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4.1 Radian and Degree Measure

What you should learn

- Describe angles.
- Use radian measure.
- Use degree measure.
- Use angles to model and solve real-life problems.

Why you should learn it

You can use angles to model and solve real-life problems. For instance, in Exercise 108 on page 293, you are asked to use angles to find the speed of a bicycle.

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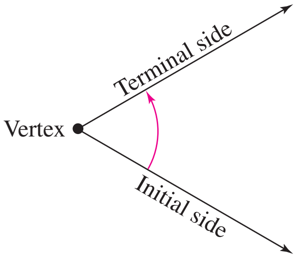
The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain additional resources related to the concepts discussed in this chapter.

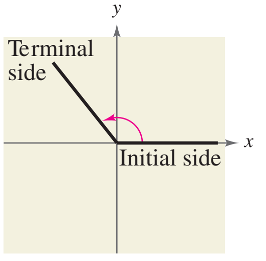
Angles

As derived from the Greek language, the word **trigonometry** means “measurement of triangles.” Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena involving rotations and vibrations. These phenomena include sound waves, light rays, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles.

The approach in this text incorporates *both* perspectives, starting with angles and their measure.



Angle
FIGURE 4.1



Angle in Standard Position
FIGURE 4.2

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.3. Angles are labeled with Greek letters α (alpha), β (beta), and θ (theta), as well as uppercase letters A, B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

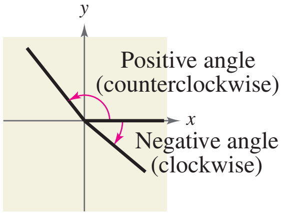


FIGURE 4.3

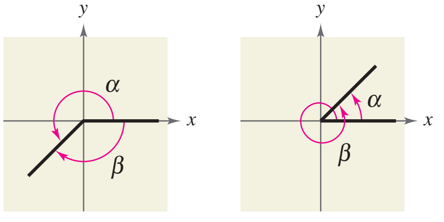


FIGURE 4.4 *Coterminal Angles*

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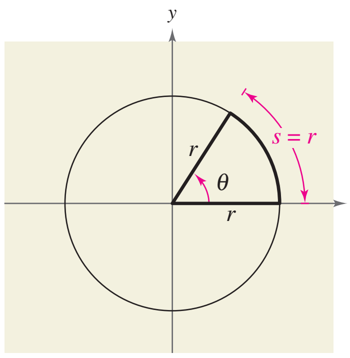

Arc length = radius when θ = 1 radian
FIGURE 4.5

Radian Measure

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*. This type of measure is especially useful in calculus. To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle, as shown in Figure 4.5.

Definition of Radian

One **radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians.

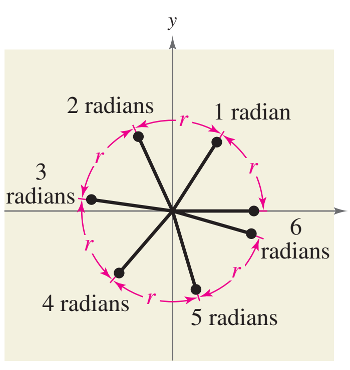

FIGURE 4.6

Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of

$$s = 2\pi r.$$

Moreover, because $2\pi \approx 6.28$, there are just over six radius lengths in a full circle, as shown in Figure 4.6. Because the units of measure for s and r are the same, the ratio s/r has no units—it is simply a real number.

Because the radian measure of an angle of one full revolution is 2π, you can obtain the following.

$$\frac{1}{2} \text{ revolution} = \frac{2\pi}{2} = \pi \text{ radians}$$

$$\frac{1}{4} \text{ revolution} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ radians}$$

$$\frac{1}{6} \text{ revolution} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}$$

These and other common angles are shown in Figure 4.7.

STUDY TIP

One revolution around a circle of radius r corresponds to an angle of 2π radians because

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians.}$$

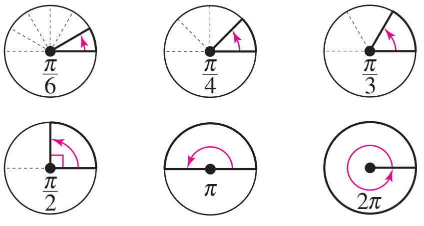

FIGURE 4.7

Recall that the four quadrants in a coordinate system are numbered I, II, III, and IV. Figure 4.8 on page 284 shows which angles between 0 and 2π lie in each of the four quadrants. Note that angles between 0 and $\pi/2$ are **acute** angles and angles between $\pi/2$ and π are **obtuse** angles.

