Unit 2: Solving Equations

- Why is it important to do the same thing to both sides of an equation?
- Why should you check a solution?
- Why might a solution not always be the appropriate answer to a problem?
- Why is it possible to solve an equation in different ways?

2.4 Variables and Equations

Goal: Solve equations with variables.

Vocabulary

Equation: _____

Solution of an equation: _____

Solving an equation: _____

Example 1 *Writing Verbal Sentences as Equations*

Verbal Sentence	Equation
a. The sum of x and 4 is 8.	
b. The difference of 7 and y is 13.	
c. The product of -2 and p is 24.	
d. The quotient of n and 3 is 5.	

Example 2 *Checking Possible Solutions*

Tell whether 7 or 8 is a solution of $x - 3 = 5$.

a. Substitute 7 for x.

$$x - 3 = 5$$
$$\boxed{} - 3 \overset{?}{=} 5$$
$$\boxed{}\boxed{} \; 5$$

Answer: 7 $\boxed{}$ a solution.

b. Substitute 8 for x.

$$x - 3 = 5$$
$$\boxed{} - 3 \overset{?}{=} 5$$
$$\boxed{}\boxed{} \; 5$$

Answer: 8 $\boxed{}$ a solution.

Checkpoint Write the verbal sentence as an equation.

1. The sum of x and 7 is 12.
2. The quotient of n and 4 is 16.
3. Tell whether 8 or 10 is a solution of $x - 4 = 6$.

Example 3 *Solving Equations Using Mental Math*

Equation	Question	Solution	Check
a. $x + 4 = 7$		☐	☐ $+ 4 = 7$
b. $12 - n = 5$		☐	$12 - $ ☐ $= 5$
c. $18 = 3t$		☐	$18 = 3($☐$)$
d. $\dfrac{y}{4} = -5$		☐	$\dfrac{\boxed{}}{4} = -5$

Checkpoint Solve the equation using mental math.

4. $x - 8 = 10$	**5.** $24 = 4m$	**6.** $\dfrac{c}{3} = 9$

2.5 Solving Equations Using Addition or Subtraction

Goal: Solve equations using addition or subtraction.

Vocabulary

Inverse operations:

Equivalent equations:

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Numbers If $x + 3 = 5$, then $x + 3 - \boxed{} = 5 - \boxed{}$, or $x = \boxed{}$.

Algebra If $x + a = b$, then $x + a - \boxed{} = b - \boxed{}$, or $x = \boxed{}$.

Example 1 *Solving an Equation Using Subtraction*

Solve $x + 5 = -2$.

Solution

> When you solve an equation, your goal is to write an equivalent equation that has the variable by itself on one side. This process is called *solving for the variable.*

Use the subtraction property of equality to solve for x.

$x + 5 = -2$ Write original equation.

$x + 5 - \boxed{} = -2 - \boxed{}$ Subtract $\boxed{}$ from each side.

$x = \boxed{}$ Simplify.

Answer: The solution is $\boxed{}$.

Check: $x + 5 = -2$ Write original equation.

$\boxed{} + 5 \stackrel{?}{=} -2$ Substitute for x.

$\boxed{}\boxed{} - 2$ $\boxed{}$.

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Numbers If $x - 3 = 5$, then $x - 3 + \boxed{} = 5 + \boxed{}$, or $x = \boxed{}$.

Algebra If $x - a = b$, then $x - a + \boxed{} = b + \boxed{}$, or

$x = \boxed{}$.

Example 2 *Solving an Equation Using Addition*

Solve $12 = y - 7$.

Solution

Use the addition property of equality to solve for y.

$$12 = y - 7 \qquad \text{Write original equation.}$$

$$12 + \boxed{} = y - 7 + \boxed{} \qquad \text{Add } \boxed{} \text{ to each side.}$$

$$\boxed{} = y \qquad \text{Simplify.}$$

Answer: The solution is $\boxed{}$.

✓ **Checkpoint** Solve the equation. Check your solution.

