Unit 1: Variables, Expressions, and Rational Numbers

- Why is there more than one way to simplify an expression?
- Why is math considered a language?
- Why are variables necessary?
- What is a reasonable answer?
- How do you complete calculations involving negative numbers?
- When is an expression in simplest form?
- Why do real-life applications sometimes require restrictions?

1.1 Expressions and Variables

Goal: Evaluate and write variable expressions.

Vocabulary

Numerical expression:

Variable:

Variable expression:

Evaluate a variable expression:

Verbal model:

When you write a variable expression involving multiplication, avoid using the symbol $\times$. It may be confused with the variable x.

Example 1 *Evaluating a Variable Expression*

Evaluate the expression $3 \cdot b$ when $b = 90$.

Solution

$3 \cdot b = 3 \cdot$ ☐ Substitute for b.

$=$ ☐ Multiply.

Example 2 *Evaluating Expressions with Two Variables*

Evaluate the expression when $x = 9$ and $y = 5$.

a. $x + y =$ ☐ $+$ ☐ Substitute for x and for y.

$=$ ☐ Add.

b. $xy =$ ☐(☐) Substitute for x and for y.

$=$ ☐ Multiply.

Checkpoint Evaluate the expression when $x = 8$ and $y = 2$.

1. $x + 12$	2. $x - y$	3. $\frac{x}{y}$

Common Words and Phrases that Indicate Operations			
Addition	**Subtraction**	**Multiplication**	**Division**
plus	minus	times	divided by
the sum of	the difference of	the product of	divided into
increased by	decreased by	multiplied by	the quotient of
total	fewer than	of	
more than	less than		
added to	subtracted from		

Example 3 *Writing a Variable Expression*

Editing You have a 350-page manuscript that needs to be edited very quickly. You are going to divide the number of pages among several editors. You want to give the same number of pages to each editor. Use a verbal model to write a variable expression for the number of pages given to each editor if you know the number of editors.

Solution

Let e represent the number of editors. The phrase *divide* indicates [ ].

[Number of pages for each editor] = [Total number of pages] ÷ [ ]

= [ ] ÷ [ ]

Answer: The number of pages for each editor is [ ] ÷ [ ], or [ ].

When you write a variable expression involving division, use a fraction bar instead of the division symbol ÷. For example, write the "quotient of n and 12" as $\frac{n}{12}$.

1.2 Powers and Exponents

Goal: Use powers to describe repeated multiplication.

Vocabulary

Power:

Base:

Exponent:

Formula:

Example 1 *Using Exponents*

Write the product using an exponent.

a. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^{\square}$ The base $\square$ is used as a factor $\square$ times.

b. $(0.4)(0.4) = (0.4)^{\square}$ The base $\square$ is used as a factor $\square$ times.

c. $a \cdot a \cdot a \cdot a = a^{\square}$ The base $\square$ is used as a factor $\square$ times.

d. $r \cdot r \cdot r \cdot r \cdot r \cdot r = r^{\square}$ The base $\square$ is used as a factor $\square$ times.

Checkpoint **Write the product using an exponent.**

1. $12 \cdot 12 \cdot 12 \cdot 12$	**2.** $(0.2)(0.2)(0.2)(0.2)(0.2)(0.2)(0.2)$
3. $x \cdot x \cdot x \cdot x \cdot x$	**4.** $y \cdot y \cdot y$

Example 2 *Evaluating Powers with Variables*

Evaluate the expression x^3 when $x = 0.4$.

$x^3 = (\quad)^3$ Substitute for x.

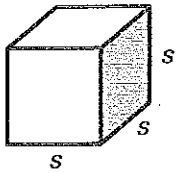
$= (\quad)(\quad)(\quad)$ Use $\square$ as a factor $\square$ times.

$= \square$ Multiply.

Area and Volume Formulas

Area A of a square

$A = s^2$



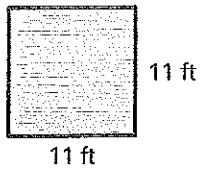
Volume V of a cube

$V = s^3$

Area is measured in square units, such as square feet (ft^2) or square centimeters (cm^2). Volume is measured in cubic units, such as cubic inches ($in.^3$) or cubic meters (m^3).

Example 3 *Using Powers in Formulas*

Room Size You are planning to put wall-to-wall carpeting in your room. To do this, you need to find the area of the square-shaped floor.



Solution

$A = s^2$ Write the formula.

$= (\quad)^2$ Substitute for s.