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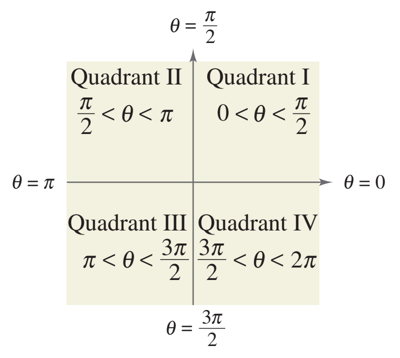

FIGURE 4.8

STUDY TIP

The phrase "the terminal side of θ lies in a quadrant" is often abbreviated by simply saying that "θ lies in a quadrant." The terminal sides of the "quadrant angles" 0, $\pi/2$, π, and $3\pi/2$ do not lie within quadrants.

Two angles are coterminal if they have the same initial and terminal sides. For instance, the angles 0 and 2π are coterminal, as are the angles $\pi/6$ and $13\pi/6$. You can find an angle that is coterminal to a given angle θ by adding or subtracting 2π (one revolution), as demonstrated in Example 1. A given angle θ has infinitely many coterminal angles. For instance, $\theta = \pi/6$ is coterminal with

$$\frac{\pi}{6} + 2n\pi$$

where n is an integer.

Example 1 Sketching and Finding Coterminal Angles

Remind your students to work in radians.

a. For the positive angle $13\pi/6$, subtract 2π to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$ See Figure 4.9.

b. For the positive angle $3\pi/4$, subtract 2π to obtain a coterminal angle

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}.$$ See Figure 4.10.

c. For the negative angle $-2\pi/3$, add 2π to obtain a coterminal angle

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$ See Figure 4.11.

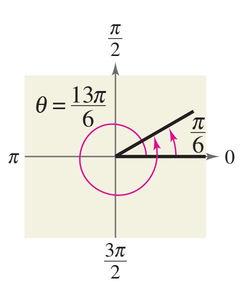

FIGURE 4.9

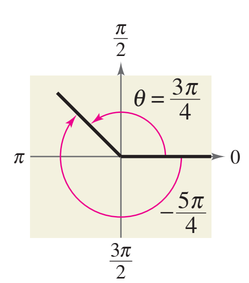

FIGURE 4.10

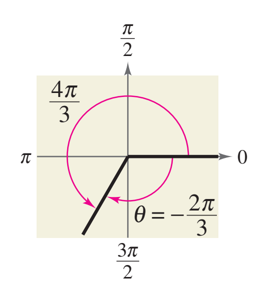

FIGURE 4.11

CHECKPOINT Now try Exercise 17.

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You might point out that complementary and supplementary angles do not necessarily share a common side. For example, the acute angles of a right triangle are complementary because the sum of their measures is $\pi/2$.

Two positive angles α and β are **complementary** (complements of each other) if their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) if their sum is π. See Figure 4.12.

Complementary Angles *Supplementary Angles*

FIGURE 4.12

Example 2 Complementary and Supplementary Angles

If possible, find the complement and the supplement of (a) $2\pi/5$ and (b) $4\pi/5$.

Solution

a. The complement of $2\pi/5$ is

$$\frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi}{10} - \frac{4\pi}{10} = \frac{\pi}{10}.$$

The supplement of $2\pi/5$ is

$$\pi - \frac{2\pi}{5} = \frac{5\pi}{5} - \frac{2\pi}{5} = \frac{3\pi}{5}.$$

b. Because $4\pi/5$ is greater than $\pi/2$, it has no complement. (Remember that complements are *positive* angles.) The supplement is

$$\pi - \frac{4\pi}{5} = \frac{5\pi}{5} - \frac{4\pi}{5} = \frac{\pi}{5}.$$

CHECKPOINT Now try Exercise 21.