1. $x + 6 = 19$	2. $-5 = y + 12$	3. $m - 3 = -11$

2.6 Solving Equations Using Multiplication or Division

Goal: Solve equations using multiplication or division.

Division Property of Equality

Words Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Numbers If $3x = 12$, then $\dfrac{3x}{\boxed{}} = \dfrac{12}{\boxed{}}$, or $x = \boxed{}$.

Algebra If $ax = b$ and $a \neq 0$, then $\dfrac{ax}{\boxed{}} = \dfrac{b}{\boxed{}}$, or $x = \boxed{}$.

> Remember that you cannot divide a number or an expression by 0.

Example 1 Solving an Equation Using Division

Solve $-7x = 42$.

Solution

$$-7x = 42 \qquad \text{Write original equation.}$$

$$\frac{-7x}{\boxed{}} = \frac{42}{\boxed{}} \qquad \text{Divide each side by } \boxed{}.$$

$$x = \boxed{} \qquad \text{Simplify.}$$

Answer: The solution is $\boxed{}$.

Check:

$$-7x = 42 \qquad \text{Write original equation.}$$

$$-7\left(\boxed{}\right) \stackrel{?}{=} 42 \qquad \text{Substitute for } x.$$

$$\boxed{}\ \boxed{}\ 42 \qquad \boxed{}.$$

✓ **Checkpoint** Solve the equation. Check your solution.

1. $5x = 45$	2. $-56 = -8y$

Multiplication Property of Equality

Words Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Numbers If $\frac{x}{3} = 12$, then $\boxed{} \cdot \frac{x}{3} = \boxed{} \cdot 12$, or $x = \boxed{}$.

Algebra If $\frac{x}{a} = b$ and $a \neq 0$, then $\boxed{} \cdot \frac{x}{a} = \boxed{} \cdot b$, or $x = \boxed{}$.

Example 2 *Solving an Equation Using Multiplication*

Solve $5 = \frac{w}{11}$.

Solution

$$5 = \frac{w}{11} \qquad \text{Write original equation.}$$

$$\boxed{} \cdot 5 = \boxed{} \cdot \frac{w}{11} \qquad \text{Multiply each side by } \boxed{}.$$

$$\boxed{} = w \qquad \text{Simplify.}$$

Answer: The solution is $\boxed{}$.

✔ *Checkpoint* **Solve the equation. Check your solution.**

3. $\frac{m}{4} = 11$	4. $-9 = \frac{c}{6}$

2.7 Decimal Operations and Equations with Decimals

Goal: Solve equations involving decimals.

Example 1 *Adding and Subtracting Decimals*

a. Find the sum $-1.7 + (-3.4)$.

Use the rule for adding numbers with the same sign. Add

$\boxed{}$ and $\boxed{}$. Both decimals are $\boxed{}$, so the

sum is $\boxed{}$.

$$-1.7 + (-3.4) = \boxed{}$$

b. Find the difference $-21.29 - (-34.62)$.

First rewrite the difference as a sum: $-21.29 + 34.62$. Then

use the rule for adding numbers with different signs. Subtract

$\boxed{}$ from $\boxed{}$. $\boxed{} > \boxed{}$, so the sum

has the same sign as $\boxed{}$.

$$-21.29 - (-34.62) = \boxed{}$$

✔ **Checkpoint** Find the sum or difference.

1. $-2.8 + (-5.9)$	2. $7.12 - (-3.46)$

Example 2 — *Multiplying and Dividing Decimals*

a. $-0.4(13.7) = \boxed{}$ Different signs: Product is $\boxed{}$.

b. $-2.5(-6.75) = \boxed{}$ Same signs: Product is $\boxed{}$.

c. $-23.49 \div (-2.9) = \boxed{}$ Same signs: Quotient is $\boxed{}$.

d. $18.05 \div (-1.9) = \boxed{}$ Different signs: Quotient is $\boxed{}$.

✓ Checkpoint Find the product or quotient.

