Unit 4: Geometry

- Why is it important to use an appropriate symbol for naming and notating points, lines, segments, rays, and angles?
- How are points, lines, segments, rays, angles and planes named and notated?
- When two parallel lines are cut by a transversal, how can one angle measure be used to determine the others?
- Why is understanding the properties of triangles so important?
- How are algebraic equations applied to geometric situations?

9.1 Square Roots

Goal: Find and approximate square roots of numbers.

Vocabulary

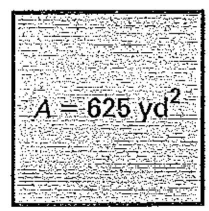
Square root:

Perfect square:

Radical expression:

Example 1 *Finding a Square Root*

Playground A community is building a playground on a square plot of land with an area of 625 square yards. What is the length of each side of the plot of land?



Solution

The plot of land is a square with an area of 625 square yards, so the length of each side of the plot of land is the __________.

Because length cannot be negative, it doesn't make sense to find the negative square root.

$\sqrt{625} =$ ___ because ___ $= 625$.

Answer: The length of each side of the plot of land is ______.

Checkpoint **Find the square roots of the number.**

1. 9	**2.** 49	**3.** 169	**4.** 196

Example 2 *Approximating a Square Root*

Approximate $\sqrt{28}$ to the nearest integer.

The perfect square closest to, but less than, 28 is ☐. The perfect square closest to, but greater than, 28 is ☐. So, 28 is between ☐ and ☐. This statement can be expressed by the *compound inequality* ☐ < 28 < ☐.

☐ < 28 < ☐	Identify perfect squares closest to 28.
☐ < $\sqrt{28}$ < ☐	Take positive square root of each number.
☐ < $\sqrt{28}$ < ☐	Evaluate square root of each perfect square.

Answer: Because 28 is closer to ☐ than to ☐, $\sqrt{28}$ is closer to ☐ than to ☐. So, to the nearest integer, $\sqrt{28} \approx$ ☐.

Checkpoint **Approximate the square root to the nearest integer.**

5. $\sqrt{46}$	**6.** $-\sqrt{125}$	**7.** $\sqrt{68.9}$	**8.** $-\sqrt{87.5}$

Example 3 *Using a Calculator*

Use a calculator to approximate $\sqrt{636}$. Round to the nearest tenth.

Keystrokes	Display	Answer
2nd [√‾] 636) =	☐	☐

Checkpoint Use a calculator to approximate the square root. Round to the nearest tenth.

9. $\sqrt{6}$	10. $-\sqrt{104}$	11. $-\sqrt{819}$	12. $\sqrt{1874}$

Example 4 *Evaluating a Radical Expression*

Evaluate $5\sqrt{a^2 - b}$ when $a = 6$ and $b = 27$.

$5\sqrt{a^2 - b} =$ ______ Substitute for a and for b.

$=$ ______ Evaluate expression inside radical symbol.

$=$ ______ Evaluate square root.

$=$ ______ Multiply.

Checkpoint Evaluate the expression when $a = 16$ and $b = 9$.

13. $-\sqrt{a + b}$	14. $\sqrt{b^2 - 2a}$	15. $2\sqrt{ab}$

9.3 The Pythagorean Theorem

Goal: Use the Pythagorean theorem to solve problems.

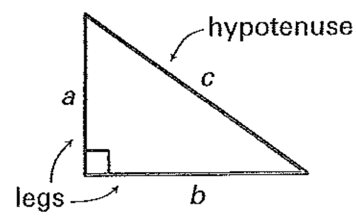
Vocabulary

Hypotenuse:

Legs:

Pythagorean Theorem

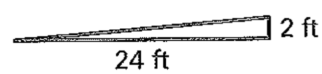
Words For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.



Algebra $a^2 + b^2 = c^2$

Example 1 ***Finding the Length of a Hypotenuse***

A building's access ramp has a horizontal distance of 24 feet and a vertical distance of 2 feet. Find the length of the ramp to the nearest tenth of a foot.

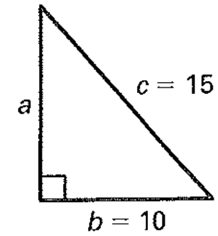


$a^2 + b^2 = c^2$	Pythagorean theorem
$\square^2 + \square^2 = c^2$	Substitute for *a* and for *b*.
$\square = c^2$	Evaluate powers and add.
$\square = c$	Take positive square root of each side.
$\square \approx c$	Simplify.

Answer: The length of the ramp is about $\square$ feet.

Example 2 *Finding the Length of a Leg*

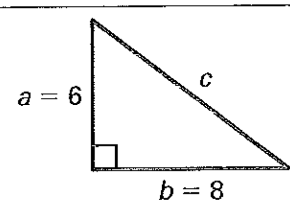
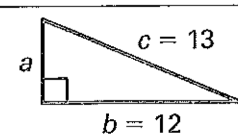
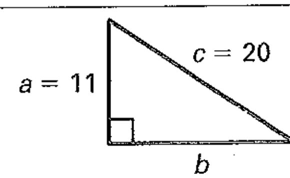
Find the unknown length *a* in simplest form.



$a^2 + b^2 = c^2$	Pythagorean theorem
$a^2 + \square^2 = \square^2$	Substitute.
$a^2 + \square = \square$	Evaluate powers.
$a^2 = \square$	Subtract $\square$ from each side.
$a = \square$	Take positive square root of each side.
$a = \square$	Simplify.

Answer: The unknown length *a* is $\square$ units.

Checkpoint **Find the unknown length. Write your answer in simplest form.**

1. 	**2.** 	**3.** 

Converse of the Pythagorean Theorem

The Pythagorean theorem can be written in "if-then" form.

Theorem: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

If you reverse the two parts of the statement, the new statement is called the *converse* of the Pythagorean theorem.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Although not all converses of true statements are true, the converse of the Pythagorean theorem is true.

Example 3 *Identifying Right Triangles*

Determine whether the triangle with the given side lengths is a right triangle.

a. $a = 8, b = 9, c = 12$

b. $a = 7, b = 24, c = 25$

Solution

a. $a^2 + b^2 = c^2$

$\square^2 + \square^2 \stackrel{?}{=} \square^2$

$\square + \square \stackrel{?}{=} \square$

$\square \; \square \; \square$

Answer: ____________

b. $a^2 + b^2 = c^2$

$\square^2 + \square^2 \stackrel{?}{=} \square^2$

$\square + \square \stackrel{?}{=} \square$

$\square \; \square \; \square$

Answer: ____________

Checkpoint **Determine whether the triangle with the given side lengths is a right triangle.**

4. $a = 12, b = 9, c = 15$	5. $a = 10, b = 25, c = 27$

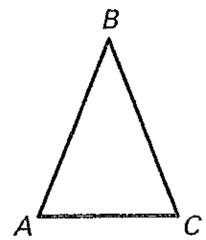
10.1 Triangles

Goal: Solve problems involving triangles.

> You can classify a triangle by its angle measures or by its side lengths. When classified by angle measures, triangles are acute, right, obtuse, or equiangular. When classified by side lengths, triangles are equilateral, isosceles, or scalene.

