PRE-CALCULUS/TRIG 3 Name: KEY Date:

3.1 Compound Interest & e & Annuities

$$e = 1+\frac{1}{1}+\frac{1}{1\bullet 2}+\frac{1}{1\bullet 2\bullet 3}+\frac{1}{1\bullet 2\bullet 3\bullet 4}+\cdots+\frac{1}{1\bullet 2\bullet 3\bullet\cdots\bullet n}$$

$$= 1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\cdots$$

$$\approx 2.71828182846$$

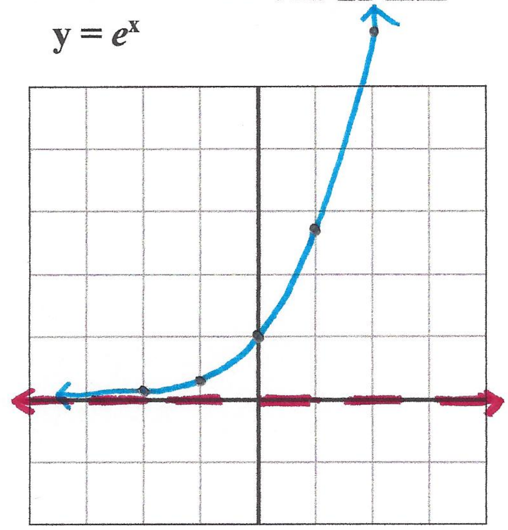
*Use the e^x button on your calculator.

$y = e^x \approx 2.72^x$

x	y
-2	0.135
-1	0.368
0	1
1	2.72
2	7.39

HA: y=0

y = e^x



Compound Interest (A) $\boxed{A = P(1+\frac{r}{n})^{nt}}$

P= Principal Investment (initial amount of money)

r= Rate (%) (APR → Annual Percentage Rate) → ★Change to a decimal!

n= # of compoundings per year

t= time (in years) ⟶ Ex: 2yrs 3months = $2\frac{3}{12}$ yrs = $2\frac{1}{4}$ yrs = 2.25 or $\frac{9}{4}$

A= Account Balance

- **$1,000, 5%, monthly, 5 years**

P=1000, r=.05, n=12, t=5

$A = P(1+\frac{r}{n})^{nt}$

$A = 1000(1+\frac{.05}{12})^{(12\times 5)}$ ⟶ In calc: $1000(1+\frac{.05}{12})\wedge(12\times 5)$

A = $1283.36

Interest = A − P
= 1283.36 − 1000
= $283.36

Compound Interest compounded continuously $\boxed{A = Pe^{rt}}$ *Use the "e^x" button in calculator.

↳ Keyword

- **$1,000, 5%, 5 years, continuously**

P=1000, r=.05, t=5

$A = Pe^{rt}$

$A = 1000e^{(.05\times 5)}$ ⟶ In calc: 1000 2nd $\overset{e}{\boxed{\div}}$ ∧ (.05 × 5)

or 1000 2nd $\overset{e^x}{\boxed{LN}}$ (.05 × 5)

A = $1284.03

Interest = 1284.03 − 1000
= $284.03

$P=500,\ r=.03,\ t=4$ $\quad A=P\left(1+\frac{r}{n}\right)^{nt}$

EXAMPLE: $500, 3%, 4 YEARS

yearly (n = 1) $\quad A=500\left(1+\frac{.03}{1}\right)^{(1\times4)}$ $\quad$ A = $562.75

quarterly (n = 4) $\quad A=500\left(1+\frac{.03}{4}\right)^{(4\times4)}$ $\quad$ A = $563.50

monthly (n = 12) $\quad A=500\left(1+\frac{.03}{12}\right)^{(12\times4)}$ $\quad$ A = $563.66

weekly (n = 52) $\quad A=500\left(1+\frac{.03}{52}\right)^{(52\times4)}$ $\quad$ A = $563.73

daily (n = 365) $\quad A=500\left(1+\frac{.03}{365}\right)^{(365\times4)}$ $\quad$ A = $563.75

continuously (e) $\quad A=500e^{(.03\times4)}$ $\quad$ A = $563.75

$A=Pe^{rt}$

Applications:

1. Determine how much money your parents would have needed to invest at the time of your birth in order for you to have $30,000 when you turn 18? (The investment would be compounded semi-annually at a 2.75% interest rate).