3. $-2.8(-5.9)$	4. $7.093 \div (-3.46)$

Example 3 — *Solving Addition and Subtraction Equations*

Solve the equation.

a. $x + 6.3 = 4.8$ b. $y - 5.74 = -3.51$

Solution

a. $x + 6.3 = 4.8$ Write original equation.

$x + 6.3 - \boxed{} = 4.8 - \boxed{}$ Subtract $\boxed{}$ from each side.

$x = \boxed{}$ Simplify.

b. $y - 5.74 = -3.51$ Write original equation.

$y - 5.74 + \boxed{} = -3.51 + \boxed{}$ Add $\boxed{}$ to each side.

$y = \boxed{}$ Simplify.

✓ Checkpoint **Solve the equation. Check your solution.**

5. $x + 5.6 = 9.4$	6. $-3.5 = y + 1.2$	7. $m - 5.3 = -7.2$

Example 4 **Solving Multiplication and Division Equations**

Solve the equation.

a. $0.8m = 4.8$ b. $\dfrac{n}{5} = -2.15$

Solution

a. $0.8m = 4.8$ Write original equation.

$\dfrac{0.8m}{\boxed{}} = \dfrac{4.8}{\boxed{}}$ Divide each side by $\boxed{}$.

$m = \boxed{}$ Simplify.

b. $\dfrac{n}{5} = -2.15$ Write original equation.

$\boxed{}\left(\dfrac{n}{5}\right) = \boxed{}(-2.15)$ Multiply each side by $\boxed{}$.

$n = \boxed{}$ Simplify.

✓ Checkpoint **Solve the equation. Check your solution.**

8. $6x = -43.2$	9. $\dfrac{y}{-3.1} = -8.4$

3.1 Solving Two-Step Equations

Goal: Solve two-step equations.

Using Subtraction and Division to Solve

Solve $4x + 9 = -7$. Check your solution.

$$4x + 9 = -7$$ Write original equation.

$$4x + 9 - \boxed{} = -7 - \boxed{}$$ Subtract $\boxed{}$ from each side.

$$4x = \boxed{}$$ Simplify.

$$\frac{4x}{\boxed{}} = \frac{-16}{\boxed{}}$$ Divide each side by $\boxed{}$.

$$x = \boxed{}$$ Simplify.

Answer: The solution is $\boxed{}$.

Check: $4x + 9 = -7$ Write original equation.

$$4\left(\boxed{}\right) + 9 \stackrel{?}{=} -7$$ Substitute for x.

$$\boxed{}\ \boxed{} -7 \quad \boxed{}$$

✓ **Checkpoint** Solve the equation. Check your solution.

1. $3x + 8 = 26$	2. $-21 = 4x + 7$

Example 2 *Using Addition and Multiplication to Solve*

Solve $\frac{x}{3} - 4 = -1$. Check your solution.

$$\frac{x}{3} - 4 = -1$$ Write original equation.

$$\frac{x}{3} - 4 + \boxed{} = -1 + \boxed{}$$ Add $\boxed{}$ to each side.

$$\frac{x}{3} = \boxed{}$$ Simplify.

$$\boxed{}\left(\frac{x}{3}\right) = \boxed{}\left(\boxed{}\right)$$ Multiply each side by $\boxed{}$.

$$x = \boxed{}$$ Simplify.

Answer: The solution is $\boxed{}$.

Check: $\frac{x}{3} - 4 = -1$ Write original equation.

$$\frac{\boxed{}}{3} - 4 \stackrel{?}{=} -1$$ Substitute for x.

$$\boxed{}\ \boxed{} -1$$ $\boxed{}$.

✅ *Checkpoint*   Solve the equation. Check your solution.

3. $\frac{x}{4} - 7 = 2$	4. $8 = \frac{b}{5} - 3$

Example 3 *Solving an Equation with Negative Coefficients*

Solve $2 - 3x = 17$. Check your solution.