Degree Measure

A second way to measure angles is in terms of **degrees,** denoted by the symbol °. A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex. To measure angles, it is convenient to mark degrees on the circumference of a circle, as shown in Figure 4.13. So, a full revolution (counterclockwise) corresponds to 360°, a half revolution to 180°, a quarter revolution to 90°, and so on.

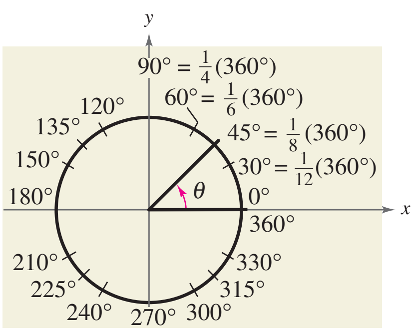

FIGURE 4.13

Because 2π radians corresponds to one complete revolution, degrees and radians are related by the equations

$$360° = 2\pi \text{ rad} \qquad \text{and} \qquad 180° = \pi \text{ rad}.$$

From the latter equation, you obtain

$$1° = \frac{\pi}{180} \text{ rad} \qquad \text{and} \qquad 1 \text{ rad} = \left(\frac{180°}{\pi}\right)$$

which lead to the conversion rules at the top of the next page.

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Converting from degrees to radians and vice versa should help your students become familiar with radian measure.

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship $\pi \text{ rad} = 180°$. (See Figure 4.14.)

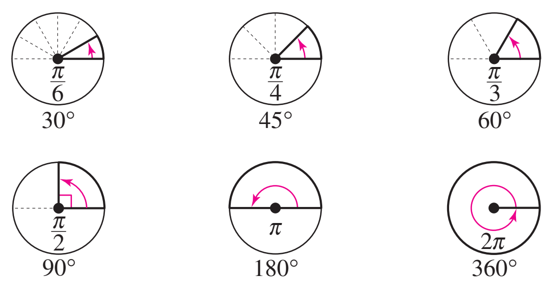

FIGURE 4.14

When no units of angle measure are specified, *radian measure is implied*. For instance, if you write $\theta = 2$, you imply that $\theta = 2$ radians.

Example 3 Converting from Degrees to Radians

a. $135° = (135 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = \dfrac{3\pi}{4} \text{ radians}$ — Multiply by $\pi/180$.

b. $540° = (540 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = 3\pi \text{ radians}$ — Multiply by $\pi/180$.

c. $-270° = (-270 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = -\dfrac{3\pi}{2} \text{ radians}$ — Multiply by $\pi/180$.

CHECKPOINT Now try Exercise 47.

Example 4 Converting from Radians to Degrees

a. $-\dfrac{\pi}{2} \text{ rad} = \left(-\dfrac{\pi}{2} \text{ rad}\right)\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = -90°$ — Multiply by $180/\pi$.

b. $\dfrac{9\pi}{2} \text{ rad} = \left(\dfrac{9\pi}{2} \text{ rad}\right)\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = 810°$ — Multiply by $180/\pi$.

c. $2 \text{ rad} = (2 \text{ rad})\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = \dfrac{360°}{\pi} \approx 114.59°$ — Multiply by $180/\pi$.

CHECKPOINT Now try Exercise 51.

If you have a calculator with a "radian-to-degree" conversion key, try using it to verify the result shown in part (c) of Example 4.

Technology

With calculators it is convenient to use *decimal* degrees to denote fractional parts of degrees. Historically, however, fractional parts of degrees were expressed in *minutes* and *seconds*, using the prime (′) and double prime (″) notations, respectively. That is,

$1' = \text{one minute} = \frac{1}{60}(1°)$

$1'' = \text{one second} = \frac{1}{3600}(1°)$

Consequently, an angle of 64 degrees, 32 minutes, and 47 seconds is represented by $\theta = 64° \, 32' \, 47''$. Many calculators have special keys for converting an angle in degrees, minutes, and seconds (D° M′ S″) to decimal degree form, and vice versa.

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Applications

The *radian measure* formula, $\theta = s/r$, can be used to measure arc length along a circle.

