Unit 3: Ratios, Proportions and Probability

- How are ratios and rates similar and different?
- Why are proportions defined as two equivalent ratios?
- When is it mathematically appropriate to use a proportion?
- Why is it important to calculate probability and odds?
- How could probability be used to predict future outcomes?

6.1 Ratios and Rates

Goal: Find ratios and unit rates.

Vocabulary

Ratio: ______

Equivalent ratios: ______

Writing Ratios

You can write the ratio of two quantities, *a* and *b*, where *b* is not equal to 0, in three ways.

a to *b* $\quad a:b \quad \frac{a}{b}$

Each ratio is read "the ratio of *a* to *b*." You should write the ratio in simplest form.

Example 1 *Writing Ratios*

In a recent baseball season, the Anaheim Angels played 81 home games. Anaheim won 54 of those games and lost 27. Write the ratio in three ways.

a. The number of losses to the number of wins

b. The number of losses to the number of games

Solution

a. ______ = ______ = ______

Three ways to write the ratio are ______, ______, and ______.

b. ______ = ______ = ______

Three ways to write the ratio are ______, ______, and ______.

Checkpoint

1. Use the information given in Example 1. Compare the number of wins to the number of games using a ratio. Write the ratio in three ways.

Example 2 ***Finding a Unit Rate***

Vacation On the first day of a family vacation, you and your family drive 392 miles. The amount of gasoline used is 16 gallons. What is the average mileage per gallon of gasoline?

Solution

First, write a rate comparing the ________ to the ________. Then write the rate so the denominator is ____.

____ = ____ Divide numerator and denominator by ____.

= ____ Simplify.

Answer: The average mileage per gallon of gasoline is ________.

Checkpoint Find the unit rate.

2. $\frac{220\text{ mi}}{4\text{ h}}$	3. $\frac{\$115}{5\text{ people}}$

Example 3 *Writing an Equivalent Rate*

Water The amount of water used in a certain home is 728 gallons per week. Write this rate in gallons per day.

Solution

To convert from gallons per week to gallons per day, multiply the rate by a conversion factor. There are 7 days in 1 week, so ______ = 1.

$\frac{728 \text{ gal}}{1 \text{ week}} =$ ______ • ______ Multiply rate by conversion factor.

= ______ • ______ Divide out common factor and unit.

= ______ Simplify.

Answer: The amount of water used is ______.

Example 4 *Using Equivalent Rates*

Weather Lightning strikes occur about 100 times per second around the world. About how many lightning strikes occur in 3 minutes?

Solution

1. Express the rate 100 times per second in times per minute.

   $\frac{100 \text{ times}}{1 \text{ sec}} =$ ______ • ______ Multiply by conversion factor. Divide out common unit.

   = ______ Simplify.

2. Find the number of times lightning strikes occur around the world in 3 minutes.

   Number of times = Rate • Time

   = ______ • ______ Substitute values. Divide out common unit.

   = ______ Multiply.

Answer: In 3 minutes, about ______ lightning strikes occur around the world.

6.3 Solving Proportions Using Cross Products

Goal: Solve proportions using cross products.

Vocabulary

Cross product: ______

Example 1 ***Determining if Ratios Form a Proportion***

Tell whether the ratios form a proportion.

a. $\frac{4}{26}, \frac{8}{42}$

b. $\frac{12}{21}, \frac{20}{35}$

Solution

a. $\frac{4}{26} \stackrel{?}{=} \frac{8}{42}$ Write proportion.

$\square \cdot \square \stackrel{?}{=} \square \cdot \square$ Form cross products.

$\square \; \square \; \square$ Multiply.

Answer: The ratios ______ a proportion.

b. $\frac{12}{21} \stackrel{?}{=} \frac{20}{35}$ Write proportion.

$\square \cdot \square \stackrel{?}{=} \square \cdot \square$ Form cross products.

$\square \; \square \; \square$ Multiply.

Answer: The ratios ______ a proportion.

Cross Products Property

Words The cross products of a proportion are equal.

Numbers Given that $\frac{2}{5} = \frac{6}{15}$, you know that $\square = \square$.

Algebra If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $\square = \square$.

Example 2 *Writing and Solving a Proportion*

Earnings You earn $68 mowing 4 lawns. How much would you earn if you mowed 7 lawns?

Solution

$$\frac{\square}{\square} = \frac{\square}{\square}$$ ← Money earned / ← Lawns mowed

$\square \cdot \square = \square \cdot \square$ Cross products property

$\square = \square$ Multiply.

$$\frac{\square}{\square} = \frac{\square}{\square}$$ Divide each side by $\square$.