$2 - 3x = 17$	Write original equation.
$2 - 3x - \boxed{} = 17 - \boxed{}$	Subtract $\boxed{}$ from each side.
$-3x = \boxed{}$	Simplify.
$\dfrac{-3x}{\boxed{}} = \dfrac{15}{\boxed{}}$	Divide each side by $\boxed{}$.
$x = \boxed{}$	Simplify.

Answer: The solution is $\boxed{}$.

Check:

$2 - 3x = 17$	Write original equation.
$2 - 3\left(\boxed{}\right) \overset{?}{=} 17$	Substitute for x.
$\boxed{}\,\boxed{}\,17$	$\boxed{}$.

✔ *Checkpoint* **Solve the equation. Check your solution.**

5. $3 - 2y = 19$	**6.** $-5 = 4 - m$

3.2 Solving Equations Having Like Terms and Parentheses

Goal: Solve equations using the distributive property.

Example 1 *Writing and Solving an Equation*

Baseball Game A group of five friends are going to a baseball game. Tickets for the game cost $12 each, or $60 for the group. The group also wants to eat at the game. Hot dogs cost $2.75 each and bottled water costs $1.25 each. The group has a total budget of $76. If the group buys the same number of hot dogs and bottles of water, how many can they afford to buy?

Solution

Let n represent the number of hot dogs and the number of bottles of water. Then $2.75n$ represents the cost of n hot dogs and $1.25n$ represents the cost of n bottles of water. Write a verbal model.

$$\boxed{} + \boxed{} + \boxed{} = \boxed{}$$

$$\boxed{} + \boxed{} + \boxed{} = \boxed{} \qquad \text{Substitute.}$$

$$\boxed{} + \boxed{} = \boxed{} \qquad \text{Combine like terms.}$$

$$\boxed{} + \boxed{} - \boxed{} = \boxed{} - \boxed{} \qquad \text{Subtract } \boxed{} \text{ from each side.}$$

$$\boxed{} = \boxed{} \qquad \text{Simplify.}$$

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \qquad \text{Divide each side by } \boxed{}.$$

$$n = \boxed{} \qquad \text{Simplify.}$$

Answer: The group can afford to buy $\boxed{}$ hot dogs and $\boxed{}$ bottles of water.

Example 2   *Solving Equations Using the Distributive Property*

Solve the equation.

a. $-24 = 6(2 - x)$ **b.** $-2(7 - 4x) = 10$

Solution

a.

$-24 = 6(2 - x)$ Write original equation.

$-24 = \boxed{}$ Distributive property

$-24 - \boxed{} = \boxed{} - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$\boxed{} = x$ Simplify.

b.

$-2(7 - 4x) = 10$ Write original equation.

$\boxed{} = 10$ Distributive property

$\boxed{} + \boxed{} = 10 + \boxed{}$ Add $\boxed{}$ to each side.

$\boxed{} = \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$x = \boxed{}$ Simplify.

Example 3 *Combining Like Terms After Distributing*

Solve $6x - 4(x - 1) = 14$.

$6x - 4(x - 1) = 14$ Write original equation.

$6x\ \boxed{} = 14$ Distributive property

$\boxed{} = 14$ Combine like terms.

$\boxed{} - \boxed{} = 14 - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$x = \boxed{}$ Simplify.

✅ *Checkpoint* **Solve the equation. Check your solution.**

1. $-20 = 5(3 - x)$	2. $4y - 14 + 3y = 28$
3. $-3(6 - 2x) = 12$	4. $5x - 2(x - 3) = 30$

3.3 Solving Equations with Variables on Both Sides

Goal: Solve equations with variables on both sides.

Example 1 *Solving an Equation with the Variable on Both Sides*

Solve $5n - 7 = 9n + 21$.

$5n - 7 = 9n + 21$ Write original equation.

$5n - 7 - \boxed{} = 9n + 21 - \boxed{}$ Subtract $\boxed{}$ from each side.

$-7 = \boxed{} + 21$ Simplify.

$-7 - \boxed{} = \boxed{} + 21 - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$\boxed{} = n$ Simplify.