Example 1 *Classifying a Triangle by Angle Measures*

In the diagram, $m\angle ABC = 44°$ and $m\angle BAC = m\angle BCA$. Find $m\angle BAC$ and $m\angle BCA$. Then classify $\triangle ABC$ by its angle measures.



Solution

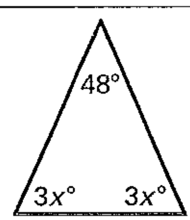
Let $x°$ represent $m\angle BAC$ and $m\angle BCA$.

$m\angle BAC + m\angle BCA + m\angle ABC = 180°$	Sum of angle measures is 180°.
$\square + \square + \square = 180°$	Substitute values.
$\square + \square = 180$	Combine like terms.
$\square = \square$	Subtract $\square$ from each side.
$x = \square$	Divide each side by $\square$.

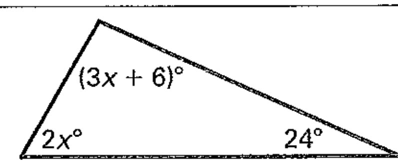
Answer: $m\angle BAC = m\angle BCA = \square$. Because $\angle BAC$, $\angle BCA$, and $\angle ABC$ are $\square$, $\triangle ABC$ is $\square$.

Checkpoint **Find the value of x. Then classify the triangle by its angle measures.**

1.



2.



Example 2 *Finding Unknown Side Lengths*

The perimeter of a scalene triangle is 45 inches. The length of the first side is twice the length of the second side. The length of the third side is 15 inches. Find the lengths of the other two sides.

Solution

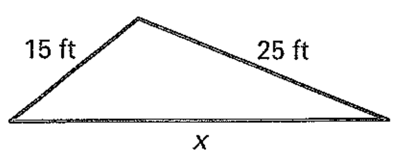
Draw the triangle. Let x and $2x$ represent the unknown side lengths. Write an equation for the perimeter P. Then solve for x.

$P = 2x + x + 15$	Formula for perimeter
$\square = 2x + x + 15$	Substitute $\square$ for P.
$\square = \square$	Combine like terms.
$\square = \square$	Subtract $\square$ from each side.
$\square = x$	Divide each side by $\square$.

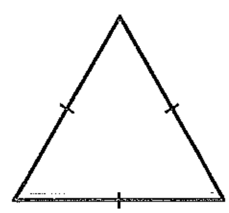
Answer: The length of the second side is $\square$ inches, and the length of the first side is $2(\square) = \square$ inches.

Checkpoint **Find the unknown side length of the triangle given the perimeter P. Then classify the triangle by its side lengths.**

3. $P = 75$ ft



4. $P = 21.6$ m



Example 3 *Finding Angle Measures Using a Ratio*

For a triangle whose angles measure 50°, 60°, and 70°, you can say that the ratio of the angle measures is 50:60:70, or 5:6:7. Therefore, if you know that the ratio of the angle measures is 5:6:7, you can say that the angle measures are $5x°$, $6x°$, and $7x°$ for some value of x.

The ratio of the angle measures of a triangle is 3 : 4 : 5. Find the angle measures. Then classify the triangle by its angle measures.

Solution.

1. Let ____, ____, and ____ represent the angle measures. Write an equation for the sum of the angle measures.

____ + ____ + ____ = 180°	Sum of angle measures is 180°.
____ = 180	Combine like terms.
x = ____	Divide each side by ____.

2. Substitute ____ for x in the expression for each angle measure.

$(3 \cdot \square)° = \square$ $(4 \cdot \square)° = \square$ $(5 \cdot \square)° = \square$

Answer: The angle measures of the triangle are ____, ____, and ____. So, the triangle is ________.

Checkpoint

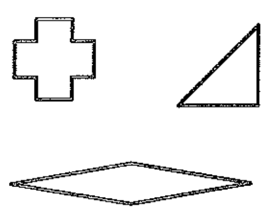
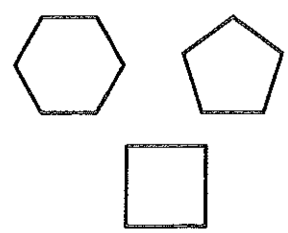
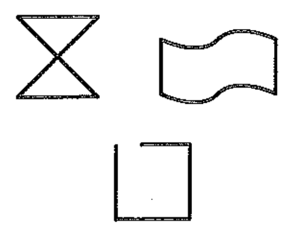
5. The ratio of the angle measures of a triangle is 7 : 12 : 17. Find the angle measures. Then classify the triangle by its angle measures.

10.2 Polygons and Quadrilaterals

Goal: Classify polygons and quadrilaterals.

Vocabulary

Polygon:	
Regular polygon:	
Convex polygon:	
Concave polygon:	
Diagonal of a polygon:	

Polygons	Regular polygons	Not polygons
		

The name *n*-gon refers to a polygon that has *n* sides. For example, a 15-gon is a polygon that has 15 sides.

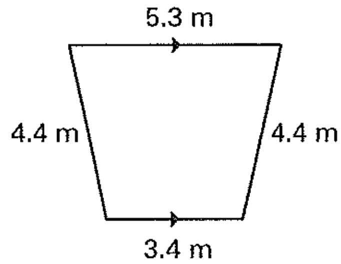
Names of Other Polygons

Polygons	Pentagon	Hexagon	Heptagon	Octagon	*n*-gon
Number of sides	5	6	7	8	n

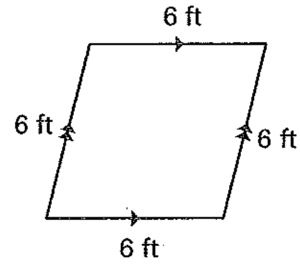
Example 2 *Classifying Quadrilaterals*

Classify the quadrilateral.

a.

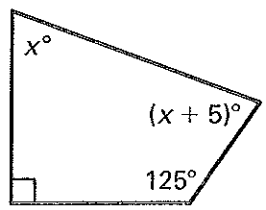


b.



Example 3 *Finding an Unknown Angle Measure*

Find the value of *x*.



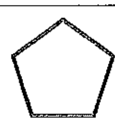
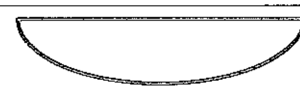
$\square + \square + \square + \square = 360°$ Sum of angle measures in quadrilateral is 360°.

$\square + \square = 360$ Combine like terms.

$\square = \square$ Subtract $\square$ from each side.

$x = \square$ Divide each side by $\square$.

Checkpoint **Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is *convex* or *concave*. If not, explain why.**

1. 	2. 

13.1 Angle Relationships

Goal: Classify special pairs of angles.

Vocabulary

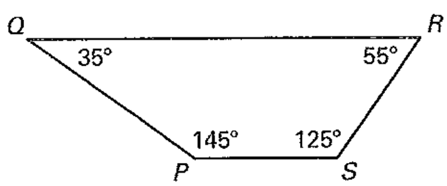
Complementary angles: ______

Supplementary angles: ______

Vertical angles: ______

Example 1 ***Identifying Complementary, Supplementary Angles***

In quadrilateral *PQRS*, identify all pairs of complementary angles and supplementary angles.



