Unit 10: Probability

- What is the difference between a permutation and combination?
- How does order affect the probability of an event?
- How does the relationship between two or more events affect the probability of their outcomes?

11.6 Permutations

Goal: Use permutations to count possibilities.

Vocabulary

Permutation: ______

n factorial: ______

0 factorial: ______

Example 1 ***Counting Permutations***

Books You have 3 new books you want to read. In how many different orders can you read the books?

Solution

You have ☐ choices for the first book, ☐ choices for the second book, and ☐ choice for the third book. So, the number of orders you can read the books is ☐.

☐ = ☐ · ☐ · ☐ = ☐

Answer: You can read the books in ☐ different orders.

Checkpoint **Evaluate the factorial.**

1. 4!	**2.** 0!	**3.** 5!	**4.** 6!

Permutations

Algebra The number of permutations of n objects taken r at a time can be written as ${}_nP_r$, where ${}_nP_r = \frac{n!}{(n-r)!}$.

Numbers ${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \overset{1}{\cancel{2}} \cdot 1}{\underset{1}{\cancel{2}} \cdot 1} = \square$

Example 2 *Counting Permutations*

Marching Bands Judges at a marching band competition are awarding prizes to the first-, second-, third-, and fourth-place finishers. The competition has 12 marching bands. How many different ways can the first-, second-, third-, and fourth-place prizes be awarded?

Solution

To find the number of ways that prizes can be awarded, find ${}_{12}P_4$.

${}_{12}P_4 = \square$ Use permutations formula.

$= \square$ Subtract.

$= \square$ Expand factorials. Divide out common factors.

$= \square \cdot \square \cdot \square \cdot \square = \square$ Multiply.

Answer: There are $\square$ ways the prizes can be awarded.

Checkpoint Find the number of permutations.

5. ${}_8P_2$	6. ${}_5P_4$	7. ${}_7P_3$	8. ${}_4P_4$

Example 3 *Finding a Probability Using Permutations*

The combination for a lock consists of the numbers 3, 4, 6, and 8. You cannot remember the order in which the four numbers are to be entered. Find the probability that you open the lock on the first try.

Solution

Each possible combination is a permutation of the digits 3, 4, 6, and 8. The number of permutations of the four digits is ☐.

☐ = ☐ • ☐ • ☐ • ☐ = ☐

Only ☐ of the possible permutations is correct, so the probability of opening the lock on the first try is ☐.

Checkpoint

9.	Timothy created a password for his e-mail account by rearranging the letters of his name. You know how he created the password, but you do not know what the password is. What is the probability that you will guess the password on the first try?
10.	A waitress takes ice cream cone orders for 5 people, but quickly forgets which person ordered which ice cream cone. If the waitress randomly chooses a person to give each ice cream cone to, what is the probability that the waitress will give the correct ice cream cone to each person?

11.7 Combinations

Goal: Use combinations to count possibilities.

Vocabulary

Combinations: ______

Example 1 *Listing Combinations*

You rent 3 movies to watch at home. You have enough time to watch 2 of the movies tonight. List and count the different possible pairs of movies you can watch tonight.

Solution

Use the letters A, B, and C to represent the 3 movies. List all possible pairs of movies. Then cross out any duplicates that represent the same pair of movies.

AB ______

______ ______

______ ______

Answer: There are ______ different pairs of movies.

Checkpoint **Find the number of combinations.**

1. For a test, you can choose any 2 essay questions to answer from the 4 questions asked. How many different pairs of essay questions could you choose?

Combinations

Words To find the number of combinations of n objects taken r at a time, divide the number of permutations of n objects taken r at a time by $r!$.

Numbers ${}_9C_5 = \frac{{}_9P_5}{5!}$ **Algebra** ${}_nC_r = \frac{{}_nP_r}{r!}$

Example 2 ***Counting Combinations***

Forming a Committee The manager of an accounting department wants to form a three-person advisory committee from the 10 employees in the department. How many different groups can the manager form?

