Unit 5: Area and Volume

- Why does are apply to two dimensional figures?
- Why does volume apply to three dimensional figures?
- How are area and volume used outside of the classroom?

10.3 Areas of Parallelograms and Trapezoids

Goal: Find the areas of parallelograms and trapezoids.

Vocabulary

Base of a parallelogram:

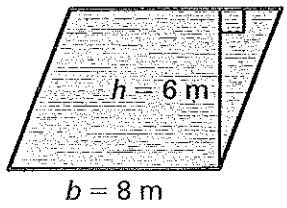
Height of a parallelogram:

Base of a trapezoid:

Height of a trapezoid:

Area of a Parallelogram

Words The area A of a parallelogram is the product of the base b and the height h.



Algebra $A = bh$

Numbers $A = \square \cdot \square = \square$

Example 1 *Finding the Area of a Parallelogram*

The base of a parallelogram is 8 yards. The height is three times the base. Find the area of the parallelogram.

1. Find the height.

$h = 3b$

$= 3(\square)$

$= \square$

2. Find the area.

$A = bh$

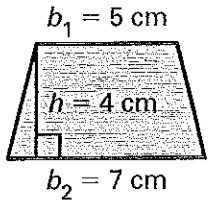
$= \square$

$= \square$

Answer: The parallelogram has an area of $\square$.

Area of a Trapezoid

Words The area A of a trapezoid is one half of the product of the sum of the bases, b_1 and b_2, and the height h.

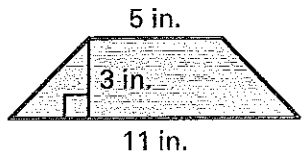


Algebra $A = \frac{1}{2}(b_1 + b_2)h$

Numbers $A = \square(\square + \square)\square = \square$

Example 2 *Finding the Area of a Trapezoid*

The diagram shows one of the trapezoids in a floor design. Find the area of the trapezoid.



Solution

$A = \frac{1}{2}(b_1 + b_2)h$ Write formula for area of a trapezoid.

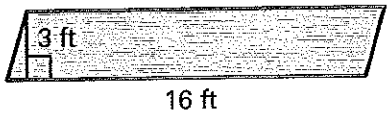
$= \square(\square + \square)\square$ Substitute values.

$= \square$ Simplify.

Answer: The trapezoid has an area of $\square$.

Checkpoint **Find the area of the parallelogram or trapezoid.**

1.



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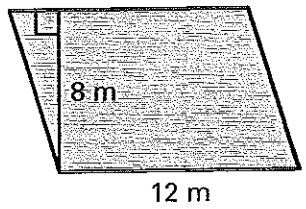

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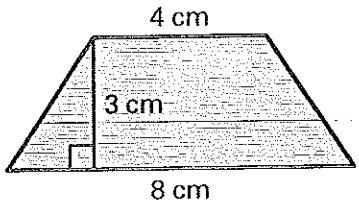

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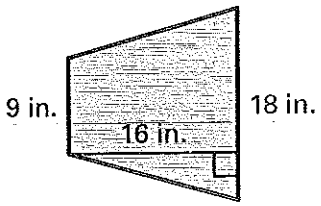


Example 3 ***Finding an Unknown Length***

The height of a trapezoid is 8 feet. One of its bases is 12 feet. The area of the trapezoid is 108 square feet. Find the other base.

$A = \frac{1}{2}(b_1 + b_2)h$ Write formula for area of a trapezoid.

$\square = \square(\square + b_2)\square$ Substitute values.

$\square = \square(\square + b_2)$ Multiply.

$\square = \square + \square$ Distributive property

$\square = \square$ Subtract $\square$ from each side.

$\square = b_2$ Divide each side by $\square$.

Answer: The other base is $\square$.

10.4 Circumference and Area of a Circle

Goal: Find the circumferences and areas of circles.

Vocabulary

Circle:

Center:

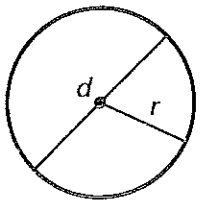
Radius:

Diameter:

Circumference:

Circumference of a Circle

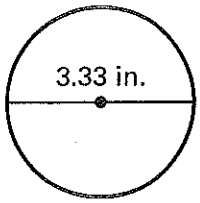
Words The circumference C of a circle is the product of π and the diameter d, or twice the product of π and the radius r.



Algebra $C = \pi d$ $C = 2\pi r$

Example 1 *Finding the Circumference of a Circle*

Labels A circular label has a diameter of 3.33 inches. Approximate the distance around the label to the nearest inch.



Solution

When the radius or diameter of a circle is divisible by 7, use $\frac{22}{7}$ to approximate π. Otherwise, use 3.14 to approximate π.