Solution

a. Because $m\angle Q + m\angle R =$ ☐ + ☐ = ☐, $\angle Q$ and $\angle R$ are ______ angles.

b. Because $m\angle P + m\angle Q =$ ☐ + ☐ = ☐, $\angle P$ and $\angle Q$ are ______ angles.

c. Because $m\angle R + m\angle S =$ ☐ + ☐ = ☐, $\angle R$ and $\angle S$ are ______ angles.

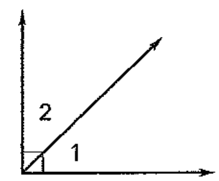
Checkpoint Tell whether the angles are *complementary*, *supplementary*, or *neither*.

1. $m\angle 1 = 37°$ $m\angle 2 = 73°$	2. $m\angle 3 = 42°$ $m\angle 4 = 48°$	3. $m\angle 5 = 127°$ $m\angle 6 = 53°$

Adjacent angles that form a right angle are complementary. Adjacent angles that form a straight angle are supplementary.

Example 2 *Finding an Angle Measure*

For the diagram shown, $\angle 1$ and $\angle 2$ are complementary angles, and $m\angle 1 = 46°$. Find $m\angle 2$.



Solution

$m\angle 1 + m\angle 2 = \square$ Definition of complementary angles

$\square + m\angle 2 = \square$ Substitute for $m\angle 1$.

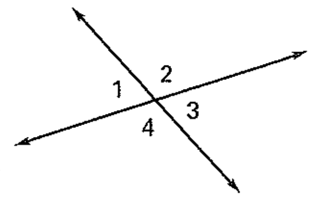
$m\angle 2 = \square$ Subtract $\square$ from each side.

Checkpoint $\angle 1$ and $\angle 2$ are complementary angles. Given $m\angle 1$, find $m\angle 2$.

4. $m\angle 1 = 64°$	5. $m\angle 1 = 13°$
6. $m\angle 1 = 82°$	7. $m\angle 1 = 7°$

Example 3 *Using Supplementary and Vertical Angles*

For the diagram shown, $m\angle 1 = 65°$.
Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



Solution

a. $m\angle 1 + m\angle 2 =$ ☐ — $\angle 1$ and $\angle 2$ are supplementary.

☐ $+ m\angle 2 =$ ☐ — Substitute for $m\angle 1$.

$m\angle 2 =$ ☐ — Subtract ☐ from each side.

b. $m\angle 3 =$ ☐ — Vertical angles have same measure.

$m\angle 3 =$ ☐ — Substitute for $m\angle 1$.

c. $m\angle 4 =$ ☐ — Vertical angles have same measure.

$m\angle 4 =$ ☐ — Substitute for $m\angle 2$.

Checkpoint

8. $\angle 1$ and $\angle 2$ are supplementary angles, and $m\angle 1 = 132°$. Find $m\angle 2$.	**9.** $\angle 3$ and $\angle 4$ are supplementary angles, and $m\angle 3 = 23°$. Find $m\angle 4$.

10. In Example 3, suppose that $m\angle 1 = 54°$. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

13.2 Angles and Parallel Lines

Goal: Identify angles when a transversal intersects lines.

Vocabulary

Transversal:

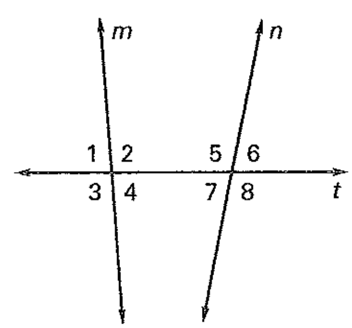
Corresponding angles:

Alternate interior angles:

Alternate exterior angles:

Example 1 *Identifying Angles*

In the diagram, line t is a transversal. Tell whether the angles are *corresponding*, *alternate interior*, or *alternate exterior* angles.



a. $\angle 1$ and $\angle 5$

b. $\angle 2$ and $\angle 7$

c. $\angle 3$ and $\angle 6$

Solution

a. $\angle 1$ and $\angle 5$ are ______ angles.

b. $\angle 2$ and $\angle 7$ are ______ angles.

c. $\angle 3$ and $\angle 6$ are ______ angles.

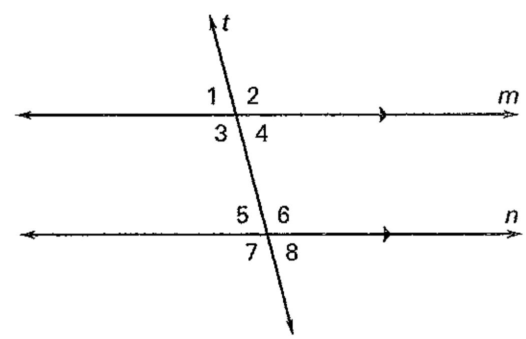
Checkpoint In Example 1, tell whether the angles are *corresponding*, *alternate interior*, or *alternate exterior* angles.

1. $\angle 4$ and $\angle 5$	2. $\angle 1$ and $\angle 8$	3. $\angle 4$ and $\angle 8$

Angles and Parallel Lines

In the diagram, transversal *t* intersects parallel lines *m* and *n*.

Corresponding angles



$m\angle 1 =$ ☐

$m\angle 2 =$ ☐

$m\angle 3 =$ ☐

$m\angle 4 =$ ☐

Alternate interior angles

$m\angle 3 =$ ☐

$m\angle 4 =$ ☐

Alternate exterior angles

$m\angle 1 =$ ☐

$m\angle 2 =$ ☐

13.3 Angles and Polygons

Goal: Find measures of interior and exterior angles.

Vocabulary

Interior angle:	
Exterior angle:	

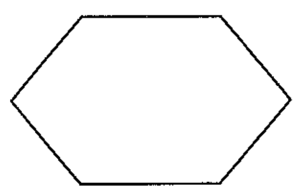
Measures of Interior Angles of a Convex Polygon

The sum of the measures of the interior angles of a convex n-gon is given by the formula $(n - 2) \cdot 180°$.

The measure of an interior angle of a regular n-gon is given by the formula $\frac{(n-2) \cdot 180°}{n}$.

Example 1 *Finding the Sum of a Polygon's Interior Angles*

Find the sum of the measures of the interior angles of the polygon.



Solution

For a convex hexagon, $n = \square$.

$$(n - 2) \cdot 180° = (\square - 2) \cdot 180°$$
$$= \square \cdot 180°$$
$$= \square$$

Example 2 *Finding the Measure of an Interior Angle*

Find the measure of an interior angle of a regular octagon.

Solution

For a regular octagon, $n = 8$.

Measure of an interior angle = [ ] Write formula.

= [ ] Substitute for *n*.

= [ ] Simplify.

Checkpoint

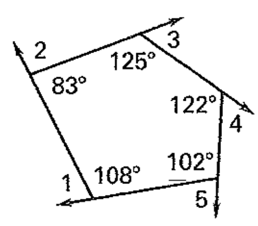
1. Find the sum of the measures of the interior angles of a convex 9-gon.

2. Find the measure of an interior angle of a regular 18-gon.

Example 3 *Finding the Measure of an Exterior Angle*

> An interior angle and an exterior angle at the same vertex form a straight angle.

Find $m\angle 1$ in the diagram.



Solution

The angle that measures [ ] forms a straight angle with $\angle 1$, which is the exterior angle at the same vertex.

$m\angle 1 +$ [ ] = [ ] Angles are supplementary.