$= \square$ Evaluate power.

Answer: The area of the floor is $\square$ square feet.

Checkpoint **Evaluate the expression when $n = 2$.**

5. n^2	**6.** n^3	**7.** n^4	**8.** n^5

Find the volume of a cube with the given side length.

9. 2 meters	**10.** 3 feet

1.3 Order of Operations

Goal: Use order of operations to evaluate expressions.

Vocabulary

Order of operations:

Grouping symbols:

Order of Operations

1. Evaluate expressions inside grouping symbols.
2. Evaluate powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Example 1 *Using Order of Operations*

Evaluate the expression $4 \cdot 20 + 8 \cdot 5 + 4.8$.

$4 \cdot 20 + 8 \cdot 5 + 4.8$ Write expression.

$= \square + \square + 4.8$ Multiply.

$= \square$ Add.

Checkpoint **Evaluate the expression.**

1. $25 - 6 \cdot 3$

2. $56 \div 8 - 4$

Example 2 *Using Grouping Symbols*

Evaluate the expression.

a. $5(14 - 3.8) = 5(\square)$ Subtract within parentheses.

$= \square$ Multiply.

b. $\frac{27 - 3}{4 + 2} = (\square) \div (\square)$ Rewrite fraction as division.

$= \square \div \square$ Evaluate within parentheses.

$= \square$ Divide.

When grouping symbols appear inside other grouping symbols, work from the innermost grouping symbols out.

c. $4 \cdot [35 - (11 + 8)] = 4 \cdot [35 - \square]$ Add within parentheses.

$= 4 \cdot \square$ Subtract within brackets.

$= \square$ Multiply.

Example 3 *Evaluating Variable Expressions*

Evaluate the expression when $x = 3$ and $y = 6$.

a. $3(x + y) = 3(\square + \square)$ Substitute for x and for y.

$= 3(\square)$ Add within parentheses.

$= \square$ Multiply.

b. $5(y - x)^2 = 5(\square - \square)^2$ Substitute for x and for y.

$= 5(\square)^2$ Subtract within parentheses.

$= 5(\square)$ Evaluate power.

$= \square$ Multiply.

Checkpoint **Evaluate the expression when $x = 4$ and $y = 5$.**

3. $y(19 - x^2)$	**4.** $\frac{6y}{x + 1}$

1.4 Comparing and Ordering Integers

Goal: Compare and order integers.

Vocabulary

Integers:

Negative integers:

Positive integers:

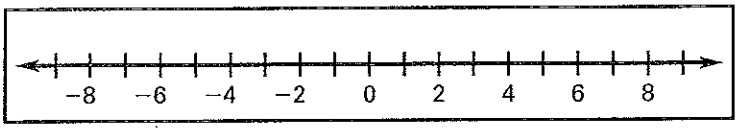
Absolute Value:

Opposites:

The expression $-a$ is always read as "the opposite of a" and *not* as "negative a." If a is a positive number, then $-a$ is a negative number. If a is a negative number, then $-a$ is a positive number.

Example 1 *Graphing and Ordering Integers*

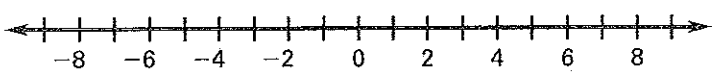
Use a number line to order these integers from least to greatest: 0, −6, −2, −8, 7, −9, 3.



Read the numbers from left to right: [ ], [ ], [ ], [ ], [ ], [ ], [ ].

✔ ***Checkpoint*** **Graph the integers on a number line. Then write the integers in order from least to greatest.**

1. 2, −7, 6, 4, 0, −4, −1



Example 2 *Finding Absolute Value*

State the absolute value of the number.

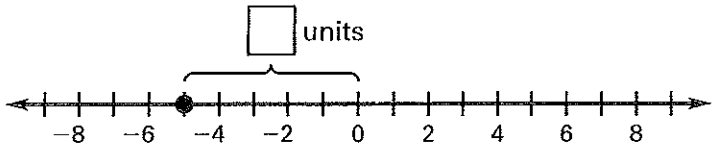
a. 7 **b.** −5

Solution

a.

The distance between 7 and ☐ is ☐. So, $|7| =$ ☐.

b.



The distance between −5 and ☐ is ☐. So, $|-5| =$ ☐.

