$.

Example 4 *Finding a New Amount*

Tuna In 1990, the average price per pound of light chunk tuna was \$2.11. By 2001, the average price per pound had decreased by 7.1%. What was the average price per pound in 2001?

Solution

To find a new amount, do one of the following.
- For a p% increase, multiply the original amount by (100% + p%).
- For a p% decrease, multiply the original amount by (100% − p%).

Price in 2001 = Price in 1990 • (100% − p%)

= [ ] • (100% − [ ]%) Substitute.

= [ ] • [ ] Subtract percents.

= [ ] • [ ] Write percent as a decimal.

= [ ] Multiply.

Answer: The average price per pound in 2001 was about \$[ ].

Checkpoint **Find the percent of increase.**

1. Original: 25 New: 31	**2.** Original: 150 New: 195

Find the new amount.

3. Increase 54 by 25%.	**4.** Decrease 78 by 40%.

7.6 Percent Applications

Goal: Find markups, discounts, sales tax, and tips.

Vocabulary

Markup: ____

Discount: ____

Example 1 *Finding a Retail Price*

Pillows A store buys decorative pillows from a manufacturer for $2 each. The store marks up the price by 400%. What is the retail price?

Solution

Method 1 Add the markup to the wholesale price.

Retail price = Wholesale price + Markup

= ☐ + ☐ • ☐	Substitute.
= ☐ + ☐ • ☐	Write ☐ as a decimal.
= ☐ + ☐	Multiply.
= ☐	Add.

Method 2 Multiply the wholesale price by (100% + Markup percent).

Retail price = Wholesale price • (100% + Markup percent)

= ☐ • (100% + ☐)	Substitute.
= ☐ • ☐	Add percents.
= ☐ • ☐	Write ☐ as a decimal.
= ☐	Multiply.

Answer: The retail price of a decorative pillow is $☐.

Example 2 *Finding a Sale Price*

Backpack You buy a backpack that is on sale for 25% off the original price of $20. What is the sale price?

Solution

Method 1 Subtract the discount from the original price.

Sale price = Original price − Discount

$= \square - \square \cdot \square$ Substitute.

$= \square - \square \cdot \square$ Write $\square$ as a decimal.

$= \square - \square$ Multiply.

$= \square$ Subtract.

Method 2 Multiply the original price by (100% − Discount percent).

Sale price = Original price • (100% − Discount percent)

$= \square \cdot (100\% - \square)$ Substitute.

$= \square \cdot \square$ Subtract percents.

$= \square \cdot \square$ Write $\square$ as a decimal.

$= \square$ Multiply.

Answer: The sale price of the backpack is $\$\square$.

Checkpoint

1. In Example 1, what is the retail price of a decorative pillow if the markup percent is 250%?

2. A pair of shorts that originally costs $15 is 40% off. Find the sale price.

Example 3 *Using Sales Tax and Tips*

Pizza You order pizza to be delivered. The bill is $18. You give the delivery person a 15% tip. The sales tax is 5%. What is the total cost of the pizza?

Solution

Sales tax and tips are calculated using a percent of the purchasing price. These amounts are then added to the purchase price.

Total = Food bill + Sales tax + Tip

$= \square + \square \cdot \square + \square \cdot \square$ Substitute.

$= \square + \square \cdot \square + \square \cdot \square$ Write percents as decimals.

$= \square$ Evaluate.

Answer: The total cost of the pizza is $\$\square$.

Checkpoint

3. In Example 3, find the total cost of the pizza if the tip is 20%.

Example 4 *Finding an Original Amount*

Blender A store marks up the wholesale price of a blender by 125%. The retail price is $30. What is the wholesale price?

Solution

Let x represent the wholesale price.

Retail price = Wholesale price • (100% + Markup percent)

$\square = x \cdot (100\% + \square)$ Substitute.

$\square = x \cdot \square$ Add percents.

$\square = x \cdot \square$ Write $\square$ as a decimal.

$\square \approx x$ Divide each side by $\square$.

Answer: The wholesale price of the blender is about $\$\square$.

7.7 Simple and Compound Interest

Goal: Calculate interest earned and account balances.

Vocabulary

Interest:

Principal:

Simple interest:

Annual interest rate:

Balance:

Compound interest:

Simple Interest Formula

Simple interest I is given by the formula

$$I = Prt$$

where P is the principal, r is the annual interest rate (written as a decimal), and t is the time in years.

Example 1 *Finding Simple Interest*

A $2000 bond earns 3% simple interest per year on its purchase price. Find the interest earned after 5 years.

Solution

$I = Prt$	Write simple interest formula.
$= \square$	Substitute.
$= \square$	Multiply.

Answer: The bond will earn $\$\square$ in interest after 5 years.

Example 2 *Finding an Interest Rate*

You deposit $900 into an account that earns simple annual interest. After 8 months, the balance is $913.20. Find the annual interest rate.

Solution

Because t in the formula $A = P(1 + rt)$ is the time in years, write 8 months as $\frac{8}{12}$, or $\frac{2}{3}$ year. Then solve for r after substituting values for A, P, and t in $A = P(1 + rt)$.

$A = P(1 + rt)$	Write formula for finding balance.
$\square = \square[1 + r\square]$	Substitute.
$\square = \square + \square r$	Distributive property
$\square = \square r$	Subtract $\square$ from each side.
$\square = r$	Divide each side by $\square$.

Answer: The annual interest rate is $\square$%.

Checkpoint

1. A $1200 bond earns 5% simple interest per year on its purchase price. Find the interest earned after 3 years.

Find the unknown quantity for an account that earns simple annual interest.

2. $A =$?, $P = \$1300$, $r = 4\%$, $t = 4$ years

3. $A = \$1116.50$, $P = \$1100$, $r = 3\%$, $t =$?

Compound Interest Formula

When an account earns interest compounded annually, the balance A is given by the formula

$$A = P(1 + r)^t$$

where P is the principal, r is the annual interest rate (written as a decimal), and t is the time in years.

Example 3 *Calculating Compound Interest*

You deposit $1250 into an account that earns 2.25% interest compounded annually. Find the balance after 4 years.

Solution

$A = P(1 + r)^t$ Write formula.

$= \square(1 + \square)^{\square}$ Substitute.

$\approx \square$ Use a calculator.

Answer: The balance of the account after 4 years is about $\$\square$.