$m\angle 1 =$ [ ] Subtract [ ] from each side.

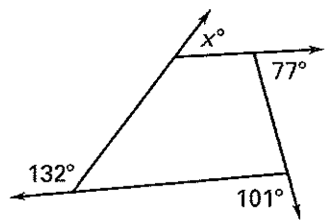
Checkpoint

3. In Example 3, find $m\angle 2$, $m\angle 3$, $m\angle 4$, and $m\angle 5$.

Each vertex of a convex polygon has two exterior angles. If you draw one exterior angle at each vertex, then the sum of the measures of these angles is 360°.

Example 4 *Using the Sum of Measures of Exterior Angles*

Find the unknown angle measure in the diagram.



Solution

$x° + 77° + 101° + 132° =$ [ ] Sum of measures of exterior angles of a convex polygon is 360°.

$x +$ [ ] $=$ [ ] Add.

$x =$ [ ] Subtract [ ] from each side.

Answer: The angle measure is [ ].

Checkpoint

4. Five exterior angles of a convex hexagon have measures 42°, 78°, 60°, 55°, and 62°. Find the measure of the sixth exterior angle.

13.4 Translations

Goal: Translate figures in a coordinate plane.

Vocabulary

Transformation:

Image:

Translation:

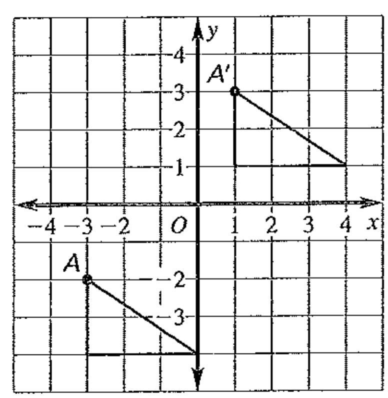
Tessellation:

In a translation, a figure and its image are congruent.

Example 1 *Describing a Translation*

For the diagram shown, describe the translation in words.

Solution



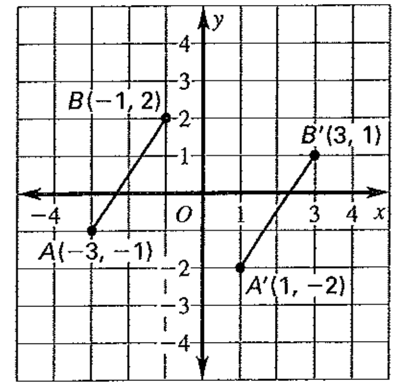
Think of moving horizontally and vertically from a point on the original figure to the corresponding point on the new figure. For instance, you move ☐ units to the ☐ and ☐ units ☐ from $A(-3, -2)$ to reach A' ☐.

Coordinate Notation

You can describe a translation of each point (x, y) of a figure using the coordinate notation

$$(x, y) \rightarrow (x + a, y + b)$$

where *a* indicates how many units a point moves horizontally, and *b* indicates how many units a point moves ______. Move the point (x, y) to the right if *a* is positive and to the ______ if *a* is ______. Move the point up if *b* is positive and ______ if *b* is ______.



(x, y) → ______

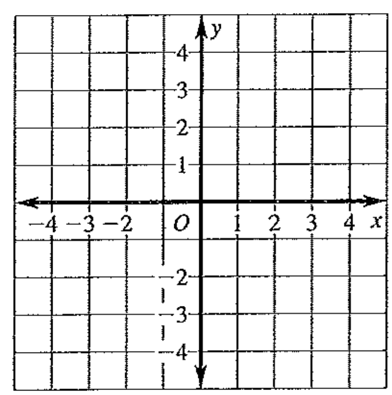
Example 2 *Translating a Figure*

Draw △*ABC* with vertices *A*(−2, 1), *B*(−1, 4), and *C*(0, 1). Then find the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 5)$, and draw the image.

Solution

First draw △*ABC*. Then, to translate △*ABC*, ______ to the *x*-coordinate and ______ from the *y*-coordinate of each vertex.

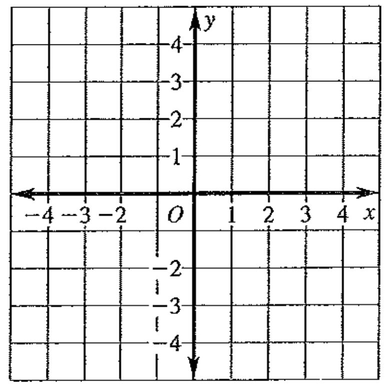
Original		**Image**
(x, y)	→	(x + 4, y − 5)
A(−2, 1)	→	A′ ______
B(−1, 4)	→	B′ ______
C(0, 1)	→	C′ ______



Finally, draw △*A′B′C′*. Notice that each point on △*ABC* moves ___ units to the ______ and ___ units ______.

Checkpoint

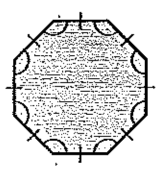
1. Draw quadrilateral *PQRS* with vertices *P*(−4, −1), *Q*(−1, 0), *R*(−2, −3), and *S*(−4, −4). Then find the coordinates of the image after the translation $(x, y) \rightarrow (x + 6, y + 5)$, and draw the image.



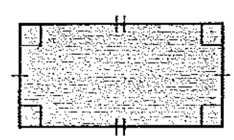
Example 3 *Creating Tessellations*

Tell whether you can create a tessellation using only translations of the given polygon. If you can, create a tessellation. If not, explain why not.

a.

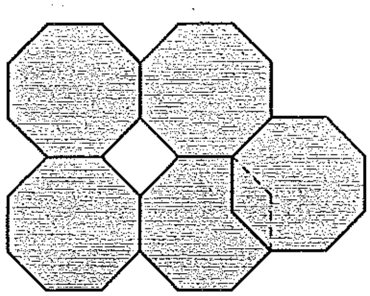


b.

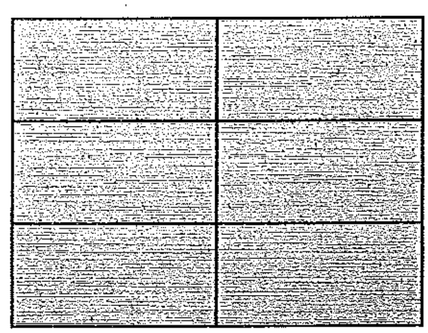


Solution

a. You ______ translate a regular octagon to create a tessellation. Notice in the design that there ______ gaps and overlaps.



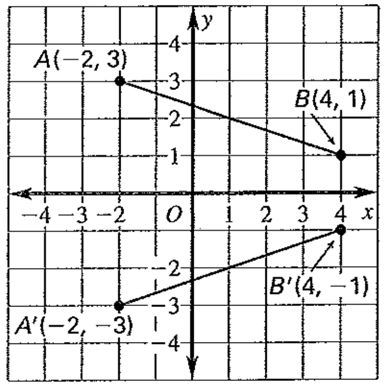
b. You ______ translate the rectangle to create a tessellation. Notice in the design that there ______ gaps or overlaps.



Coordinate Notation

You can use coordinate notation to describe the images of figures after reflections in the axes of a coordinate plane.