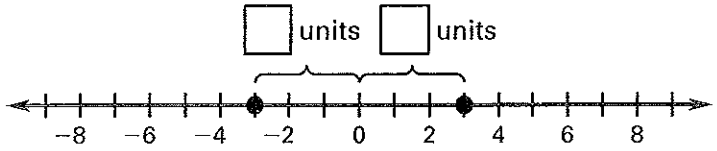
Example 3 *Finding Opposites*

State the opposite of the number.

a. 3 **b.** −8

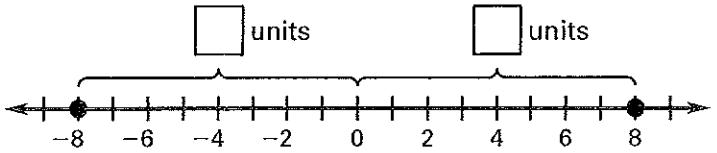
Solution

a.



The opposite of 3 is ☐.

b.



The opposite of −8 is ☐.

✓ ***Checkpoint*** **State the absolute value and the opposite of the number.**

2. 9	3. −12

Adding Integers

Words	Numbers
1. Same Sign Add the absolute values and use the [ ].	$8 + 12 =$ [ ] $-6 + (-4) =$ [ ]
2. Different Signs Subtract the [ ] absolute value from the [ ] absolute value and use the sign of the number with the [ ] absolute value.	$5 + (-8) =$ [ ] $-11 + 13 =$ [ ]
3. Opposites The sum of a number and its opposite is [ ].	$7 + (-7) =$ [ ]

Example 2 *Adding Two Integers*

a. $-35 + (-18) =$ [ ]

Same sign: Add $|$[ ]$|$ and $|$[ ]$|$.

Both integers are [ ], so the sum is [ ].

b. $27 + (-13) =$ [ ]

Different signs: Subtract $|$[ ]$|$ from $|$[ ]$|$.

Because $|$[ ]$| > |$[ ]$|$, the sum has the same sign as [ ].

Example 3 *Adding More Than Two Integers*

Find the sum $-7 + (-41) + 32$.

$-7 + (-41) + 32 =$ [ ] $+ 32$ Add -7 and -41.

$=$ [ ] Add [ ] and 32.

Checkpoint **Find the sum.**

3. $-19 + 36$	**4.** $-29 + (-31) + 47$

1.5 Adding Integers

Goal: Add integers.

Vocabulary

Additive inverse:

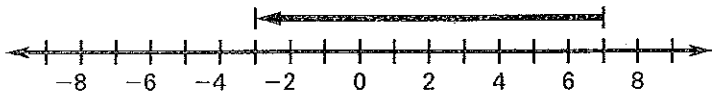
Additive inverse property:

Example 1 *Adding Integers Using a Number Line*

Use a number line to find the sum.

a. $7 + (-10)$

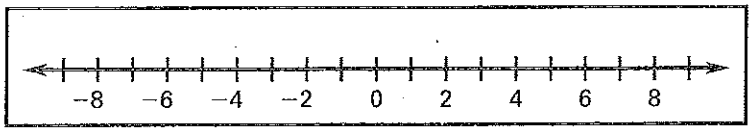
Start at [ ]. Then move [ ] units to the [ ].



Answer: The final position is [ ]. So, $7 + (-10) =$ [ ].

b. $-6 + 5$

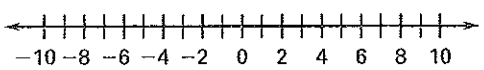
Start at [ ]. Then move [ ] units to the [ ].



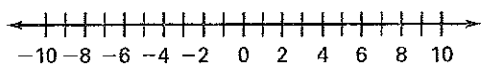
Answer: The final position is [ ]. So, $-6 + 5 =$ [ ].

Checkpoint **Use a number line to find the sum.**

1. $9 + (-6)$



2. $-2 + (-6)$



Adding Integers

Words	Numbers
1. Same Sign Add the absolute values and use the ☐.	$8 + 12 =$ ☐ $-6 + (-4) =$ ☐
2. Different Signs Subtract the ☐ absolute value from the ☐ absolute value and use the sign of the number with the ☐ absolute value.	$5 + (-8) =$ ☐ $-11 + 13 =$ ☐
3. Opposites The sum of a number and its opposite is ☐.	$7 + (-7) =$ ☐

Example 2 *Adding Two Integers*

a. $-35 + (-18) =$ ☐

Same sign: Add $|\square|$ and $|\square|$.