Answer: The solution is $\boxed{}$.

Example 2 *An Equation with No Solution*

Solve $3(2x + 1) = 6x$.

$3(2x + 1) = 6x$ Write original equation.

$\boxed{} = 6x$ Distributive property

Notice that this statement $\boxed{}$ true because the number $6x$

$\boxed{}$. The equation has $\boxed{}$

$\boxed{}$. As a check, you can continue solving the equation.

$\boxed{} = 6x \, \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = \boxed{}$ Simplify.

The statement $\boxed{}\,\boxed{}$ true, so the equation has

$\boxed{}$.

Example 3 *Solving an Equation with All Numbers as Solutions*

Solve $4(x + 2) = 4x + 8$.

$4(x + 2) = 4x + 8$ Write original equation.

$\boxed{} = 4x + 8$ Distributive property

Notice that for all values of x, the statement $\boxed{} = 4x + 8$ is

$\boxed{}$. The equation has $\boxed{}$.

✅ **Checkpoint**   **Solve the equation. Check your solution.**

1. $3n - 6 = 5n + 20$	**2.** $12x = 4(3x - 1)$
3. $3(2n + 4) = 2(3n + 6)$	**4.** $2x + 7 = -2x - 13$

Example 4 *Solving an Equation to Find a Perimeter*

Geometry Find the perimeter of the square.

$x + 6$

$3x$
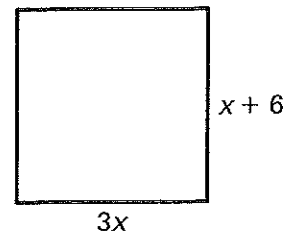
Solution

1. A square has four sides of equal length. Write an equation and solve for x.

$\boxed{} = \boxed{}$ Write equation.

$\boxed{} - \boxed{} = \boxed{} - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} = \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$x = \boxed{}$ Simplify.

2. Find the length of one side by substituting $\boxed{}$ for x in either expression.

$3x = 3\left(\boxed{}\right) = \boxed{}$ Substitute for x and multiply.

3. To find the perimeter, multiply the length of one side by $\boxed{}$.

$\boxed{} \cdot \boxed{} = \boxed{}$

Answer: The perimeter of the square is $\boxed{}$ units.

✓ *Checkpoint* **Find the perimeter of the square.**

5.

$3x + 8$

$5x$
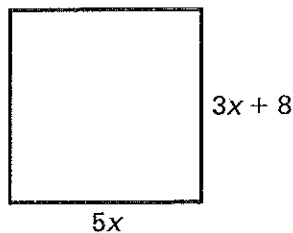
5.7 Equations and Inequalities with Rational Numbers

Goal: Use the LCD to solve equations and inequalities.

Example 1 *Solving an Equation by Clearing Fractions*

$$\frac{1}{4}x + \frac{3}{10} = \frac{2}{5}$$ Original equation

$$\boxed{}\left(\frac{1}{4}x + \frac{3}{10}\right) = \boxed{}\left(\frac{2}{5}\right)$$ Multiply each side by LCD of fractions.

$$\boxed{}\left(\boxed{}\right) + \boxed{}\left(\boxed{}\right) = \boxed{}\left(\frac{2}{5}\right)$$ Use distributive property.

$$\boxed{} + \boxed{} = \boxed{}$$ Simplify.

$$\boxed{} + \boxed{} - \boxed{} = \boxed{} - \boxed{}$$ Subtract $\boxed{}$ from each side.

$$\boxed{} = \boxed{}$$ Simplify.

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$ Divide each side by $\boxed{}$.

$$x = \boxed{}$$ Simplify.

✔ **Checkpoint** Solve the equation by first clearing the fractions.

1. $\frac{1}{3}x + \frac{5}{6} = \frac{7}{9}$	2. $\frac{3}{10} - \frac{7}{15}x = \frac{2}{3}$

Example 2   *Solving an Equation by Clearing Decimals*

Solve the equation 2.75 = 6.15 + 0.4m.