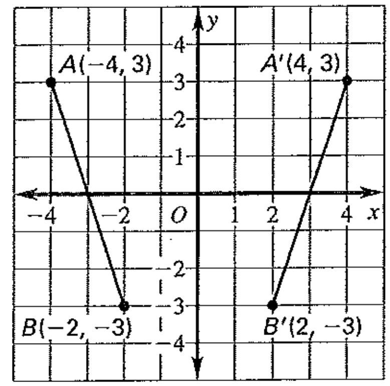
Reflection in the *x*-axis



Multiply the *y*-coordinate by −1.

$(x, y) \rightarrow$ ______

Reflection in the *y*-axis



Multiply the *x*-coordinate by −1.

$(x, y) \rightarrow$ ______

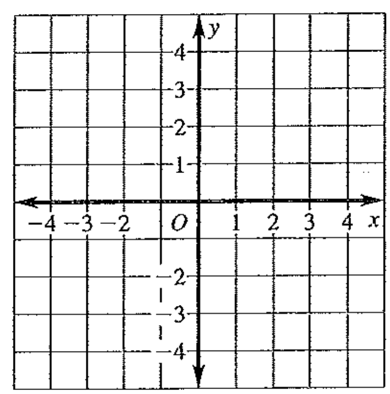
Example 2 *Reflecting a Triangle*

Draw △*ABC* with vertices *A*(2, 2), *B*(2, 5), and *C*(4, 1). Then find the coordinates of the vertices of the image after a reflection in the *x*-axis, and draw the image.

Solution

First draw △*ABC*. Then, to reflect △*ABC* in the *x*-axis, multiply the *y*-coordinate of each vertex by ______.

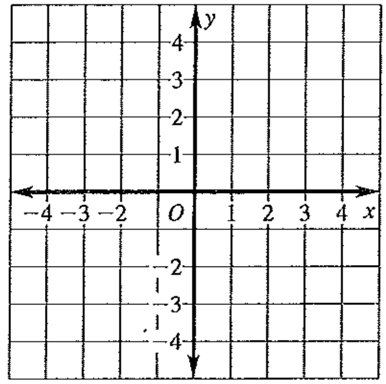
Original		Image
(x, y)	→	______
$A(2, 2)$	→	A' ______
$B(2, 5)$	→	B' ______
$C(4, 1)$	→	C' ______



Finally, draw △*A′B′C′*.

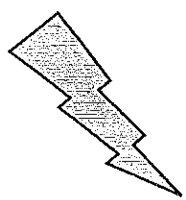
Checkpoint

1. Draw $\triangle ABC$ with vertices $A(-4, -3)$, $B(-4, 4)$, and $C(-1, -3)$. Then find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the y-axis, and draw the image.

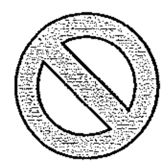


Example 3 *Identifying Lines of Symmetry*

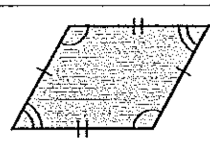
Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.

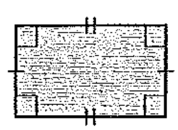
a. 

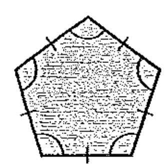
b. 

c. 

Checkpoint **Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.**

2. 

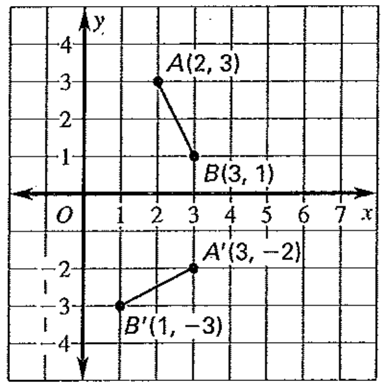
3. 

4. 

90° Rotations

In this lesson, all rotations in the coordinate plane are centered at the origin. You can use coordinate notation to describe a 90° rotation of a figure about the origin.

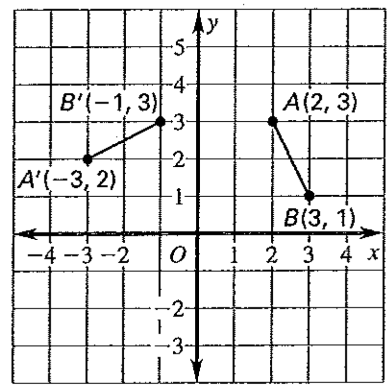
90° clockwise rotation



Switch the coordinates, then multiply the new y-coordinate by −1.

$(x, y) \rightarrow$ ______

90° counterclockwise rotation



Switch the coordinates, then multiply the new x-coordinate by −1.

$(x, y) \rightarrow$ ______

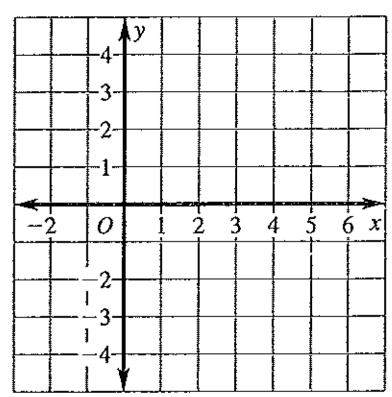
Example 2 *Rotating a Triangle*

Draw $\triangle ABC$ with vertices $A(1, 1)$, $B(3, 4)$, and $C(4, 0)$. Then find the coordinates of the vertices of the image after a 90° clockwise rotation, and draw the image.

Solution

First draw $\triangle ABC$. Then, to rotate $\triangle ABC$ 90° clockwise, switch the coordinates and multiply the new y-coordinate by −1.

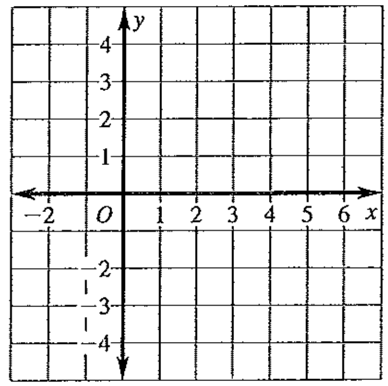
Original		**Image**
(x, y)	$\rightarrow$	______
$A(1, 1)$	$\rightarrow$	A' ______
$B(3, 4)$	$\rightarrow$	B' ______
$C(4, 0)$	$\rightarrow$	C' ______



Finally, draw $\triangle A'B'C'$.

Checkpoint

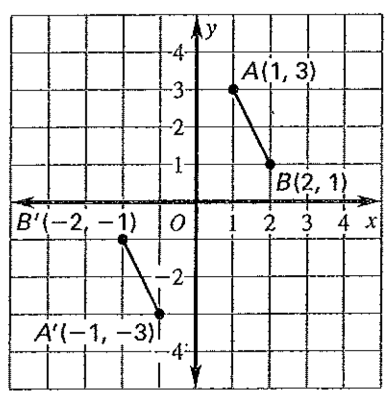
1. Draw $\triangle ABC$ with vertices $A(1, -1)$, $B(3, -1)$, and $C(4, -4)$. Then find the coordinates of the vertices of the image after a 90° counterclockwise rotation, and draw the image.



180° Rotations

To rotate a point 180° about the origin, multiply each coordinate by −1. The image is the same whether you rotate the figure ______ or ______.