$C = \pi d$	Write formula for circumference of a circle.
$\approx$ ☐(☐)	Substitute for π and for d.
$=$ ☐	Multiply.

Answer: The distance around the label is about ☐.

Example 2 *Finding the Radius of a Circle*

The circumference of a circle is 56 feet. Find the radius of the circle to the nearest foot.

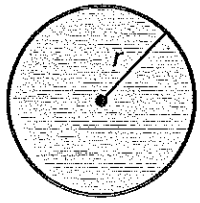
$C = 2\pi r$	Write formula for circumference of a circle.
☐ $\approx$ ☐(☐)r	Substitute for C and for π.
☐ $\approx$ ☐r	Multiply.
☐ $\approx r$	Divide each side by ☐. Use a calculator.

Answer: The radius of the circle is about ☐.

Area of a Circle

Words The area A of a circle is the product of π and the square of the radius r.

Algebra $A = \pi r^2$

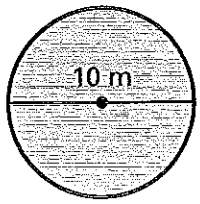


Example 3 *Finding the Area of a Circle*

Find the area of the circle to the nearest square meter.

1. Find the radius.

$$r = \frac{d}{2} = \frac{\square}{2} = \square$$



2. Find the area.

$A = \pi r^2$ Write formula for area of a circle.

$\approx \square(\square)^2$ Substitute for π and for r.

$= \square$ Simplify.

Answer: The area of the circle is about $\square$.

Example 4 *Finding the Radius of a Circle*

The area of a circle is 39.25 square yards. Find the radius of the circle to the nearest tenth of a yard.

$A = \pi r^2$ Write formula for area of a circle.

$\square \approx \square r^2$ Substitute for A and for π.

$\square \approx r^2$ Divide each side by $\square$.

$\square \approx r$ Take positive square root of each side.

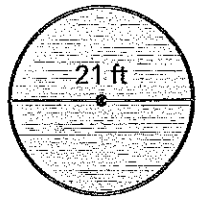
$\square \approx r$ Use a calculator to approximate square root.

Answer: To the nearest tenth of a yard, the radius of the circle is about $\square$.

Checkpoint **Find the circumference and the area of the circle. Round to the nearest whole number.**

1.

2.



10.5 Surface Areas of Prisms and Cylinders

Goal: Find the surface areas of prisms and cylinders.

In your textbook, every prism is a *right prism*, which means that the edges connecting the bases are perpendicular to the bases.

Vocabulary

Surface area of a solid:

Net:

Lateral faces of a prism:

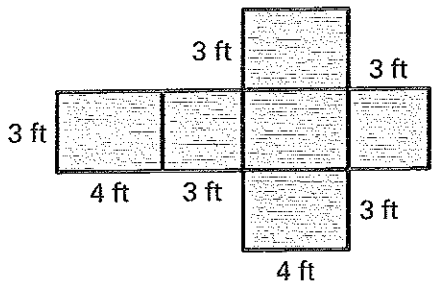
Lateral area of a prism:

Lateral surface of a cylinder:

Lateral area of a cylinder:

Example 1 *Using a Net to Find Surface Area*

A storage chest has the shape of a rectangular prism. The net represents the storage chest. Use the net to find the surface area of the storage chest.



1. Find the area of each face.

   Area of a rectangular face: ____ = ____

   Area of a square face: ____ = ____

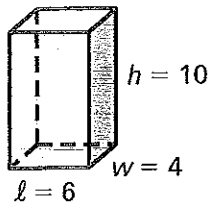
2. Find the sum of the areas of the faces.

   ____ = ____

Answer: The surface area of the storage chest is ____.

Surface Area of a Prism

Words The surface area S of a prism is the sum of twice the base area B and the product of the base perimeter P and the height h.



Algebra $S = 2B + Ph$

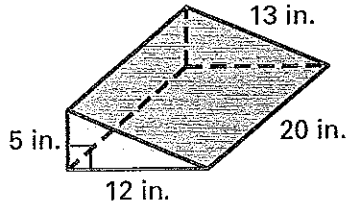
Numbers $S = 2(6 \cdot 4) + [2(6) + 2(4)]10 = 148$ square units

The formula applies to any prism. When finding the surface area of a rectangular prism, you can substitute ℓw for B and $2\ell + 2w$ for P. So, the formula becomes $S = 2\ell w + (2\ell + 2w)h$.

Example 2 *Using a Formula to Find Surface Area*

Find the surface area of the prism.

The bases of the prism are right triangles.



$S = 2B + Ph$ Write formula for surface area.

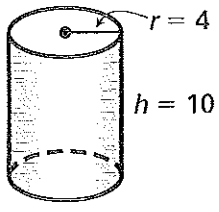
$= 2($ ☐ $) + ($ ☐ $)($ ☐ $)$ Substitute.