Because the greatest number of decimal places in any of the terms with decimals is ☐ , multiply each side of the equation by ☐ , or ☐ .

$$2.75 = 6.15 + 0.4m$$ Original equation

☐ (2.75) = ☐ (6.15 + 0.4m)   Multiply each side by ☐ .

☐ = ☐ + ☐   Use distributive property. Simplify.

☐ − ☐ = ☐ + ☐ − ☐   Subtract ☐ from each side.

☐ = ☐   Simplify.

$$\frac{☐}{☐} = \frac{☐}{☐}$$ Divide each side by ☐ .

☐ = m   Simplify.

Example 3   *Solving an Inequality with Fractions*

Geometry Describe the possible values of *x* if the area of the rectangle is at least 24 square inches.

6

$\frac{2}{5}x + 2$
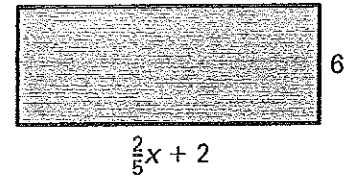
Solution

| Length | · | Width | ≥ | Area |

□ · □ ≥ □   Substitute.

□ + □ ≥ □   Use distributive property.

□ + □ − □ ≥ □ − □   Subtract □ from each side.

□ ≥ □   Simplify.

□(□) ≥ □(□)   Multiply each side by multiplicative inverse of □.

□ ≥ □   Simplify.

Answer: The possible values of *x* are □ .

Example 4   *Solving an Inequality by Clearing Fractions*

$$-\frac{1}{6}m - \frac{5}{12} \leq -\frac{5}{6}$$ Original inequality

$$\boxed{}\left(-\frac{1}{6}m - \frac{5}{12}\right) \leq \boxed{}\left(-\frac{5}{6}\right)$$ Multiply each side by LCD of fractions.

$$\boxed{}\left(\boxed{}\right) - \boxed{}\left(\boxed{}\right) \leq \boxed{}\left(-\frac{5}{6}\right)$$ Use distributive property.

$$\boxed{} - \boxed{} \leq \boxed{}$$ Simplify.

$$\boxed{} - \boxed{} + \boxed{} \leq \boxed{} + \boxed{}$$ Add $\boxed{}$ to each side.

$$\boxed{} \leq \boxed{}$$ Simplify.

$$\frac{\boxed{}}{\boxed{}} \boxed{} \frac{\boxed{}}{\boxed{}}$$ Divide each side by $\boxed{}$. $\boxed{}$ the inequality symbol.

$$m \boxed{} \boxed{}$$ Simplify.

✅ *Checkpoint*   **Solve the inequality by first clearing the fractions.**

3. $\dfrac{4}{11}x + 1 < \dfrac{2}{3}$	4. $\dfrac{3}{7}x + \dfrac{1}{4} < \dfrac{1}{2}$

3.4 Solving Inequalities Using Addition or Subtraction

Goal: Solve inequalities using addition or subtraction.

Vocabulary

Inequality:

Solution of an inequality:

Equivalent inequalities:

Example 1 *Writing and Graphing an Inequality*

Air Travel An airline allows passengers to carry on-board one piece of luggage. Luggage that exceeds 40 pounds cannot be carried on-board. Write an inequality that gives the weight of luggage that cannot be carried on-board.

Solution

Let *w* represent the weight of the luggage. Because the weight cannot exceed 40 pounds, the weight of luggage that cannot be carried on-board must be [   ].

Answer: The inequality is [   ]. Draw the graph below.

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  <--+---+---+---+---+---+---+---+---+-->
     0  10  20  30  40  50  60  70  80
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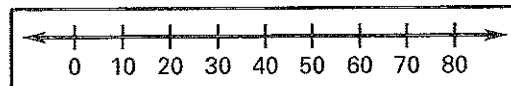
Addition and Subtraction Properties of Inequality

Words Adding or subtracting the same number on each side of an inequality produces an equivalent inequality.