$(x, y) \rightarrow$ ______



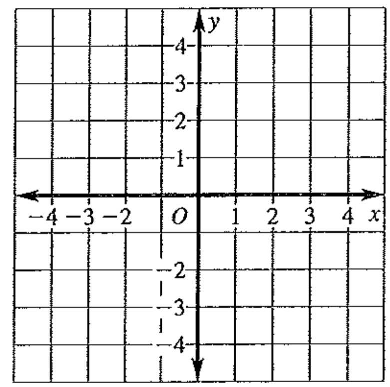
Example 3 *Rotating a Triangle*

Draw $\triangle MNP$ with vertices $M(-4, -4)$, $N(-3, -1)$, and $P(-1, -2)$. Then find the coordinates of the vertices of the image after a 180° rotation, and draw the image.

Solution

First draw $\triangle MNP$. Then, to rotate $\triangle MNP$ 180°, multiply the coordinates by -1.

Original		Image
(x, y)	$\rightarrow$	
$M(-4, -4)$	$\rightarrow$	M'
$N(-3, -1)$	$\rightarrow$	N'
$P(-1, -2)$	$\rightarrow$	P'

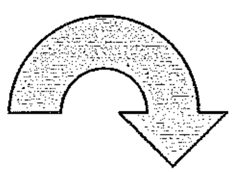


Finally, draw $\triangle M'N'P'$.

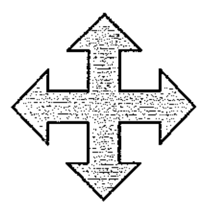
Example 4 *Identifying Rotational Symmetry*

Tell whether the figure has rotational symmetry. If so, give the angle and direction of rotation.

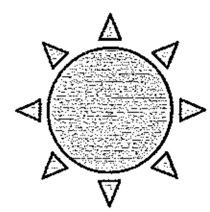
a.



b.



c.



Solution

a. The figure ______ rotational symmetry.

b. The figure ______ rotational symmetry.

c. The figure ______ ______ rotational symmetry.

13.5 Reflections and Symmetry

Goal: Reflect figures and identify lines of symmetry.

Vocabulary

Reflection:

Line of reflection:

Line symmetry:

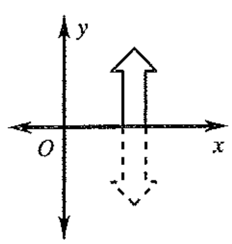
Line of symmetry:

In a reflection, a figure and its image are congruent.

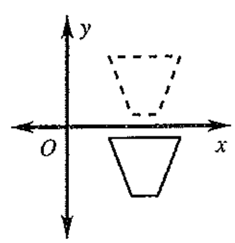
Example 1 *Identifying Reflections*

Tell whether the transformation is a reflection. If so, identify the line of reflection.

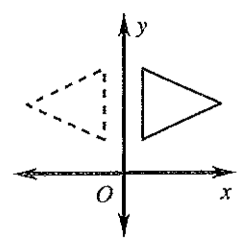
a.



b.



c.



Solution

a.

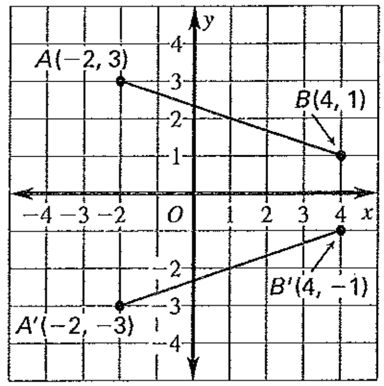
b.

c.

Coordinate Notation

You can use coordinate notation to describe the images of figures after reflections in the axes of a coordinate plane.

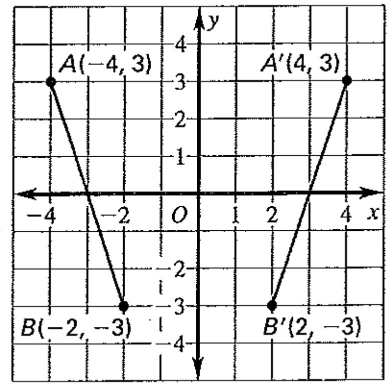
Reflection in the *x*-axis



Multiply the *y*-coordinate by −1.

$(x, y) \rightarrow$ ______

Reflection in the *y*-axis



Multiply the *x*-coordinate by −1.

$(x, y) \rightarrow$ ______

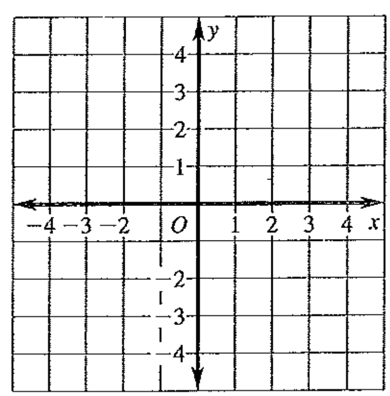
Example 2 *Reflecting a Triangle*

Draw $\triangle ABC$ with vertices $A(2, 2)$, $B(2, 5)$, and $C(4, 1)$. Then find the coordinates of the vertices of the image after a reflection in the *x*-axis, and draw the image.

Solution

First draw $\triangle ABC$. Then, to reflect $\triangle ABC$ in the *x*-axis, multiply the *y*-coordinate of each vertex by ______.

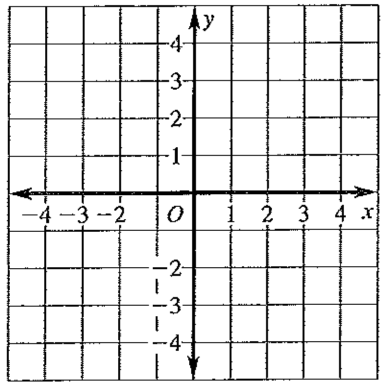
Original		Image
(x, y)	$\rightarrow$	______
$A(2, 2)$	$\rightarrow$	A' ______
$B(2, 5)$	$\rightarrow$	B' ______
$C(4, 1)$	$\rightarrow$	C' ______



Finally, draw $\triangle A'B'C'$.

Checkpoint

1. Draw $\triangle ABC$ with vertices $A(-4, -3)$, $B(-4, 4)$, and $C(-1, -3)$. Then find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the y-axis, and draw the image.



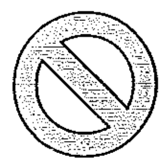
Example 3 *Identifying Lines of Symmetry*

Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.

a.

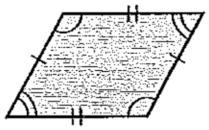
b.

c.

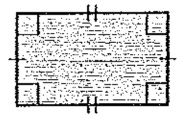


Checkpoint **Draw the lines of symmetry on the figure. Tell how many lines of symmetry the figure has.**

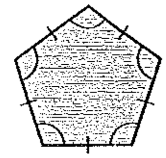
2.



3.



4.



13.6 Rotations and Symmetry

Goal: Rotate figures and identify rotational symmetry.