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$s = r\theta$ Length of circular arc

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

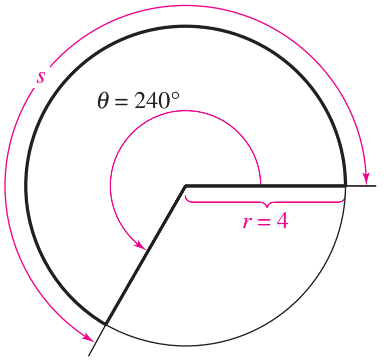

FIGURE 4.15

Example 5 Finding Arc Length

A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of 240°, as shown in Figure 4.15.

Solution

To use the formula $s = r\theta$, first convert 240° to radian measure.

$$240° = (240 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) = \frac{4\pi}{3} \text{ radians}$$

Then, using a radius of $r = 4$ inches, you can find the arc length to be

$$s = r\theta = 4\left(\frac{4\pi}{3}\right) = \frac{16\pi}{3} \approx 16.76 \text{ inches.}$$

Note that the units for $r\theta$ are determined by the units for r because θ is given in radian measure, which has no units.

CHECKPOINT Now try Exercise 87.

Because radian measure is so often used, you may want to encourage your students to be as familiar with the radian measure of angles as they are with degree measure. Measuring arc length along a circle is one of many applications in which radian measure is used.

The formula for the length of a circular arc can be used to analyze the motion of a particle moving at a *constant speed* along a circular path.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

STUDY TIP

Linear speed measures how fast the particle moves, and angular speed measures how fast the angle changes. By dividing the formula for arc length by t, you can establish a relationship between linear speed v and angular speed ω, as shown.

$$s = r\theta$$

$$\frac{s}{t} = \frac{r\theta}{t}$$

$$v = r\omega$$

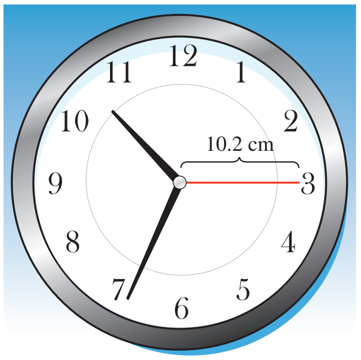

FIGURE 4.16

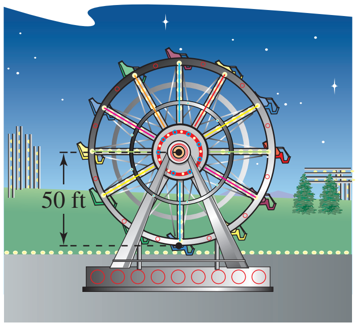

FIGURE 4.17

Activities

1. Find the supplement of the angle $\theta = \frac{5\pi}{7}$.

   Answer: $\frac{2\pi}{7}$

2. Convert 60° from degrees to radians.

   Answer: $\frac{\pi}{3}$

3. On a circle with a radius of 9 inches, find the length of the arc intercepted by a central angle of 140°.

   Answer: $7\pi \approx 22$ inches

Example 6 Finding Linear Speed

The second hand of a clock is 10.2 centimeters long, as shown in Figure 4.16. Find the linear speed of the tip of this second hand as it passes around the clock face.

Solution

In one revolution, the arc length traveled is

$$s = 2\pi r$$

$$= 2\pi(10.2) \qquad \text{Substitute for } r.$$

$$= 20.4\pi \text{ centimeters.}$$

The time required for the second hand to travel this distance is

$$t = 1 \text{ minute} = 60 \text{ seconds.}$$

So, the linear speed of the tip of the second hand is

$$\text{Linear speed} = \frac{s}{t}$$

$$= \frac{20.4\pi \text{ centimeters}}{60 \text{ seconds}}$$

$$\approx 1.068 \text{ centimeters per second.}$$

CHECKPOINT Now try Exercise 103.

Example 7 Finding Angular and Linear Speeds

A Ferris wheel with a 50-foot radius (see Figure 4.17) makes 1.5 revolutions per minute.

a. Find the angular speed of the Ferris wheel in radians per minute.

b. Find the linear speed of the Ferris wheel.