$=$ ☐ Simplify.

Answer: The surface area of the prism is ☐.

Surface Area of a Cylinder

Words The surface area S of a cylinder is the sum of twice the base area B and the product of the base circumference C and the height h.



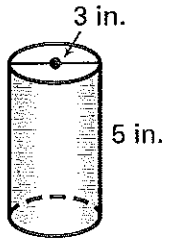
Algebra $S = 2B + Ch = 2\pi r^2 + 2\pi rh$

Numbers $S = 2\pi(4)^2 + 2\pi(4)(10) \approx 352$ square units

In your textbook, all cylinders are *right cylinders*, which means that the segment connecting the centers of the bases is perpendicular to the bases.

Example 3 *Using a Formula to Find Surface Area*

Soup Can Find the surface area of the can of soup. Round your answer to the nearest tenth of an inch.



Solution

The radius is one half of the diameter, so $r = 1.5$ inches.

Although you used 3.14 as an approximation for π previously, you can obtain a more accurate approximation by using the π key on a calculator.

$S = 2\pi r^2 + 2\pi rh$ Write formula for surface area of a cylinder.

$= 2\pi(\quad)^2 + 2\pi(\quad)(\quad)$ Substitute.

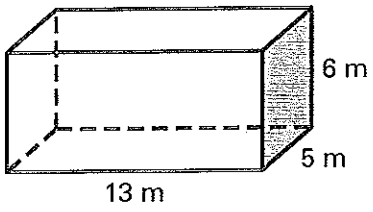
$= \quad \pi$ Simplify.

$\approx$ Evaluate. Use a calculator.

Answer: The surface area of the can of soup is about _____.

Checkpoint **Find the surface area of the prism or cylinder. Round to the nearest whole number.**

1.



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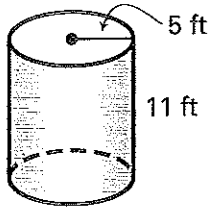


10.7 Volumes of Prisms and Cylinders

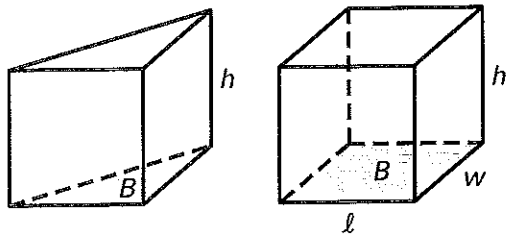
Goal: Find the volumes of prisms and cylinders.

Volume of a Prism

Words The volume V of a prism is the product of the base area B and the height h.

Algebra $V = Bh$

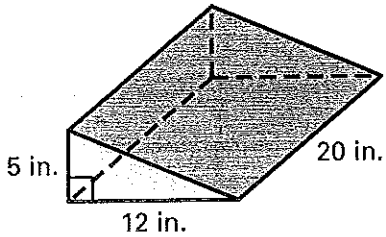
This formula applies to any prism.



Example 1 *Finding the Volume of a Prism*

Find the volume of the prism shown.

The bases of the prism are triangles, so use the formula for the area of a triangle to find B.



$V = Bh$ Write formula for volume of a prism.

$= \square(\square)(\square)(\square)$ Substitute values.

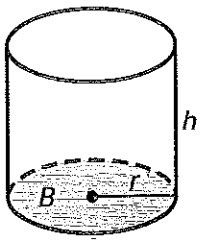
$= \square$ Multiply.

Answer: The volume of the prism is $\square$.

Volume of a Cylinder

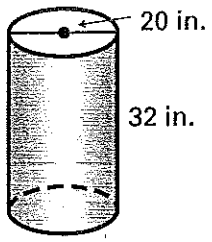
Words The volume V of a cylinder is the product of the base area B and the height h.

Algebra $V = Bh = \pi r^2 h$



Example 2 *Finding the Volume of a Cylinder*

Find the capacity (in gallons) of the recycling bin shown. Round to the nearest whole number. (Use the fact that $1 \text{ in.}^3 \approx 0.004$ gal.)



Solution

1. The radius is one half of the diameter. So, $r = 10$.

$V = \pi r^2 h$ — Write formula for volume of a cylinder.

$= \pi(\square)^2(\square)$ — Substitute values.

$= \square$ — Simplify. Leave in terms of π.

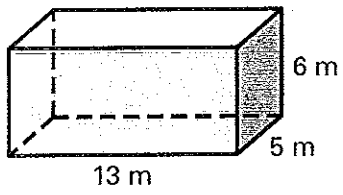
2. Use a conversion factor that converts cubic inches to gallons.

$\square \cdot \square \approx \square$ — Evaluate. Use a calculator.