Vocabulary

Rotation:

Center of rotation:

Angle of rotation:

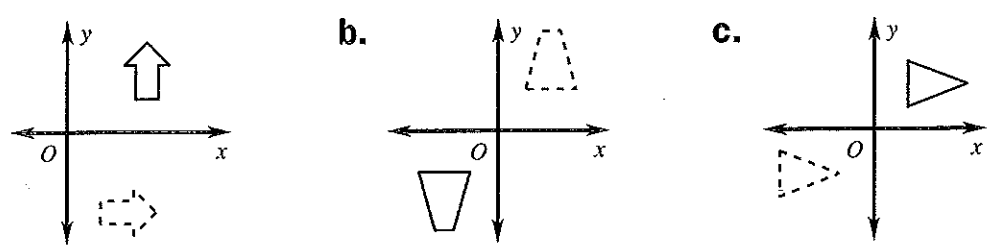
Rotational symmetry:

In a rotation, a figure and its image are congruent.

Example 1 *Identifying Rotations*

Tell whether the transformation is a rotation about the origin. If so, give the angle and direction of the rotation.

a. **b.** **c.**



Solution

a.

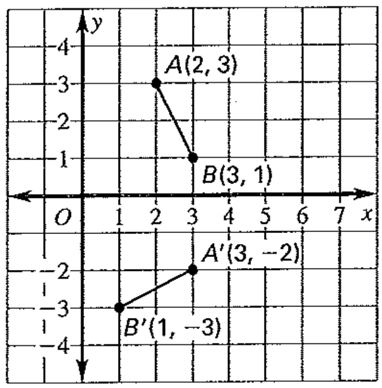
b.

c.

90° Rotations

In this lesson, all rotations in the coordinate plane are centered at the origin. You can use coordinate notation to describe a 90° rotation of a figure about the origin.

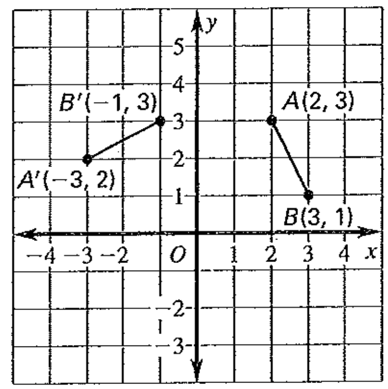
90° clockwise rotation



Switch the coordinates, then multiply the new *y*-coordinate by −1.

$(x, y) \rightarrow$ ______

90° counterclockwise rotation



Switch the coordinates, then multiply the new *x*-coordinate by −1.

$(x, y) \rightarrow$ ______

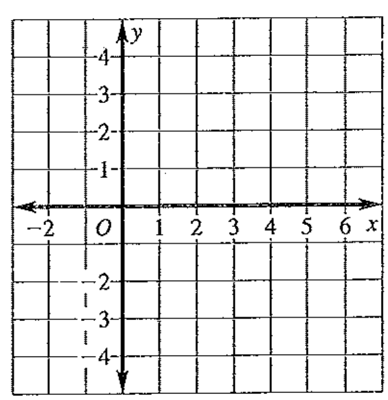
Example 2 ***Rotating a Triangle***

Draw $\triangle ABC$ with vertices $A(1, 1)$, $B(3, 4)$, and $C(4, 0)$. Then find the coordinates of the vertices of the image after a 90° clockwise rotation, and draw the image.

Solution

First draw $\triangle ABC$. Then, to rotate $\triangle ABC$ 90° clockwise, switch the coordinates and multiply the new *y*-coordinate by −1.

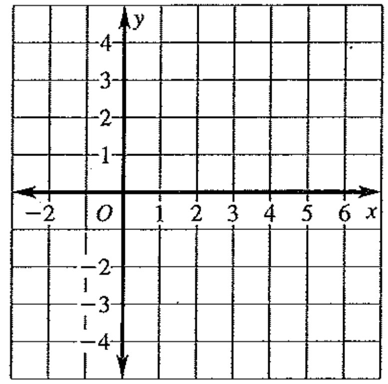
Original		**Image**
(x, y)	$\rightarrow$	______
$A(1, 1)$	$\rightarrow$	A' ______
$B(3, 4)$	$\rightarrow$	B' ______
$C(4, 0)$	$\rightarrow$	C' ______



Finally, draw $\triangle A'B'C'$.

Checkpoint

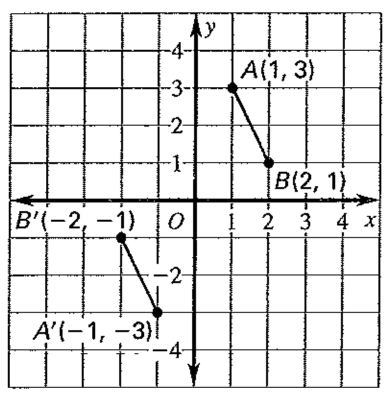
1. Draw $\triangle ABC$ with vertices $A(1, -1)$, $B(3, -1)$, and $C(4, -4)$. Then find the coordinates of the vertices of the image after a 90° counterclockwise rotation, and draw the image.



180° Rotations

To rotate a point 180° about the origin, multiply each coordinate by -1. The image is the same whether you rotate the figure ________ or ________.

$(x, y) \rightarrow$ ________



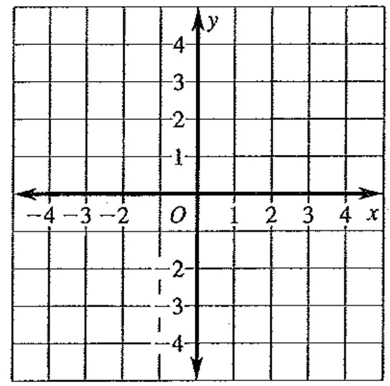
Example 3 *Rotating a Triangle*

Draw $\triangle MNP$ with vertices $M(-4, -4)$, $N(-3, -1)$, and $P(-1, -2)$. Then find the coordinates of the vertices of the image after a 180° rotation, and draw the image.

Solution

First draw $\triangle MNP$. Then, to rotate $\triangle MNP$ 180°, multiply the coordinates by -1.

Original		Image
(x, y)	$\rightarrow$	
$M(-4, -4)$	$\rightarrow$	M'
$N(-3, -1)$	$\rightarrow$	N'
$P(-1, -2)$	$\rightarrow$	P'

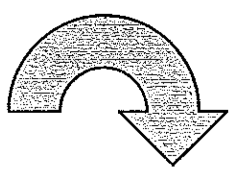


Finally, draw $\triangle M'N'P'$.

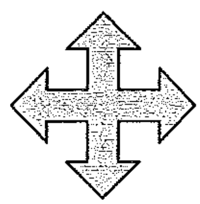
Example 4 *Identifying Rotational Symmetry*

Tell whether the figure has rotational symmetry. If so, give the angle and direction of rotation.

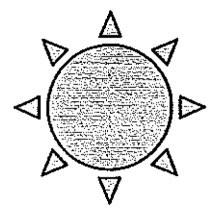
a.



b.



c.



Solution

a. The figure ______ rotational symmetry.

b. The figure ______ rotational symmetry.

c. The figure ______ ______ rotational symmetry.

13.7 Dilations

Goal: Dilate figures in a coordinate plane.

Vocabulary

Dilation:

Center of dilation:

Scale factor:

In a dilation, a figure and its image are similar.