Algebra If $a < b$, then $a + c < b + c$ and $a - c < b - c$.

If $a > b$, then $a + c > b + c$ and $a - c > b - c$.

> The addition and subtraction properties of inequality are also true for inequalities involving $\leq$ and $\geq$.

Example 2 *Solving an Inequality Using Subtraction*

Solve $m + 9 \leq 12$. Graph and check your solution.

$m + 9 \leq 12$ Write original inequality.

$m + 9 - \boxed{} \leq 12 - \boxed{}$ Subtract $\boxed{}$ from each side.

$m \leq \boxed{}$ Simplify.

Answer: The solution is $m \leq \boxed{}$. Draw the graph below.
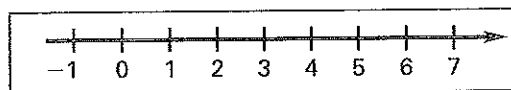
Check: Choose any number less than or equal to $\boxed{}$. Substitute the number into the original inequality.

$m + 9 \leq 12$ Write original inequality.

$\boxed{} + 9 \overset{?}{\leq} 12$ Substitute 0 for m.

$\boxed{}\boxed{} 12$ $\boxed{}$.

Example 3 *Solving an Inequality Using Addition*

Solve $-7 < x - 11$. Graph your solution.

$-7 < x - 11$ Write original inequality.

$-7 + \boxed{} < x - 11 + \boxed{}$ Add $\boxed{}$ to each side.

$\boxed{} < x$ Simplify.

Answer: The solution is $\boxed{} < x$, or $\boxed{}$. Draw the graph below.
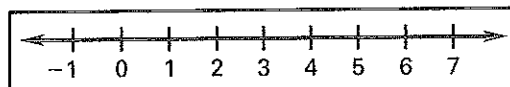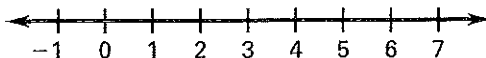
> You can read an inequality from left to right as well as from right to left. For instance, "2 is greater than x" can also be read "x is less than 2." Algebraically, this means that $2 > x$ can also be written as $x < 2$.

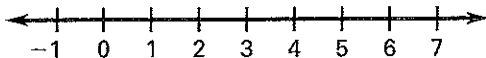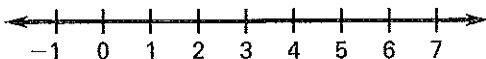**Checkpoint** Solve the inequality. Graph and check your solution.

1. $m + 7 < 13$
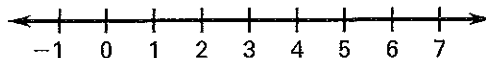
2. $a + 4 \geq 5$

3. $x - 2 \leq 3$

4. $-6 < z - 7$

3.5 Solving Inequalities Using Multiplication or Division

Goal: Solve inequalities using multiplication or division.

Multiplication Property of Inequality

Words Multiplying each side of an inequality by a *positive* number produces an equivalent inequality.

Multiplying each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

Algebra If $a < b$ and $c > 0$, then $ac \ \boxed{}\ bc$.

If $a < b$ and $c < 0$, then $ac \ \boxed{}\ bc$.

> The multiplication properties of inequality are also true for inequalities involving $>$, $\leq$, and $\geq$.

Example 1 *Solving an Inequality Using Multiplication*

Solve $\dfrac{m}{-4} > 2$.

$\dfrac{m}{-4} > 2$ Write original inequality.

$\boxed{}\left(\dfrac{m}{-4}\right)\boxed{}\boxed{} \cdot 2$ Multiply each side by $\boxed{}$. Reverse inequality symbol.

$m \boxed{}\boxed{}$ Simplify.

✓ **Checkpoint** Solve the inequality. Graph your solution.