Both integers are ☐, so the sum is ☐.

b. $27 + (-13) =$ ☐

Different signs: Subtract $|\square|$ from $|\square|$.

Because $|\square| > |\square|$, the sum has the same sign as ☐.

Example 3 *Adding More Than Two Integers*

Find the sum $-7 + (-41) + 32$.

$-7 + (-41) + 32 =$ ☐ $+ 32$ Add -7 and -41.

$=$ ☐ Add ☐ and 32.

Checkpoint **Find the sum.**

3. $-19 + 36$	**4.** $-29 + (-31) + 47$

1.6 Subtracting Integers

Goal: Subtract integers.

Subtracting Integers

Words To subtract an integer, add its [ ].

Numbers $3 - 7 = 3 + ([\ ]) = [\ ]$

Algebra $a - b = a + ([\ ])$

Example 1 *Subtracting Integers*

a. $5 - 9 = 5 + ([\ ])$ To subtract 9, add its opposite, [ ].

$= [\ ]$ Add 5 and [ ].

b. $3 - (-8) = 3 + [\ ]$ To subtract -8, add its opposite, [ ].

$= [\ ]$ Add 3 and [ ].

c. $-4 - (-10) = -4 + [\ ]$ To subtract -10, add its opposite, [ ].

$= [\ ]$ Add -4 and [ ].

Checkpoint **Find the difference.**

1. $3 - 8$	2. $-2 - 9$	3. $6 - (-3)$

Example 2 *Evaluating Variable Expressions*

Evaluate the expression when $x = -8$.

a. $x - (-22) = \square - (-22)$ Substitute for x.

$= \square + \square$ To subtract -22, add $\square$.

$= \square$ Add $\square$ and $\square$.

b. $9 - x = 9 - (\square)$ Substitute for x.

$= 9 + \square$ To subtract $\square$, add $\square$.

$= \square$ Add 9 and $\square$.

Checkpoint **Evaluate the expression when $y = -12$.**

4. $y - 6$	5. $19 - y$	6. $-7 - y$

Example 3 *Evaluating Change*

Write a verbal model to find the change in temperature given the start temperature and the end temperature. Use the model to find the change in temperature from $-5°F$ to $12°F$.

Solution

You can use subtraction to find the change in a variable quantity such as elevation or temperature. Subtract the original value of the quantity from the value after the change.

Change in temperature	=	End temperature	−	Start temperature

$= \square - (\square)$ Substitute values.

$= \square + \square$ To subtract $\square$, add $\square$.

$= \square$ Add $\square$ and $\square$.

Answer: The temperature $\square$ by $\square$°F.

Checkpoint **Find the change in temperature.**

7. From $-3°F$ to $8°F$	8. From $-15°C$ to $-2°C$

1.7 Multiplying and Dividing Integers

Goal: Multiply and divide integers.

Multiplying Integers

Words	Numbers
The product of two integers with [ ] sign is [ ].	$2(4) =$ [ ] $-2(-4) =$ [ ]
The product of two integers with [ ] signs is [ ].	$2(-4) =$ [ ] $-2(4) =$ [ ]
The product of any integer and 0 is [ ].	$2(0) =$ [ ] $-2(0) =$ [ ]

Example 1 *Multiplying Integers*

a. $-5(-8) =$ [ ] Same sign: Product is [ ].

b. $-8(7) =$ [ ] Different signs: Product is [ ].

c. $-51(0) =$ [ ] The product of any integer and 0 is [ ].

Checkpoint **Find the product.**

1. $7(-12)$	**2.** $-9(-5)$
3. $-250(0)$	**4.** $-4(11)$

Dividing Integers

Words	Numbers
The quotient of two integers with ☐ sign is ☐.	$8 \div 4 =$ ☐ $-8 \div (-4) =$ ☐
The quotient of two integers with ☐ signs is ☐.	$-8 \div 4 =$ ☐ $8 \div (-4) =$ ☐
The quotient of 0 and any nonzero integer is ☐.	$0 \div 4 =$ ☐ $0 \div (-4) =$ ☐

Example 2 ***Dividing Integers***

a. $-63 \div (-9) =$ ☐ Same sign: Quotient is ☐.

b. $24 \div (-4) =$ ☐ Different signs: Quotient is ☐.

c. $0 \div (-2) =$ ☐ The quotient of 0 and any nonzero integer is ☐.

✔ ***Checkpoint*** **Find the quotient.**

5. $0 \div (-43)$	**6.** $32 \div (-4)$
7. $-28 \div 7$	**8.** $-38 \div (-19)$

2.1 Properties and Operations

Goal: Use properties of addition and multiplication.