$\square = x$ Simplify.

Answer: If you mowed 7 lawns, you would earn $\square$.

Checkpoint **Tell whether the ratios form a proportion.**

1. $\frac{9}{39} = \frac{15}{65}$	**2.** $\frac{12}{45} = \frac{6}{28}$

Use the cross products property to solve the proportion.

3. $\frac{14}{42} = \frac{x}{6}$	**4.** $\frac{4}{9} = \frac{16}{x}$

6.4 Similar and Congruent Figures

Goal: Identify similar and congruent figures.

Vocabulary

Similar figures:

Corresponding parts:

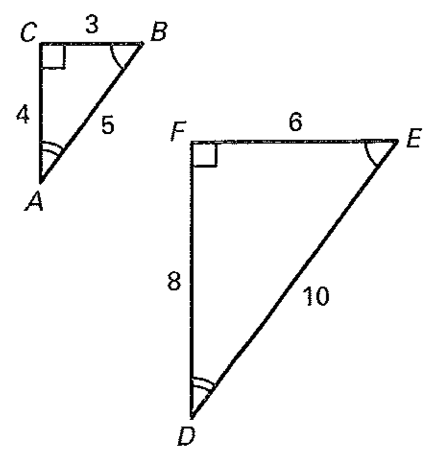
Congruent figures:

When naming similar figures, list the letters of the corresponding vertices in the same order. For the diagram at the right, it is not correct to say $\triangle CBA \sim \triangle EFD$, because $\angle C$ and $\angle E$ are not corresponding angles.

Properties of Similar Figures

$\triangle ABC \sim \triangle DEF$

The symbol $\sim$ indicates that two figures are similar.



1. Corresponding angles of similar figures are congruent.

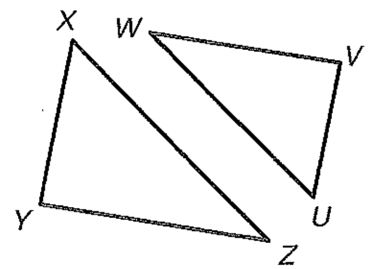
   $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

2. The ratios of the lengths of corresponding sides of similar figures are equal.

   $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

Example 1 *Identifying Corresponding Parts of Similar Figures*

Given $\triangle XYZ \sim \triangle UVW$, name the corresponding angles and the corresponding sides.



Solution

Corresponding angles:

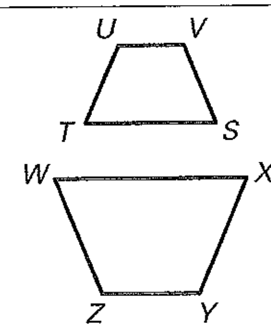
[ ]

Corresponding sides:

[ ]

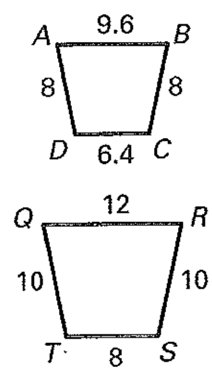
Checkpoint

1. Given $STUV \sim WXYZ$, name the corresponding angles and the corresponding sides.



Example 2 *Finding the Ratio of Corresponding Side Lengths*

Given $ABCD \sim QRST$, find the ratio of the lengths of the corresponding sides of $ABCD$ to $QRST$.



Write a ratio comparing the lengths of a pair of corresponding sides. Then substitute the lengths of the sides and simplify.

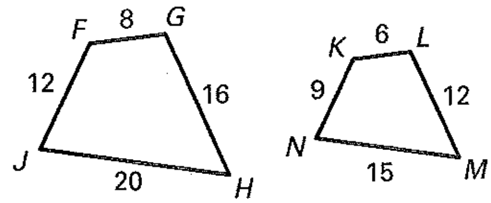
Because all the ratios of the lengths of corresponding sides of the figure in Example 2 are equal, you can use any pair of lengths of corresponding sides to write the ratio. To check the solution, choose another pair of lengths of corresponding sides.

$$\frac{AD}{QT} = \square = \square$$

Answer: The ratio of the lengths of the corresponding sides is [ ].