Solution

a. Because each revolution generates 2π radians, it follows that the wheel turns $(1.5)(2\pi) = 3\pi$ radians per minute. In other words, the angular speed is

$$\text{Angular speed} = \frac{\theta}{t}$$

$$= \frac{3\pi \text{ radians}}{1 \text{ minute}} = 3\pi \text{ radians per minute.}$$

b. The linear speed is

$$\text{Linear speed} = \frac{s}{t}$$

$$= \frac{r\theta}{t}$$

$$= \frac{50(3\pi) \text{ feet}}{1 \text{ minute}} \approx 471.2 \text{ feet per minute.}$$

CHECKPOINT Now try Exercise 105.

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A **sector** of a circle is the region bounded by two radii of the circle and their intercepted arc (see Figure 4.18).

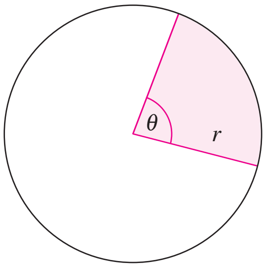

FIGURE 4.18

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is given by

$$A = \frac{1}{2}r^2\theta$$

where θ is measured in radians.

Example 8 Area of a Sector of a Circle

A sprinkler on a golf course fairway is set to spray water over a distance of 70 feet and rotates through an angle of 120° (see Figure 4.19). Find the area of the fairway watered by the sprinkler.

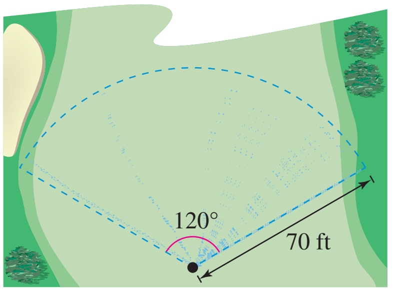

FIGURE 4.19

Solution

First convert 120° to radian measure as follows.

$$\theta = 120°$$

$$= (120 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) \quad \text{Multiply by } \pi/180.$$

$$= \frac{2\pi}{3} \text{ radians}$$

Then, using $\theta = 2\pi/3$ and $r = 70$, the area is

$$A = \frac{1}{2}r^2\theta \quad \text{Formula for the area of a sector of a circle}$$

$$= \frac{1}{2}(70)^2\left(\frac{2\pi}{3}\right) \quad \text{Substitute for } r \text{ and } \theta.$$

$$= \frac{4900\pi}{3} \quad \text{Simplify.}$$

$$\approx 5131 \text{ square feet.} \quad \text{Simplify.}$$

CHECKPOINT Now try Exercise 107.

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4.1 Exercises

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

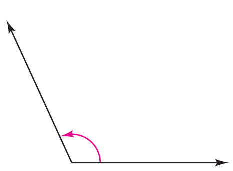
VOCABULARY CHECK: Fill in the blanks.

1. ________ means "measurement of triangles."
2. An ________ is determined by rotating a ray about its endpoint.
3. Two angles that have the same initial and terminal sides are ________.
4. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.
5. Angles that measure between 0 and $\pi/2$ are ________ angles, and angles that measure between $\pi/2$ and π are ________ angles.
6. Two positive angles that have a sum of $\pi/2$ are ________ angles, whereas two positive angles that have a sum of π are ________ angles.
7. The angle measure that is equivalent to $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
8. The ________ speed of a particle is the ratio of the arc length traveled to the time traveled.
9. The ________ speed of a particle is the ratio of the change in the central angle to time.
10. The area of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, estimate the angle to the nearest one-half radian.

1.



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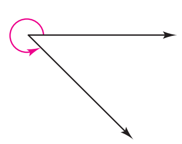

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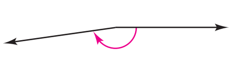

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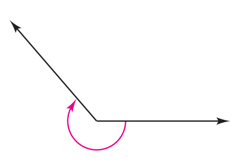

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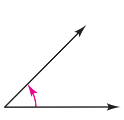

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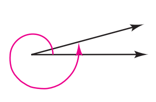


In Exercises 7–12, determine the quadrant in which each angle lies. (The angle measure is given in radians.)

7. (a) $\frac{\pi}{5}$ (b) $\frac{7\pi}{5}$

8. (a) $\frac{11\pi}{8}$ (b) $\frac{9\pi}{8}$

9. (a) $-\frac{\pi}{12}$ (b) -2

10. (a) -1 (b) $-\frac{11\pi}{9}$

11. (a) 3.5 (b) 2.25

12. (a) 6.02 (b) -4.25

In Exercises 13–16, sketch each angle in standard position.