Solution

The order in which the manager chooses people for the committee is not important. So, to find the number of different ways to choose 3 employees from 10, find ${}_{10}C_3$.

${}_{10}C_3 = \square$ Use combinations formula.

$= \square$ Write $\square$ and $\square$ as products.

$= \square$ Divide out common factors.

$= \square$ Simplify.

Answer: There are $\square$ different groups the manager can form.

Checkpoint **Find the number of combinations.**

2. ${}_5C_2$	**3.** ${}_9C_4$	**4.** ${}_6C_6$	**5.** ${}_{12}C_9$

Example 3 *Choosing Between Permutations and Combinations*

Tell whether the possibilities can be counted using *permutations* or *combinations.*

a. A survey asks people to rank comedy, drama, action, and science fiction according to how much they enjoy watching each type of movie. How many possible responses are there?

b. A literary magazine editor must choose 5 short stories for this month's issue from 30 submissions. How many different groups of 5 short stories can the editor choose?

Solution

a. Because the types of movies can be ranked first, second, third, or fourth, order is ______. So, the possibilities can be counted using ______.

b. The order in which the editor chooses the short stories ______ ______ matter. So, the possibilities can be counted using ______.

Example 4 *Finding a Probability Using Combinations*

A jury consists of 3 men and 9 women. Three jurors are selected at random for an interview. Find the probability that all 3 jurors chosen are men.

Solution

The order in which the jurors are chosen is not important. So, find ${}_{12}C_3$.

${}_{12}C_3 =$ ______ Use combinations formula.

$=$ ______ Write ______ and ______ as products.

$=$ ______ $=$ ______ Divide out common factors and simplify.

Answer: There are ______ different combinations of 3 jurors that can be chosen for the interview. Only one of the combinations can have 3 men. So, the probability is ______.

11.9 Independent and Dependent Events

Goal: Find the probability that event A *and* event B occur.

Vocabulary

Independent events:

Dependent events:

Example 1 *Identifying Independent and Dependent Events*

Tell whether the events are *independent* or *dependent*.

a. In a class of 30 students, a gym teacher randomly chooses a student for a demonstration. From the remaining students, the teacher randomly chooses another student.

b. You roll a number cube. Then you roll the number cube again.

Solution

a. Because the teacher does not include the first student for the second selection, there is one fewer student to choose from. This affects the results of the second draw. So, the events are ________.

b. The result of the first roll does not affect the result of the second roll. So, the events are ________.

Checkpoint

1. Tell whether the events are *independent* or *dependent*.

   You randomly draw a numbered ball from a bowl. Then you put it back in the bowl and randomly draw another ball from the bowl.

You can extend the formula for the probability of independent events to include more than two events. For example, the probability that independent events A, B, and C occur is the product $P(A) \cdot P(B) \cdot P(C)$.

Probability of Independent Events

Words For two independent events, the probability that both events occur is the product of the probabilities of the events.

Algebra If A and B are independent events, then

$P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$.

Example 2 *Finding the Probability of Independent Events*

States Each student in your class is to write a report on one of the first 13 states of the United States. Your teacher is allowing each state to be randomly chosen by more than one student. What is the probability that you and the student who sits next to you choose the same state?

Solution

Each state can be chosen more than once, so the choices are ________ events. The probability of each event is ____.

$P(\text{state and same state}) = P(\text{state}) \cdot P(\text{same state})$

$= \square \cdot \square$

$= \square$

Answer: The probability that you both pick the same state is ____.

Probability of Dependent Events

Words For two dependent events, the probability that both events occur is the product of the probability that the first event occurs and the probability that the second event occurs given that the first event has occurred.

Algebra If A and B are dependent events, then

P(A and B) = P(A) • P(B given A).

Example 3 ***Finding the Probability of Dependent Events***

In a box of 15 parts, 4 of the parts are defective. You randomly choose a part. Then you randomly draw a second part without replacing the first part. Find the probability that both parts are defective.

Solution

Because you don't replace the first part, the events are ______.

So, P(defective and then defective)
= P(defective) • P(defective given defective).

P(defective) = ______ There are ___ defective parts and ___ total parts.