✓ Checkpoint

2. Given $FGHJ \sim KLMN$, find the ratio of the lengths of the corresponding sides of $FGHJ$ to $KLMN$.

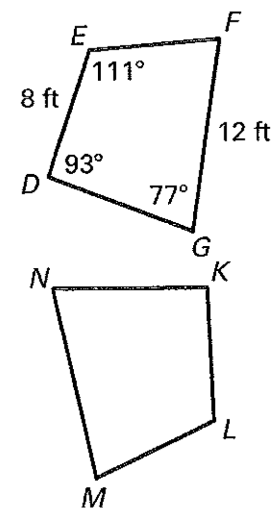


Example 3 *Finding Measures of Congruent Figures*

Given $DEFG \cong KLMN$, find the indicated measure.

a. KL

b. $\angle L$



Solution

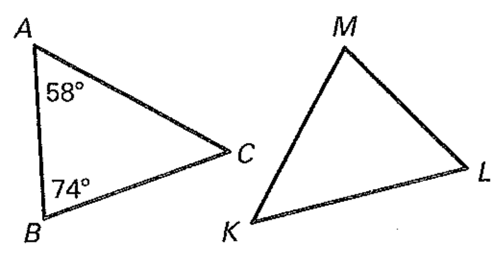
Because the quadrilaterals are congruent, the corresponding angles are congruent and the corresponding sides are congruent.

a. $\overline{KL} \cong$ ____. So, $KL =$ ____ $=$ ____

b. $\angle L \cong$ ____. So, $m\angle L =$ ____ $=$ ____

✓ Checkpoint

3. Given $\triangle ABC \cong \triangle LMK$, find $m\angle L$.

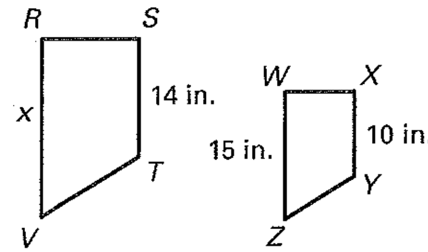


6.5 Similarity and Measurement

Goal: Find unknown side lengths of similar figures.

Example 1 *Finding an Unknown Side Length in Similar Figures*

Given $RSTV \sim WXYZ$, find VR.



Solution

Use the ratios of the lengths of corresponding sides to write a proportion involving the unknown length, VR.

$\frac{XY}{ST} = \square$ Write proportion involving VR.

$\square = \square$ Substitute.

$\square \cdot \square = \square \cdot \square$ Cross products property

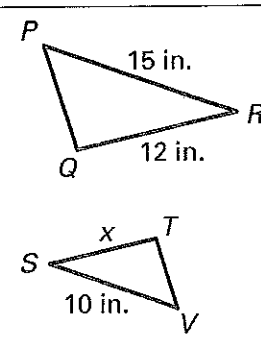
$\square = \square$ Multiply.

$x = \square$ Divide each side by $\square$.

Answer: The length of $\overline{VR}$ is $\square$ inches.

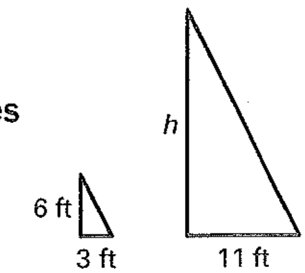
Checkpoint

1. Given $\triangle PQR \sim \triangle VTS$, find TS.



Example 2 *Using Indirect Measurement*

Height At a certain time of day, a person who is 6 feet tall casts a 3-foot shadow. At the same time, a tree casts an 11-foot shadow. The triangles formed are similar. Find the height of the tree.



Solution

Write and solve a proportion to find the height h of the tree.

$\frac{\text{Height of tree}}{\text{Height of person}} =$ ____

____ = ____ Substitute values.

____ • ____ = ____ • ____ Cross products property

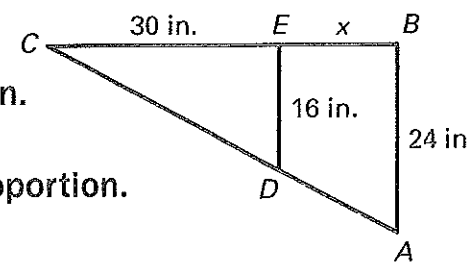
____ = ____ Multiply.

$h =$ ____ Divide each side by ____.

Answer: The tree has a height of ____ feet.

Example 3 *Using Algebra and Similar Triangles*

Given $\triangle ABC \sim \triangle DEC$, find BE.



To find BE, write and solve a proportion.

$\frac{AB}{DE} =$ ____ Write proportion.