Commutative and Associative Properties	
Commutative Property of Addition **Words** In a sum, you can add the numbers in any order. **Numbers** $4 + (-7) = -7 + 4$ **Algebra** $a + b = b + a$	**Commutative Property of Multiplication** **Words** In a product, you can multiply the numbers in any order. **Numbers** $8(-5) = -5(8)$ **Algebra** $ab = ba$
Associative Property of Addition **Words** Changing the grouping of the numbers in a sum does not change the sum. **Numbers** $(9 + 6) + 2 = 9 + (6 + 2)$ **Algebra** $(a + b) + c = a + (b + c)$	**Associative Property of Multiplication** **Words** Changing the grouping of the numbers in a product does not change the product. **Numbers** $(3 \cdot 10) \cdot 4 = 3 \cdot (10 \cdot 4)$ **Algebra** $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Example 1 *Using Properties of Addition*

Distance This week, you rode in a car for 42 miles, rode a bike for 5 miles, and rode in a bus for 23 miles. Find the total distance.

Solution

The total distance is the sum of the three distances.

Use properties of addition to group together distances that are easy to add mentally.

$42 + 5 + 23 = (42 + 5) + 23$ Use order of operations.

$= (\square + \square) + 23$ Commutative property of addition

$= \square + (\square + 23)$ Associative property of addition

$= \square + \square$ Add $\square$ and $\square$.

$= \square$ Add $\square$ and $\square$.

Answer: The total distance is $\square$ miles.

Example 2 *Using Properties of Multiplication*

Evaluate $4xy$ when $x = -8$ and $y = 15$.

$4xy = 4(\quad)(\quad)$ Substitute for x and for y.

$= [4(\quad)](\quad)$ Use order of operations.

$= [\quad(\quad)](\quad)$ Commutative property of multiplication

$= \quad[(\quad)(\quad)]$ Associative property of multiplication

$= \quad(\quad)$ Multiply $\square$ and $\square$.

$= \quad$ Multiply $\square$ and $\square$.

Checkpoint **Evaluate the expression when $x = 7$ and $y = 25$.**

1. $(2x + y) + 46$	**2.** $4x^2y$

Example 3 *Using Properties to Simplify Variable Expressions*

Simplify the expression.

a. $x + 5 + 2 = (x + 5) + 2$ Use order of operations.

$= x + (5 + 2)$ $\square$ property of addition

$= x + \square$ Add 5 and 2.

b. $3(9y) = (3 \cdot 9)y$ $\square$ property of multiplication

$= \square$ Multiply 3 and 9.

Checkpoint **Simplify the expression.**

3. $n + 6 + 7$	**4.** $(4r)(-3)$

Identity Properties	
Identity Property of Addition **Words** The sum of a number and the **additive identity**, 0, is the number. **Numbers** $-6 + 0 = -6$ **Algebra** $a + 0 = a$	**Identity Property of Multiplication** **Words** The product of a number and the **multiplicative identity**, 1, is the number. **Numbers** $4 \cdot 1 = 4$ **Algebra** $a \cdot 1 = a$

Example 4 *Identifying Properties*

Statement	Property Illustrated
a. $(3 + 2) + 4 = 3 + (2 + 4)$	
b. $0 + b = b$	
c. $1(-7) = -7$	
d. $cd = dc$	

Checkpoint **Identify the property that the statement illustrates.**

5. $(2 \cdot 6) \cdot 3 = 2 \cdot (6 \cdot 3)$	**6.** $q + (-r) = -r + q$

2.2 The Distributive Property

Goal: Use the distributive property.

Vocabulary

Equivalent numerical expressions:

Equivalent variable expressions:

The Distributive Property

Algebra		**Numbers**	
$a(b + c) = ab + ac$		$4(6 + 3) =$	
$(b + c)a = ba + ca$		$(6 + 3)4 =$	
$a(b - c) = ab - ac$		$5(7 - 2) =$	
$(b - c)a = ba - ca$		$(7 - 2)5 =$	

Example 1 *Using the Distributive Property*

Crafts You are buying beads for a craft project. You need gold, silver, and white beads. A bag of each type of bead costs $3.99. Use the distributive property and mental math to find the total cost of the beads.