1. $\dfrac{t}{5} < 3$	**2.** $\dfrac{b}{-8} \leq 1$

9 10 11 12 13 14 15 16

−9 −8 −7 −6 −5 −4 −3 −2
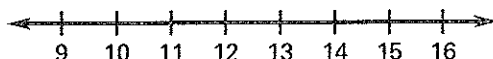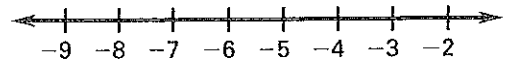
Division Property of Inequality

Words Dividing each side of an inequality by a *positive* number produces an equivalent inequality.

Dividing each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

Algebra If $a < b$ and $c > 0$, then $\dfrac{a}{c}\ \boxed{}\ \dfrac{b}{c}$.

If $a < b$ and $c < 0$, then $\dfrac{a}{c}\ \boxed{}\ \dfrac{b}{c}$.

The division properties of inequality are also true for inequalities involving $>$, $\leq$, and $\geq$.

Example 2 *Solving an Inequality Using Division*

Solve $-11t \geq 33.$

$-11t \geq 33$ Write original inequality.

$\dfrac{-11t}{\boxed{}}\ \boxed{}\ \dfrac{33}{\boxed{}}$ Divide each side by $\boxed{}$.
Reverse inequality symbol.

$t\ \boxed{}\ \boxed{}$ Simplify.

Checkpoint Solve the inequality. Graph your solution.
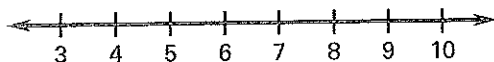
3. $4y \leq 36$	4. $-3x > 12$
      3 4 5 6 7 8 9 10	 −9 −8 −7 −6 −5 −4 −3 −2
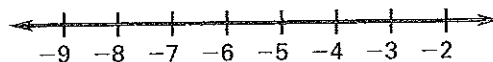
3.6 Solving Multi-Step Inequalities

Goal: Solve multi-step inequalities.

Example 1 *Writing and Solving a Multi-Step Inequality*

Charity Walk You are participating in a charity walk. You want to raise at least $500 for the charity. You already have $175 by asking people to pledge $25 each. How many more $25 pledges do you need?

Solution

Let p represent the number of additional pledges. Write a verbal model.

$$\boxed{} + \boxed{} \cdot \boxed{} \ge \boxed{}$$

$\boxed{} + \boxed{} \ge \boxed{}$ Substitute.

$\boxed{} + \boxed{} - \boxed{} \ge \boxed{} - \boxed{}$ Subtract $\boxed{}$ from each side.

$\boxed{} \ge \boxed{}$ Simplify.

$\dfrac{\boxed{}}{\boxed{}} \ge \dfrac{\boxed{}}{\boxed{}}$ Divide each side by $\boxed{}$.

$p \ge \boxed{}$ Simplify.

Answer: You need at least $\boxed{}$ more $25 pledges.

✓ *Checkpoint*

1. Look back at Example 1. Suppose you wanted to raise at least $620 and you already have raised $380 by asking people to pledge $20 each. How many more $20 pledges do you need?

Example 2 **Solving a Multi-Step Inequality**

$$\frac{x}{-3} - 9 < -7$$ Original inequality

$$\frac{x}{-3} - 9 + \boxed{} < -7 + \boxed{}$$ Add $\boxed{}$ to each side.

$$\frac{x}{-3} < \boxed{}$$ Simplify.

$$\boxed{}\left(\frac{x}{-3}\right)\boxed{}\boxed{} \cdot \boxed{}$$ Multiply each side by $\boxed{}$.
Reverse inequality symbol.

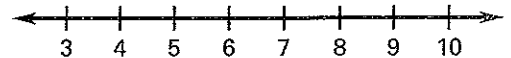$$x \boxed{}\boxed{}$$ Simplify.

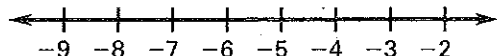✓ **Checkpoint** **Solve the inequality. Then graph the solution.**

2. $2x + 9 < 25$

3. $-3 \geq \frac{x}{-4} - 2$

4. $2 \geq -4 - x$

5. $\frac{x}{2} + 4 \leq 9$