$A=30000$
$P=?$
$r=.0275$
$n=2$
$t=18$

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$
$$30000=P\left(1+\frac{.0275}{2}\right)^{(2\times18)}$$
$$\frac{30000}{1.01375^{36}}=\frac{P(1.01375)^{36}}{\cancel{1.01375^{36}}}$$

$\boxed{P=\$18,348.90}$

2. Will Mike have enough money to buy a used car costing $3000 with the $1000 investment his grandparents made for him 16 years ago if the rate is 6.8% compounded continuously?

$A=?$
$P=1000$
$r=.068$
$t=16$

$$A=Pe^{rt}$$
$$A=1000e^{(.068\times16)}$$
$A=\$2,968.33 \longrightarrow \boxed{\text{No}}$ Mike needs $31.67 more.

3. Determine the principal P that must be invested at a 6.25% rate, compounded continuously, so that $450,000 will be available upon retirement in 35 years?

$A=450000$
$P=?$
$r=.0625$
$t=35$

$$A=Pe^{rt}$$
$$450000=Pe^{(.0625\times35)}$$
$$\frac{450000}{e^{2.1875}}=\frac{Pe^{(2.1875)}}{\cancel{e^{2.1875}}}$$

$\boxed{P=\$50,488.60}$

Annuity: a fixed sum of money paid to someone/bank each year.

- Examples: Retirement Plans, Mortgage Loans, Student Loans, and Car Payments
- Investment/Borrowing Terms: IRA (Individual Retirement Account): 401K, 403B; Mortgage Loan

Types of Annuities:

- Present Value: You are borrowing money now and paying it back over time later.
- Future Value: You are putting money away now and saving it for a later use.

Present Value:

$$P_n = p\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$P_n = Present\ Value\ of\ an\ Annuity\ (loan)$
$p = repeated\ payment\ (monthly\ loan\ payment)$
$i = interest\ rate \div \#\ of\ payments\ per\ year$ (12 $if\ paying\ monthly$)
$n = \#\ of\ payments\ over\ the\ life\ of\ the\ loan$
$(12\ payments\ a\ year \times \#\ of\ years)$

1. A monthly mortgage payment consists of an amount paid towards the principal (loan balance) and the interest on the loan. It may also contain an amount for the property taxes (school, county, municipality) that the mortgage holder will pay from an escrow (A financial document held by a third party on behalf of the other two parties in a transaction) and an amount for insurance that protects the mortgage holder in case of default on the loan. Jeremy purchased a house for $157,000. He paid an 18% down payment. He gets a 25 year mortgage with an interest rate of 8.9%. Down Payment = how much $ Jeremy has saved to buy his house.

   a. How much was his down payment? 18% of the cost of the home ($157,000)

   $157000 \times 0.18 = \boxed{\$28{,}260}$

   b. How much of the house is financed? → How much $ Jeremy must borrow from bank (P_n) (mortgage loan)

   Cost of home − Down Payment = P_n

   $157000 - 28260 = \boxed{\$128{,}740 = P_n}$

   c. What will his monthly payment for principal and interest be? Find P using formula.

   $P_n = 128740$
   $p = ?$
   $i = \frac{.089}{12}$ *don't round!
   $n = 12 \times 25 = 300$

   $$P_n = p\left[\frac{1-(1+i)^{-n}}{i}\right]$$

   $$128740 = p\left[\frac{1-\left(1+\frac{.089}{12}\right)^{-300}}{\left(\frac{.089}{12}\right)}\right]$$

   *Do all work in your calc so you aren't rounding until the end.

   $\boxed{p = \$1{,}071.58}$

   d. How much will he pay in total interest over the life of the loan?

   $p \times n - P_n$

   $1071.58 \times 300 - 128740 = \boxed{\$192{,}734}$

   e. How much did the house cost him in total? $p \times n$ + down payment

   $1071.58 \times 300 + 28260 = \boxed{\$349{,}734}$

2. When she purchased her house in 2011, Mary took out a 30-year mortgage for $230,000 with a fixed interest rate of 6.375%. *No down payment

a. What will be the monthly payment for the principal and interest?

$P_n = 230000$

$P = ?$

$i = \frac{.06375}{12}$ *do not round!

$n = 12 \times 30 = 360$

$$P_n = P\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$230000 = P\left[\frac{1-\left(1+\frac{.06375}{12}\right)^{-360}}{\left(\frac{.06375}{12}\right)}\right]$$

*Do all work in calculator.

$\boxed{P = \$1{,}434.90}$

b. After 30 years, how much money will Mary have paid to her mortgage lender?

$P \times n$

$1434.90 \times 360 = \boxed{\$516{,}564}$

c. How much interest did she end up paying her mortgage lender?

$P \times n - P_n$

$1434.90 \times 360 - 230000 = \boxed{\$286{,}564}$