Think: How many defective parts are remaining and how many total parts are remaining?

P(defective given defective) = ______

P(defective and then defective) = ___ • ___ Substitute probabilities.

= ___ ≈ ___ Multiply. Write as a decimal.

Answer: The probability that both parts are defective is about ___%.

11.8 Probabilities of Disjoint and Overlapping Events

Goal: Find the probability that event A *or* event B occurs.

Vocabulary

Disjoint, or mutually exclusive, events: ____________

Overlapping events: ____________

Complementary events: ____________

Example 1 *Identifying Disjoint and Overlapping Events*

Tell whether the events are *disjoint* or *overlapping*.

a. Roll a number cube.

Event A: Roll an odd number.

Event B: Roll a 3.

b. Randomly select a book.

Event A: Select a fiction book.

Event B: Select a math textbook.

Solution

a. The outcomes for event A are ____________. The outcome for event B is 3. The events have ____________ in common.

Answer: The events are ____________.

b. Because fiction books and math textbooks are not the same, there are ____________ in common.

Answer: The events are ____________.

Probability of Disjoint Events

Words For two disjoint events, the probability that either of the events occurs is the sum of the probabilities of the events.

Algebra If A and B are disjoint events, then

$P(\text{A or B}) = P(\text{A}) + P(\text{B})$.

Example 2 *Finding the Probability of Disjoint Events*

Your school's varsity basketball team has 4 seniors, 5 juniors, and 3 sophomores. You randomly select 1 player to interview for the school newspaper. What is the probability that you select a sophomore or a junior?

Solution

The events are disjoint because a player cannot be both a sophomore and a junior.

Event A: Select a sophomore.

Event B: Select a junior.

$P(\text{A or B}) = P(\text{A}) + P(\text{B})$		Probability of disjoint events
$= \square + \square$		Substitute probabilities.
$= \square = \square$		Add. Then simplify.

Answer: The probability that you select a sophomore or a junior is $\square$.

Checkpoint

1. Refer to Example 2. What is the probability that you select a senior or a sophomore?

Probability of Overlapping Events

Words For two overlapping events, the probability that either of the events occurs is the sum of the probabilities of the events minus the probability of both events.

Algebra If A and B are overlapping events, then

$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$.

Example 3 *Finding the Probability of Overlapping Events*

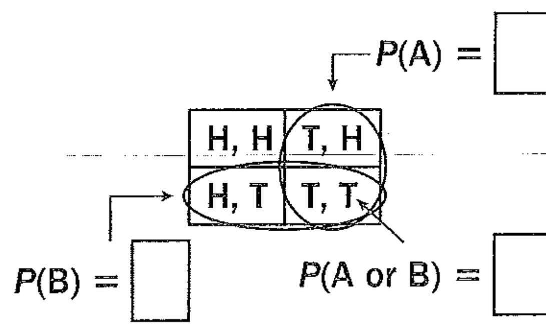
You toss a quarter and a nickel. What is the probability that at least one of the coins shows tails?

Solution

The table lists all the possible outcomes of tossing two coins.

Event A: The quarter shows tails.

Event B: The nickel shows tails.



$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$	Probability of overlapping events
$= \square + \square - \square$	Substitute probabilities.
$= \square$	Simplify.

Answer: The probability that at least one of the coins shows tails is ☐.

Example 4 *Finding the Probability of Complementary Events*

Restaurant You and 8 friends are deciding where to go out for dinner. Each person's name is put in a hat. The person whose name is drawn picks the restaurant. What is the probability that your name is *not* chosen?

Solution

The events chosen and not chosen are ☐ events because one or the other must occur.

$P(\text{not chosen}) = 1 - P(\text{chosen})$	Probability of complementary events
$= 1 - \square$	Substitute for P(chosen).
$= \square$	Subtract.

Answer: The probability that your name is not chosen is ☐.

Checkpoint Given $P(A)$, find $P(\text{not }A)$.

1. $P(A) = 24\%$	2. $P(A) = \frac{14}{37}$