____ = ____ Use fact that $BC =$ ____.

____ = ____ Substitute.

____ = ____ Cross products property

____ = ____ Multiply.

____ = ____ Subtract ____ from each side.

____ $= x$ Divide each side by ____.

Answer: The length of $\overline{BE}$ is ____ inches.

6.6 Scale Drawings

Goal: Use proportions with scale drawings.

Vocabulary

Scale drawing:	
Scale model:	
Scale:	

Example 1 *Using a Scale Drawing*

On a map, the distance between two cities is 3 inches. What is the actual distance (in miles) between the two cities if the map's scale is 1 in. : 125 mi?

Solution

Let x represent the actual distance (in miles) between the two cities. The ratio of the map distance between the two cities to the actual distance x is equal to the scale of the map. Write and solve a proportion using this relationship.

$\square = \square$ ← Map distance / ← Actual distance

$\square = \square$ Cross products property

$x = \square$ Multiply.

Answer: The actual distance is $\square$.

Checkpoint

1. On a map, the distance between two cities is 4 inches. What is the actual distance (in miles) between the two cities if the map's scale is 1 in. : 80 mi?

Example 2 *Finding the Scale of a Drawing*

Architecture In a scale drawing, a wall is 2 inches long. The actual wall is 12 feet long. Find the scale of the drawing.

Solution

Write a ratio using corresponding side lengths of the scale drawing and the actual wall. Then simplify the ratio so that the numerator is ☐.

$\frac{2 \text{ in.}}{12 \text{ ft}}$ ← Length of wall in scale drawing / ← Length of actual wall

$\frac{2 \text{ in.}}{12 \text{ ft}} = \square$ Simplify.

Answer: The drawing's scale is ☐.

The scale of a scale drawing or scale model can be written without units if the measurements have the same unit. For example, the scale 1 cm : 2 m can be written without units as follows.

1 cm : 2 m — Scale with units

$\frac{1 \text{ cm}}{2 \text{ m}}$

$\frac{1 \text{ cm}}{200 \text{ cm}}$

1 : 200 — Scale without units

Example 3 *Finding a Dimension of a Scale Model*

A model of the Sears Tower in Chicago has a scale of 1 : 103. The height of the Sears Tower's observation deck is about 412 meters. Find the height of the observation deck of the model.

Solution

Write a proportion using the scale.

$\square = \square$ ← Dimension of model / ← Dimension of Sears Tower

$\square = \square$ Cross products property

$\square = x$ Divide each side by ☐.

Answer: The height of the model's observation deck is ☐.

Checkpoint

2. The height of one antenna on the Sears Tower is about 521.1 meters. Find the height of the antenna on the model to the nearest tenth of a meter.

6.7 Probability and Odds

Goal: Find probabilities.

Vocabulary

Outcomes:	
Event:	
Favorable outcomes:	
Probability:	
Theoretical probability:	
Experimental probability:	
Odds in favor:	
Odds against:	

Probability of an Event

The probability of an event when all outcomes are equally likely is:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

Example 1 *Finding a Probability*

Suppose you roll a number cube. What is the probability that you roll an odd number?

Solution

Rolls of ______ are odd, so there are ___ favorable outcomes.

There are ___ possible outcomes.

P(______) = ______

= ___

= ___

Checkpoint

1. Suppose you roll a number cube. What is the probability that you roll a number less than 5?

2. Suppose you roll a number cube. What is the probability that you roll a number that is a multiple of 3?

Experimental Probability

The experimental probability of an event is:

$$P(\text{event}) = \frac{\text{Number of successes}}{\text{Number of trials}}$$

Example 2 ***Finding Experimental Probability***

You plant 32 seeds of a certain flower and 18 of them sprout. Find the experimental probability that the next flower seed planted will sprout.

Solution

$P(\text{flower seed will sprout}) = \square$ ← Number of successes / ← Number of trials

$= \square$ Simplify.

Answer: The experimental probability that a flower seed will sprout is $\square$, or $\square$.

Example 3 ***Finding the Odds***

Suppose you randomly choose a number between 1 and 16.

a. What are the odds in favor of choosing a prime number?

b. What are the odds against choosing a prime number?

Solution

a. There are $\square$ favorable outcomes ($\square$) and $16 - \square = \square$ unfavorable outcomes.

$$\text{Odds in favor} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \square = \square$$

The odds are $\square$, or $\square$ to $\square$, that you choose a prime number.

b. The odds against choose a prime number are $\square$, or $\square$ to $\square$.