13. (a) $\frac{5\pi}{4}$ (b) $-\frac{2\pi}{3}$

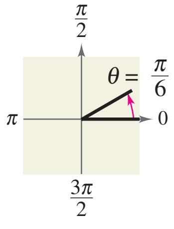
14. (a) $-\frac{7\pi}{4}$ (b) $\frac{5\pi}{2}$

15. (a) $\frac{11\pi}{6}$ (b) -3

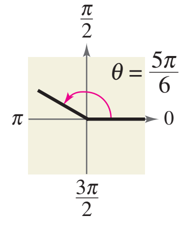
16. (a) 4 (b) 7π

In Exercises 17–20, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

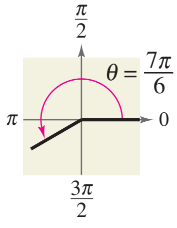
17. (a)



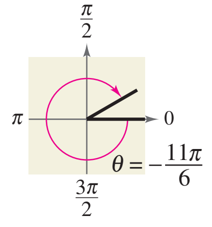
(b)



18. (a)



(b)



19. (a) $\theta = \frac{2\pi}{3}$ (b) $\theta = \frac{\pi}{12}$

20. (a) $\theta = -\frac{9\pi}{4}$ (b) $\theta = -\frac{2\pi}{15}$

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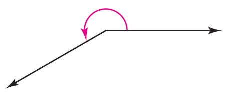
In Exercises 21–24, find (if possible) the complement and supplement of each angle.

21. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ **22.** (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$

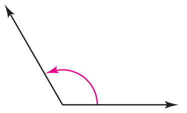
23. (a) 1 (b) 2 **24.** (a) 3 (b) 1.5

In Exercises 25–30, estimate the number of degrees in the angle.

25.



26.



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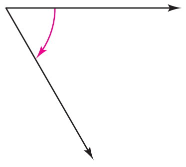
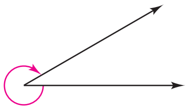
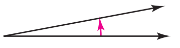


28.



29.

30.



In Exercises 31–34, determine the quadrant in which each angle lies.

31. (a) $130°$ (b) $285°$

32. (a) $8.3°$ (b) $257° 30'$

33. (a) $-132° 50'$ (b) $-336°$

34. (a) $-260°$ (b) $-3.4°$

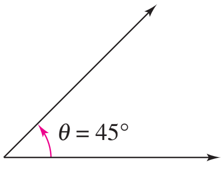
In Exercises 35–38, sketch each angle in standard position.

35. (a) $30°$ (b) $150°$ **36.** (a) $-270°$ (b) $-120°$

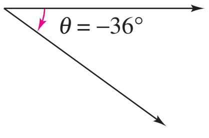
37. (a) $405°$ (b) $480°$ **38.** (a) $-750°$ (b) $-600°$

In Exercises 39–42, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

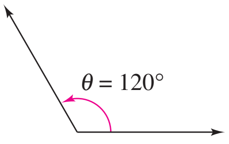
39. (a)



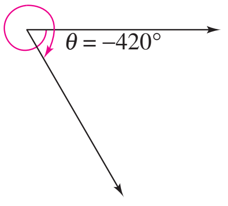
(b)



40. (a)



(b)



41. (a) $\theta = 240°$ (b) $\theta = -180°$

42. (a) $\theta = -420°$ (b) $\theta = 230°$

In Exercises 43–46, find (if possible) the complement and supplement of each angle.

43. (a) $18°$ (b) $115°$ **44.** (a) $3°$ (b) $64°$

45. (a) $79°$ (b) $150°$ **46.** (a) $130°$ (b) $170°$

In Exercises 47–50, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

47. (a) $30°$ (b) $150°$ **48.** (a) $315°$ (b) $120°$

49. (a) $-20°$ (b) $-240°$ **50.** (a) $-270°$ (b) $144°$

In Exercises 51–54, rewrite each angle in degree measure. (Do not use a calculator.)

51. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$ **52.** (a) $-\frac{7\pi}{12}$ (b) $\frac{\pi}{9}$

53. (a) $\frac{7\pi}{3}$ (b) $-\frac{11\pi}{30}$ **54.** (a) $\frac{11\pi}{6}$ (b) $\frac{34\pi}{15}$

In Exercises 55–62, convert the angle measure from degrees to radians. Round to three decimal places.

55. $115°$ **56.** $87.4°$

57. $-216.35°$ **58.** $-48.27°$

59. $532°$ **60.** $345°$

61. $-0.83°$ **62.** $0.54°$

In Exercises 63–70, convert the angle measure from radians to degrees. Round to three decimal places.

63. $\frac{\pi}{7}$ **64.** $\frac{5\pi}{11}$

65. $\frac{15\pi}{8}$ **66.** $\frac{13\pi}{2}$

67. -4.2π **68.** 4.8π

69. -2 **70.** -0.57

In Exercises 71–74, convert each angle measure to decimal degree form.

71. (a) $54° 45'$ (b) $-128° 30'$

72. (a) $245° 10'$ (b) $2° 12'$

73. (a) $85° 18' 30''$ (b) $330° 25''$

74. (a) $-135° 36''$ (b) $-408° 16' 20''$

In Exercises 75–78, convert each angle measure to D° M′ S″ form.

75. (a) $240.6°$ (b) $-145.8°$

76. (a) $-345.12°$ (b) $0.45°$

77. (a) $2.5°$ (b) $-3.58°$

78. (a) $-0.355°$ (b) $0.7865°$

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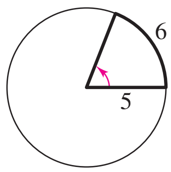
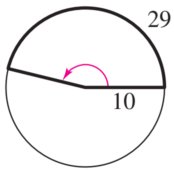
In Exercises 79–82, find the angle in radians.

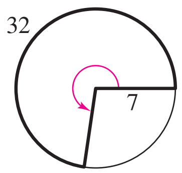
79.



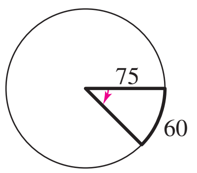
80.



81.



82.



In Exercises 83–86, find the radian measure of the central angle of a circle of radius *r* that intercepts an arc of length *s*.

	Radius r	*Arc Length s*
83.	27 inches	6 inches
84.	14 feet	8 feet
85.	14.5 centimeters	25 centimeters
86.	80 kilometers	160 kilometers

In Exercises 87–90, find the length of the arc on a circle of radius *r* intercepted by a central angle θ.

	Radius r	*Central Angle* θ
87.	15 inches	180°
88.	9 feet	60°
89.	3 meters	1 radian
90.	20 centimeters	$\pi/4$ radian

In Exercises 91–94, find the area of the sector of the circle with radius *r* and central angle θ.

	Radius r	*Central Angle* θ
91.	4 inches	$\frac{\pi}{3}$
92.	12 millimeters	$\frac{\pi}{4}$
93.	2.5 feet	225°
94.	1.4 miles	330°

Distance Between Cities **In Exercises 95 and 96, find the distance between the cities. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other).**

	City	*Latitude*
95.	Dallas, Texas	$32^\circ\,47'\,39''$ N
	Omaha, Nebraska	$41^\circ\,15'\,50''$ N
96.	San Francisco, California	$37^\circ\,47'\,36''$ N
	Seattle, Washington	$47^\circ\,37'\,18''$ N

97. ***Difference in Latitudes*** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Syracuse, New York and Annapolis, Maryland, where Syracuse is 450 kilometers due north of Annapolis?

98. ***Difference in Latitudes*** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia and Myrtle Beach, South Carolina, where Lynchburg is 400 kilometers due north of Myrtle Beach?

99. ***Instrumentation*** The pointer on a voltmeter is 6 centimeters in length (see figure). Find the angle through which the pointer rotates when it moves 2.5 centimeters on the scale.