Solution

Total cost = 3(3.99)	Write expression for total cost.
$= 3(\square - \square)$	Rewrite 3.99 as $\square - \square$.
$= 3(\square) - 3(\square)$	Distributive property
$= \square - \square$	Multiply using mental math.
$= \square$	Subtract using mental math.

Answer: The total cost of the beads is $\$\square$.

Checkpoint **Use the distributive property to evaluate the expression.**

1. $2(9 + 4)$	2. $(12 - 3)3$	3. $(4 - 11)(-4)$

Evaluate the expression using the distributive property and mental math.

4. $5(103)$	5. $4(3.8)$	6. $3(6.03)$

Example 2 ***Writing Equivalent Variable Expressions***

Use the distributive property to write an equivalent variable expression.

a. $2(x + 10) =$ ☐ Distributive property

$=$ ☐ Multiply.

b. $(m + 3)(-4) =$ ☐ Distributive property

$=$ ☐ Multiply.

$=$ ☐ Definition of subtraction

c. $-3(2y - 6) =$ ☐ Distributive property

$=$ ☐ Multiply.

$=$ ☐ Definition of subtraction

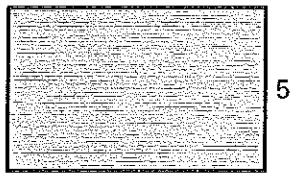
Checkpoint **Use the distributive property to write an equivalent variable expression.**

7. $(x + 7)4$	**8.** $-3(4m - 7)$

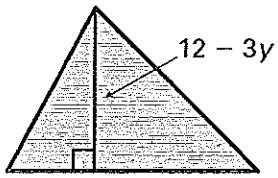
Example 3 ***Finding Areas of Geometric Figures***

Find the area of the rectangle or triangle.

a.



b.



Solution

a. Use the formula for the area of a rectangle.

$A = \ell w$

$= (\square)(\square)$

$= \square(\square) + \square(\square)$

$= \square$

Answer: The area is $\square$ square units.

b. Use the formula for the area of a triangle.

$A = \frac{1}{2}bh = \frac{1}{2}(\square)(\square)$

$= \square(\square)$

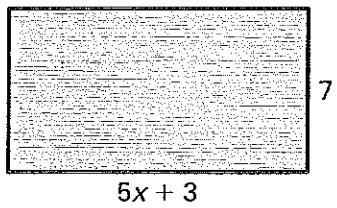
$= \square(\square) - \square(\square)$

$= \square$

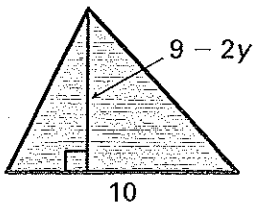
Answer: The area is $\square$ square units.

Checkpoint **Find the area of the rectangle or triangle.**

9.



10.



2.3 Simplifying Variable Expressions

Goal: Simplify variable expressions.

Vocabulary

Terms of an expression:

Coefficient of a term:

Constant term:

Like terms:

Example 1 *Identifying Parts of an Expression*

Identify the terms, like terms, coefficients, and constant terms of the expression $5 - 2x - 3 + x$.

Solution

1. Write the expression as a sum: _____.
2. Identify the parts of the expression. Note that because $x =$ ___ x, the coefficient of x is ___.

Terms: _____ **Like terms:** _____

Coefficients: _____ **Constant terms:** _____

✓ ***Checkpoint*** **Identify the terms, like terms, coefficients, and constant terms of the expression.**

1. $4y - 6 + 3y$	2. $9 + w - 5 - 8w$

Example 2 ***Simplifying an Expression***

$5m + 8 - 3m - 7 = 5m + 8 + (\square) + (\square)$ Write as a sum.

$= 5m + (\square) + \square + (\square)$ Commutative property

$= [\square + (\square)]m + \square + (\square)$ Distributive property

$= \square$ Simplify.

Example 3 ***Simplifying Expressions with Parentheses***

a. $3(x + 2) - x + 9 = \square - x + 9$ Distributive property

$= \square$ Group like terms.

$= \square$ Combine like terms.

b. $2k - 5(k + 4) = 2k - \square$ Distributive property

$= \square$ Combine like terms.

c. $5a - (5a - 7) = 5a - \square(5a - 7)$ Identity property

$= 5a - \square$ Distributive property

$= \square$ Combine like terms.

$= \square$ Simplify.

Checkpoint **Simplify the expression.**

3. $4y - 6 + 3y$	**4.** $9 + w - 5 - 8w$
5. $4(x - 1) - 2x - 7$	**6.** $-6(k + 3) + 5k$