Answer: The capacity of the recycling bin is about $\square$.

Checkpoint **Find the volume of the prism or cylinder. Round to the nearest whole number.**

1.



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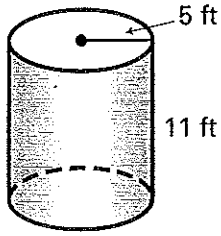
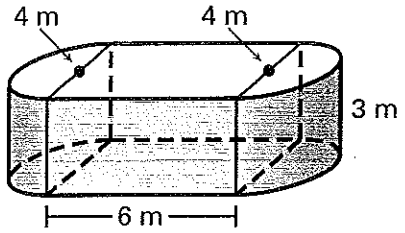


Example 3 *Finding the Volume of a Solid*

The solid shown is composed of a rectangular prism and two half cylinders. Find the volume of the solid. Round to the nearest cubic meter.



Solution

1. Find the area of a base. Each end of a base is a half circle with a radius of ☐. Together, the ends form a complete circle.

$B =$ Area of rectangle $+$ Area of circle

$= \ell w + \pi r^2$ — Use formulas for area of a rectangle and area of a circle.

$=$ ☐(☐) $+ \pi$(☐)^2 — Substitute values.

$=$ ☐ — Simplify. Leave in terms of π.

2. $V = Bh$ — Write formula for volume of a prism.

$=$ (☐)☐ — Substitute values.

$=$ ☐ — Use distributive property.

$\approx$ ☐ — Evaluate. Use a calculator.

Answer: The volume of the solid is about ☐.

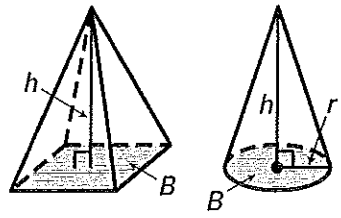
10.8 Volumes of Pyramids and Cones

Goal: Find the volumes of pyramids and cones.

Volume of a Pyramid or a Cone

Words The volume V of a pyramid or a cone is one third of the product of the base area B and the height h.

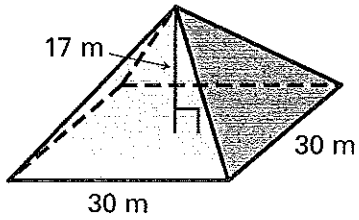
Algebra $V = \frac{1}{3}Bh$



Unlike the formula for the surface area of a pyramid, the formula for the volume of a pyramid can also be used for pyramids whose bases are not regular.

Example 1 *Finding the Volume of a Pyramid*

Find the volume of the pyramid.



$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

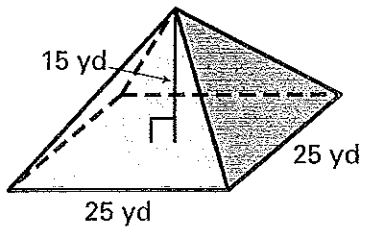
$= \frac{1}{3}(\square)(\square)$ Substitute.

$= \square$ Simplify.

Answer: The volume of the pyramid is $\square$.

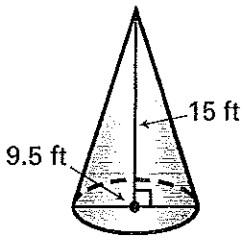
Checkpoint

1. Find the volume of the pyramid.



Example 2 *Finding the Volume of a Cone*

Find the volume of the cone. Round to the nearest cubic foot.



The radius is one half of the diameter, so $r =$ ☐ feet.

$V = \frac{1}{3}Bh$ Write formula for volume of a cone.

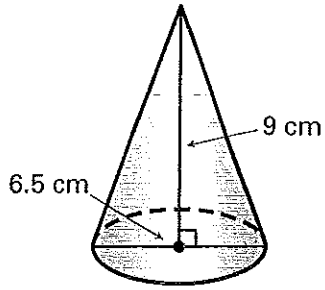
$= \frac{1}{3}$☐(☐)(☐) Substitute.

$\approx$ ☐ Evaluate. Use a calculator.

Answer: The volume of the cone is about ☐.

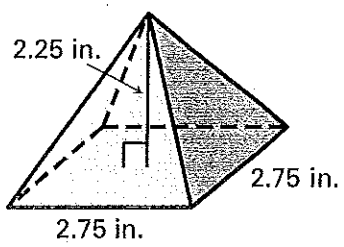
✔ *Checkpoint*

2. Find the volume of the cone. Round to the nearest cubic centimeter.



Example 3 *Finding the Volume of a Solid*

Award The shape of an award is a pyramid. Find the volume of the award to the nearest tenth of a cubic inch.



Solution

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

$= \frac{1}{3}$(☐)(☐) Substitute.

$\approx$ ☐ Simplify.

Answer: The volume of the award is about ☐.