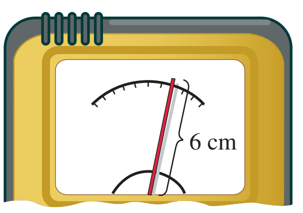


FIGURE FOR 99

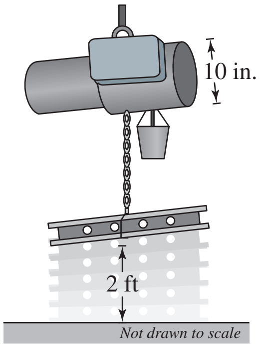


FIGURE FOR 100

100. ***Electric Hoist*** An electric hoist is being used to lift a beam (see figure). The diameter of the drum on the hoist is 10 inches, and the beam must be raised 2 feet. Find the number of degrees through which the drum must rotate.

101. ***Angular Speed*** A car is moving at a rate of 65 miles per hour, and the diameter of its wheels is 2.5 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

(b) Find the angular speed of the wheels in radians per minute.

102. ***Angular Speed*** A two-inch-diameter pulley on an electric motor that runs at 1700 revolutions per minute is connected by a belt to a four-inch-diameter pulley on a saw arbor.

(a) Find the angular speed (in radians per minute) of each pulley.

(b) Find the revolutions per minute of the saw.

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103. ***Linear and Angular Speeds*** A $7\frac{1}{4}$-inch circular power saw rotates at 5200 revolutions per minute.

(a) Find the angular speed of the saw blade in radians per minute.

(b) Find the linear speed (in feet per minute) of one of the 24 cutting teeth as they contact the wood being cut.

104. ***Linear and Angular Speeds*** A carousel with a 50-foot diameter makes 4 revolutions per minute.

(a) Find the angular speed of the carousel in radians per minute.

(b) Find the linear speed of the platform rim of the carousel.

105. ***Linear and Angular Speeds*** The diameter of a DVD is approximately 12 centimeters. The drive motor of the DVD player is controlled to rotate precisely between 200 and 500 revolutions per minute, depending on what track is being read.

(a) Find an interval for the angular speed of a DVD as it rotates.

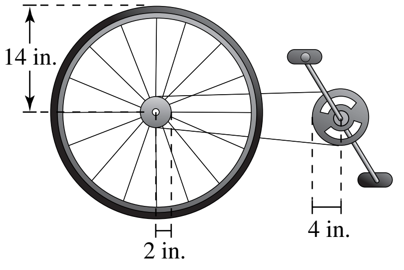
(b) Find an interval for the linear speed of a point on the outermost track as the DVD rotates.

106. ***Area*** A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and wipes the windshield over a distance of 14 inches. Find the area covered by the wiper.

107. ***Area*** A sprinkler system on a farm is set to spray water over a distance of 35 meters and to rotate through an angle of 140°. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

Model It

108. ***Speed of a Bicycle*** The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

Model It (continued)

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

(d) Classify the types of functions you found in parts (b) and (c). Explain your reasoning.

Synthesis

True or False? **In Exercises 109–111, determine whether the statement is true or false. Justify your answer.**

109. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

110. The difference between the measures of two coterminal angles is always a multiple of 360° if expressed in degrees and is always a multiple of 2π radians if expressed in radians.

111. An angle that measures $-1260°$ lies in Quadrant III.

112. ***Writing*** In your own words, explain the meanings of (a) an angle in standard position, (b) a negative angle, (c) coterminal angles, and (d) an obtuse angle.

113. ***Think About It*** A fan motor turns at a given angular speed. How does the speed of the tips of the blades change if a fan of greater diameter is installed on the motor? Explain.

114. ***Think About It*** Is a degree or a radian the larger unit of measure? Explain.

115. ***Writing*** If the radius of a circle is increasing and the magnitude of a central angle is held constant, how is the length of the intercepted arc changing? Explain your reasoning.

116. ***Proof*** Prove that the area of a circular sector of radius r with central angle θ is $A = \frac{1}{2}\theta r^2$, where θ is measured in radians.

Skills Review

In Exercises 117–120, simplify the radical expression.

117. $\dfrac{4}{4\sqrt{2}}$

118. $\dfrac{5\sqrt{5}}{2\sqrt{10}}$

119. $\sqrt{2^2 + 6^2}$

120. $\sqrt{17^2 - 9^2}$

In Exercises 121–124, sketch the graphs of $y = x^5$ and the specified transformation.

121. $f(x) = (x - 2)^5$

122. $f(x) = x^5 - 4$

123. $f(x) = 2 - x^5$

124. $f(x) = -(x + 3)^5$