6.8 The Counting Principle

Goal: Use the counting principle to find probabilities.

Vocabulary

Tree diagram:

Counting principle:

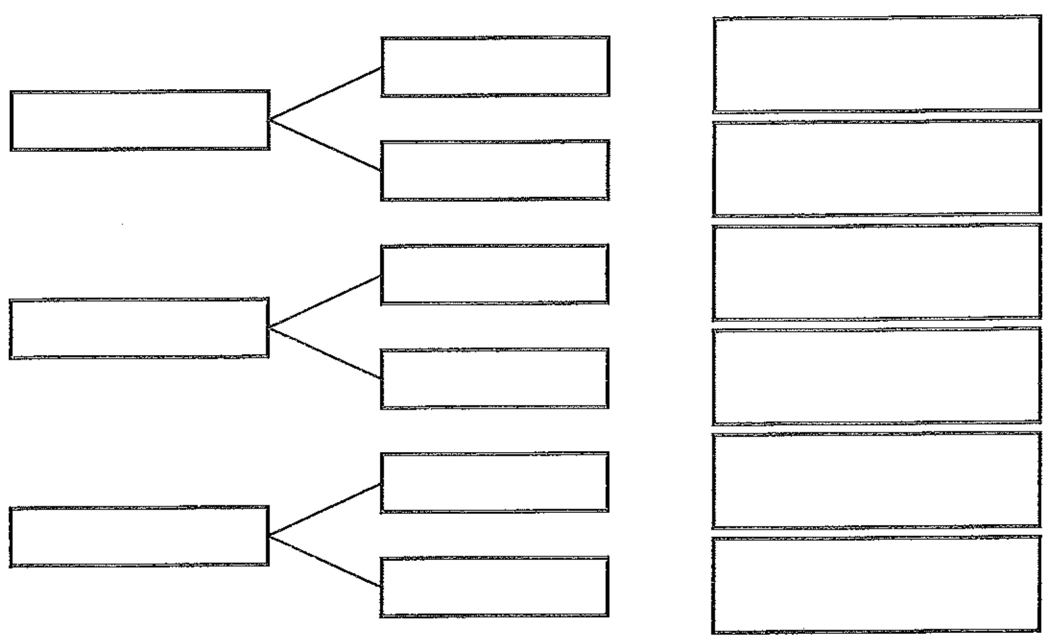
Example 1 ***Making a Tree Diagram***

At a picnic lunch, you can choose one sandwich and one salad. The choices include the following: veggie burger, hamburger, tuna sandwich, garden salad, and fruit salad. How many different picnic lunches are possible?

Solution

To count the number of possible picnic lunches, you can make a tree diagram.

List the sandwiches. **List the salads for each sandwich.** **List the possibilities for each picnic lunch.**



Answer: ☐ different picnic lunches are possible.

Checkpoint

1. Suppose for each picnic lunch in Example 1 you also get to choose one fruit. The choices include the following: apple, banana, orange, and grapefruit. Copy the tree diagram in Example 1 and add the new choices. How many possible picnic lunch choices are there?

The Counting Principle

If one event can occur in m ways, and for each of these ways a second event can occur in n ways, then the number of ways that the two events can occur together is $m \cdot n$.

The counting principle can be extended to three or more events.

Example 2 *Use the Counting Principle*

You roll a number cube and randomly draw a marble from a bag. There is one marble for each of the following colors: red, blue, green, and yellow. Use the counting principle to find the number of different outcomes that are possible.

Number of outcomes for the number cube		Number of outcomes for the marble		Total number of possible outcomes
☐	$\cdot$	☐	=	☐

Answer: There are ☐ different possible outcomes.

Checkpoint

2. You roll a number cube, randomly draw a marble from a bag, and flip a coin. There is one marble for each of the following colors: red, blue, and yellow. Use the counting principle to find the number of different outcomes that are possible.

Example 3 *Finding a Probability*

Car Security The access code for a car's security system consists of 4 digits. You randomly enter 4 digits. What is the probability that you choose the correct code?

Solution

First find the number of different codes.

[ ] = [ ] Use the counting principle.

Then find the probability that you choose the correct code.

P(correct code) = [ ]

Answer: The probability that you choose the correct code is [ ].

Checkpoint

3. Your computer password has 2 lowercase letters followed by 6 digits. Your friend randomly chooses 2 lowercase letters and 6 digits. Use a calculator to find the probability that your friend chooses your password.