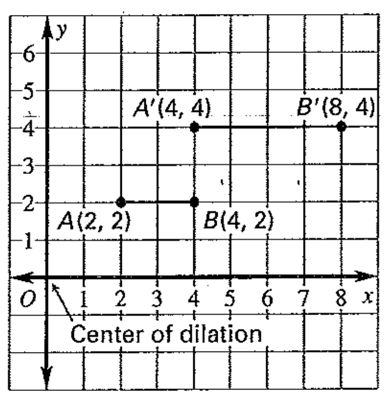
Dilation

In this lesson, the origin of the coordinate plane is the center of dilation.

In the diagram, $\overline{A'B'}$ is the image of $\overline{AB}$ after a dilation. Because $\frac{A'B'}{AB} = 2$, the scale factor is ☐. You can describe a dilation with respect to the origin using the notation

$(x, y) \rightarrow (kx, ky)$

where k is the ☐.



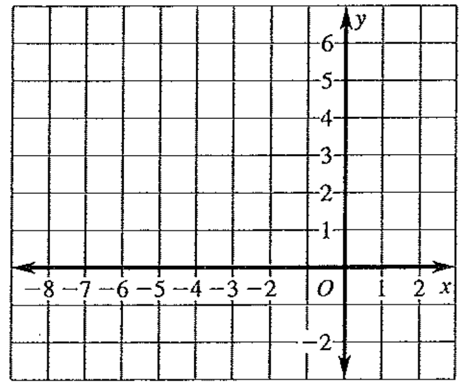
Example 1 *Dilating a Quadrilateral*

Draw quadrilateral with vertices $A(-4, 1)$, $B(1, 3)$, $C(1, -1)$, and $D(-3, -1)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 2, and draw the image.

Solution

First draw quadrilateral $ABCD$. Then, to dilate $ABCD$, multiply the x- and y-coordinates of each vertex by ____.

Original		Image
(x, y)	$\rightarrow$	
$A(-4, 1)$	$\rightarrow$	A'
$B(1, 3)$	$\rightarrow$	B'
$C(1, -1)$	$\rightarrow$	C'
$D(-3, -1)$	$\rightarrow$	D'

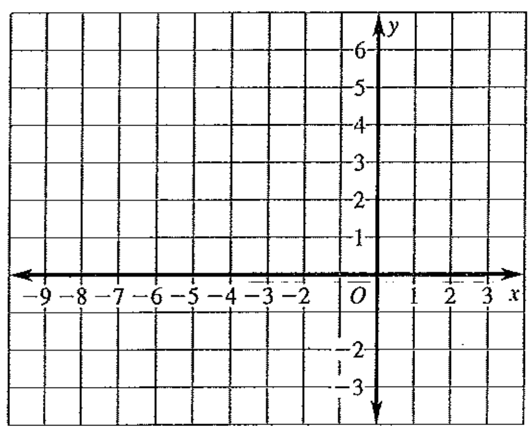


Notice in Example 1 that when $k > 1$, the new figure is an enlargement of the original figure.

Finally, draw quadrilateral $A'B'C'D'$.

Checkpoint

1. Draw $\triangle DEF$ with vertices $D(-3, 2)$, $E(1, 2)$, and $F(1, -1)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 3, and draw the image.



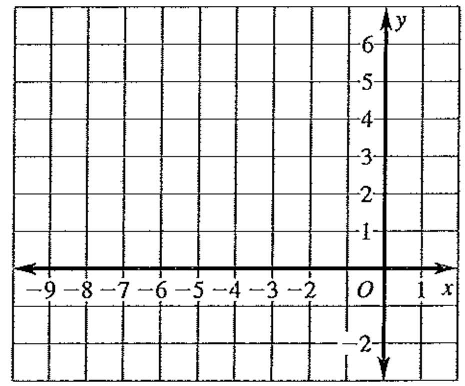
Example 2 *Using a Scale Factor Less than 1*

Draw $\triangle PQR$ with vertices $P(-8, 4)$, $Q(-6, 6)$, and $R(-4, -2)$. Then find the coordinates of the vertices of the image after a dilation having a scale factor of 0.5, and draw the image.

Solution

Draw $\triangle PQR$. Then, to dilate $\triangle PQR$, multiply the *x*- and *y*-coordinates of each vertex by ______.

Original		Image
(x, y)	$\rightarrow$	______
$P(-8, 4)$	$\rightarrow$	P' ______
$Q(-6, 6)$	$\rightarrow$	Q' ______
$R(-4, -2)$	$\rightarrow$	R' ______

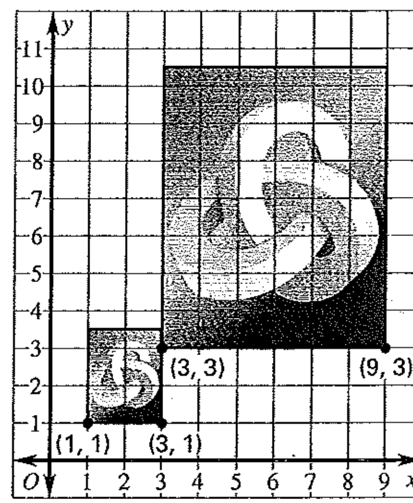


Finally, draw $\triangle P'Q'R'$.

Notice in Example 2 that when $k < 1$, the new figure is a reduction of the original figure.

Example 3 *Finding a Scale Factor*

Computer Graphics An artist uses a computer program to enlarge a design, as shown. What is the scale factor of the dilation?



Solution

The width of the original design is ______ = ______ units. The width of the image is ______ = ______ units. So, the scale factor $\frac{\text{\_\_\_ units}}{\text{\_\_\_ units}}$ is, or ______.

Checkpoint

2. Given $\overline{CD}$ with endpoints $C(6, -9)$ and $D(-3, 1)$, let $\overline{C'D'}$ with endpoints $C'(2, -3)$ and $D'\left(-1, \frac{1}{3}\right)$ be the image of $\overline{CD}$ after a dilation. Find the scale factor.

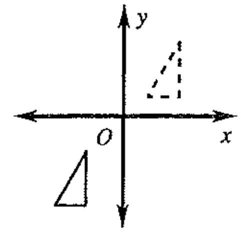
Summary

Transformations in a Coordinate Plane

Translations

In a translation, each point of a figure is moved the ______ in the ______.

$(x, y) \rightarrow$ ______

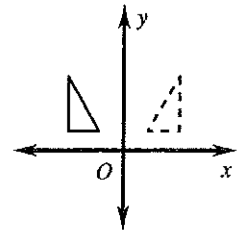


Reflections

In a reflection, a figure is ______ over a line.

Reflection in x-axis: $(x, y) \rightarrow$ ______

Reflection in y-axis (shown): $(x, y) \rightarrow$ ______



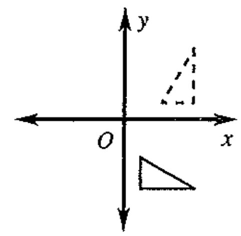
Rotations

In a rotation, a figure is turned about the origin through a given ______ and ______.

90° clockwise rotation (shown): $(x, y) \rightarrow$ ______

90° counterclockwise rotation: $(x, y) \rightarrow$ ______

180° rotation: $(x, y) \rightarrow$ ______



Dilations

In a dilation, a figure ______ or ______ with respect to the origin.

$(x, y) \rightarrow$ ______, where k is the ______.